
STUDIO · ET · VIGILANTIA

ELEMENTS

OF

PHYSICAL MANIPULATION.

BY

EDWARD C. PICKERING,

Thayer Professor of Physics in the Massachusetts Institute of Technology.

PART II.

London:

MACMILLAN AND CO.

1876.

PREFACE.

Since the publication of the first Volume of this work, its scope has been greatly enlarged. It is now made to include, not only Physics proper, but several kindred branches, and aims to describe the principal methods of experiment with which every physicist should be familiar. As in the first volume, each Experiment is divided into two parts, a description of the *Apparatus*, intended mainly for the instructor, and the details of the *Experiment*, for the student.

The subject of Electricity is perhaps better adapted than any other to the laboratory system, and a large amount of space is therefore devoted to it. Heat follows, not being introduced earlier on account of its difficulty, The student may thus first acquire the requisite skill, and this subject then furnishes an excellent test of his proficiency. To attain accurate results, not only is great care needed, but very carefully constructed apparatus, and numerous precautions must be taken, and corrections applied. It was therefore deemed better to select simple and inexpensive forms of apparatus which should show the student the principles involved, although not capable of giving very accurate results. If the latter are desired, the student should be referred at once to the original memoirs.

The experiments headed Mechanical Engineering, though more special in their nature, are yet of a kind which is important to

every physicist. Physical problems which are to be solved on a large scale require a thorough knowledge of mechanical engineering. To this class belong also those which have the greatest pecuniary value. The methods of conducting such experiments also are often so faulty, that a brief description of how they should be performed will not seem out of place.

The next section is devoted to Meteorology, and contains a brief description of the principal meteorological and magnetic instruments. No special description of self-registering instruments is given, on account of the great space and elaborate engravings required, and because the use of such instruments is, from their nature, of little value as a means of education, and in general, only involves replacing a sheet of paper every day; proper directions also usually accompany every instrument.

One of the most important features of this volume is the introduction of a chapter on Astronomy. This subject is so seldom taught practically, except to single individuals as assistants in an Observatory, that its value as a means of training appears to have been overlooked. A careful examination of the subject seems to show that the laboratory method may be used to teach Astronomy as successfully as Chemistry or Physics. A promising field is open to any College or School of Science where the attempt shall be made to teach Practical Astronomy to classes in a systematic manner; and surely nothing can be more valuable to the civil engineer or explorer than to be able to determine his latitude, longitude and time, by the sextant or transit. As this book is intended to be used in this way, a portion of the smaller corrections are omitted, as sufficient accuracy is thus attained for ordinary purposes, and the chapter is not intended as a guide in an Astronomical Observatory where the greatest possible accuracy is demanded. The methods of Astronomy, especially as regards the discussion of results, the determination of errors and the applica-

tion of corrections, are so much superior to those commonly employed in purely physical work, that they deserve a careful study. The more exact physical measurements, especially those involving the accurate determination of angles and time, are, moreover, so closely akin to those of Astronomy that every physicist should have some acquaintance with the latter science. In so brief a description of so vast a science there was little opportunity to add anything new, and the standard works are so complete that the professional astronomer would go at once to them. Most of the methods here given will therefore be found treated more fully in the works of Chauvenet, Loomis, Coffin and Webb.

As every lecturer on science may derive great aid from the Lantern, in the projection of illustrations, or in rendering experiments readily visible to an audience, a chapter is added on Lantern Projections. The aim has been to show how, with simple and inexpensive means, excellent results may be obtained.

Several subjects of general importance remained which could not be inserted in the body of the work. These have been incorporated in three Appendices, *A*, *B* and *C*. The author, having experienced a great want of a brief description of the principles of electrical measurements, prepared a pamphlet for his own students which forms Appendix *A*.

Appendix *B* gives a series of tables of the numerical constants most used in physical work. The tables of powers, logarithms and trigonometrical functions are arranged in a way which is more brief, and is believed to be more convenient, than that ordinarily adopted. A saving of nearly one-third of the time is effected by the fact that each table covers only two opposite pages, so it is not necessary to turn the leaves in its use. The method of using them is also nearly the same for all. The trigonometrical functions are given to tenths of a degree, as the circles used in the galvanometers, and most of the other instruments described in

this work, are divided into degrees and read by the eye to tenths. If the readings are made by verniers to minutes, as in the optical circle and astronomical instruments, the more extended five or six place logarithms are required. A great saving of space is effected by bringing all the principal constants for the metals into a single table. Similar tables are given for liquids and gases. The blanks clearly indicate where additional determinations are needed.

In Appendix *C* are given with some detail directions for the establishment of Physical Laboratories, on the plan of that under the charge of the writer. In this Laboratory, about a hundred students are instructed every year. It has been in operation with but little change, except to enlarge its work, for the past six years, and has therefore had a practical trial on a large scale. A brief list of works of reference is also added, and a short description of a hundred additional experiments. These are especially intended to aid both teacher and student in what should be the aim of every scientific man, the encouragement of original research. The wide range of subjects now included in the term physics, shows that this work is addressed to no narrow circle of readers. The attention of all persons interested in experimental work of any kind is solicited, and of all who believe that the practical method of teaching science, now so largely adopted, is a step in the right direction.

In conclusion, hearty thanks are tendered to Profs. Trowbridge and Cross, and especially to Mr. Holman, for careful examination and revision of the proof sheets of this work.

E. C. P.

February 22nd, 1876.

CONTENTS.

ELECTRICITY.

HEAT.

MECHANICAL ENGINEERING.

METEOROLOGY.

PRACTICAL ASTRONOMY.

LANTERN PROJECTIONS.

APPENDIX A. ELECTRICITY.

APPENDIX B. TABLES.

APPENDIX C. PHYSICAL LABORATORIES.

ELECTRICITY.

CERTAIN instruments are employed in almost all the applications of dynamie electricity. The following description of them should therefore be read carefully before performing any experiments in this subject.

Batteries. The most common method of generating a current of electricity is by the unequal action of an acid liquid on two metals. This may be effeeted in a great variety of ways, but the most common forms of galvanic batteries may be divided into the three following elasses. The first elass eontains those in whieh a single liquid is used, the seeond those of which the Daniell's battery is the type, with zine and eopper as eleetrodes, and sulphurie aeid and sulphate of copper as liquids. The third class contains the other two-fluid batteries like the Grove and Bunsen, a more intense action being seeured by nitrie, chromic or other strong acids. The action of other batteries is readily understood from them, and therefore need not be described here.

If any two metals, as copper and zinc, are immersed in dilute sulphuric aeid, a decomposition of the liquid takes place, and the zinc is found to be positively, the copper negatively, eleetrified. If they are connected by a wire or other eonductor, a continuous current of electricity will pass through them from the eopper to the zine. A chemieal action now takes place, by which the acid unites with the zine, and the hydrogen set free is deposited in bubbles on the eopper. As soon as the cireuit, as it is called, is broken, by removing the wire, this action ceases, and the metals return to their former feebly electrified condition. If the experiment is made with commercial zinc which eontains particles of iron and other impurities, local action takes place, that is, eaeh partiele aets like the eopper plate, and the liquid is deeomposed without sending any current through the wire. To obviate this

difficulty the zinc must be amalgamated, that is, covered with mercury, which gives it a uniform surface so that it will act only when the circuit is closed. To amalgamate a plate of zinc, first scrape off any lumps of salts or dirt that may adhere to it, and then clean it by immersing in dilute sulphuric acid (1 part to 8 of water). Local action at once ensues, accompanied with a rapid disengagement of hydrogen with a hissing sound. Then lay the plate in a wooden trough which will just contain it, and pour mercury over it, repeating the operation and aiding the action with a stiff brush. If the mercury does not adhere at once, dip the plate again in the acid, and repeat until the surface has a bright silvery lustre, and no effect is produced on immersion in the acid. Zinc may also be amalgamated by dipping in a solution of chloride of mercury. The presence of the hydrogen on the platinum is very objectionable, both because it increases the resistance, and because it tends with the zinc to form a current in the opposite direction. The surface of the platinum should therefore be roughened either mechanically, or better, by covering it with a coating of platinum black, to which the bubbles cannot adhere. To save expense, the plate may be made of silver or lead covered with platinum black, or a plate of gas carbon may be employed. The carbon should be first pulverized, then mixed with molasses, molded under a high pressure, and finally heated to redness.

A battery known as a Smee, is then easily made by filling a glass jar half full of dilute sulphuric acid (1 part in 10) and immersing in it two or more plates side by side of amalgamated zinc and platinum, or carbon. Wires are then connected with the two plates, and a current sent through any pieces of apparatus by merely connecting them with it. Such a battery is very clean and convenient for many purposes, but it is not very powerful, and rapidly grows weaker when the circuit is closed.

Instead of sulphuric acid, chromic acid is sometimes used, formed by mixing one part of concentrated sulphuric acid with four or five parts by volume of a saturated solution of bichromate of potash; as the mixture will become very hot, the acid should be added slowly and well stirred. The zinc is commonly made so that it can be lifted out of the liquid, as if left standing in it, it is

gradually dissolved. Powerful batteries are made by using several cells of this form, connecting the plates with a rack and pinion, or with a wire rope and windlass, so that they may be lifted simultaneously by a crank. This battery has a large electromotive force and small resistance; it therefore gives a very strong current, but like the preceding runs down rapidly.

To remedy the difficulty arising from the weakening of batteries with a single liquid, an important improvement was introduced by Daniell, in a battery containing two liquids, sulphate of copper and dilute sulphuric acid, separated by a porous earthenware diaphragm. A plate of copper is placed in the former liquid, amalgamated zinc in the latter. In one form of this battery the jar itself is of copper, in which is placed the sulphate of copper, and in this the earthenware cylinder, called the porous cell. The latter is filled with dilute sulphuric acid, so that the liquids shall stand at about the same height inside and out, and the zinc is immersed in it. When the circuit is closed, the chemical action takes place through the porous cell, so that the hydrogen set free decomposes the sulphate of copper and deposits the copper on the copper plate. The latter therefore becomes heavier and heavier, instead of being used up. Some crystals of sulphate of copper are placed in the jar to replace that which is decomposed, otherwise the solution would grow weaker and weaker. Such a battery gives a very steady or constant current with an electromotive force a little over 1 volt. Sometimes the copper is placed in the interior, instead of outside the acid, and sometimes a glass balloon, like a Florence flask, filled with sulphate of copper, is inverted over the solution in the jar, to replace that which is used up. These batteries work best with the circuit closed when not in use, as the electricity thus wasted costs less than the injury sometimes done by a broken circuit. In the latter case, the copper is often deposited on the porous cell in curious lumps not easily removed.

To avoid the trouble arising from a porous cell, gravity batteries are used, in which the cell is dispensed with, and the two liquids kept apart by the difference in their specific gravity. One of the best forms of gravity battery is the Callaud, in which the copper plate is placed at the bottom, and the zinc suspended some inches above it. Water is then poured in until the zinc is just

covered, and some crystals of sulphate of copper dropped in. The circuit is then closed over night, and the next day the battery will be in good working order. By dropping in a little sulphate of zinc the action may be hastened. The solution of sulphate of copper being heavier than the acid, will remain at the bottom, as is easily seen by its color. When the circuit has been closed for some time the line of demarcation will be well marked, and will gradually descend as the copper is used up. More crystals must then be dropped in. On an open circuit the copper, by diffusion, slowly ascends, the color gradually fading off. Such a battery will remain in action for months with very little attention, and gives a very constant current. To prevent evaporation, the surface of the liquid is frequently covered with oil. It, of course, must not be moved or the liquids will mix.

Another form of two fluid battery is the Grove, in which the porous cell contains strong nitric acid and a plate of platinum; the outer liquid being dilute sulphuric acid, in which is a plate of amalgamated zinc. Carbon is, for cheapness, often substituted for the platinum, the battery then being called a Bunsen. When the circuit is closed the nitric acid is decomposed, a part of the oxygen uniting with the hydrogen and setting free binoxide of nitrogen, which on contact with the air forms dark red fumes of hyponitric acid. These fumes are very objectionable, as they are hurtful to breathe, and at the same time rapidly corrode brass, and other metal surfaces. Various plans have been tried to remedy this difficulty. For instance, they are rapidly absorbed by alcohol or by quicklime, and one of these substances is therefore sometimes placed in flat dishes in the same box as the battery. A mixture of bichromate of potash and sulphuric acid is sometimes added to the nitric acid, the chromic acid at once oxidyzing the nitric fumes. The principal objection to this arrangement is that the chromium permeates the carbons, forming a disagreeable green mass in the inner cell. The electromotive force is also less than that of a Bunsen or Grove cell.

The best remedy is to place the battery out doors, or under a well ventilated flue, or in an adjacent battery room, where the fumes will do no harm.

Another form of battery much used at the present time 's the

Léelanché. This consists of a porous cell containing a rod of carbon tightly packed with pulverized binoxide of manganese and placed in a glass jar containing a saturated solution of chloride of ammonium. The negative electrode is a rod of amalgamated zinc. These cells are very good when the circuit is closed only for a few minutes at a time, as the electromotive force is high and the resistance small, but they run down very rapidly when the circuit is closed. Many other forms of battery are also in use, but generally their action is readily understood from the above examples.

The advantages of a Smee battery are its cheapness, convenience, cleanliness and the rapidity with which it will work. Its objections, its small electromotive force, and that it rapidly grows weaker on a closed circuit. It is, however, much used for electric clocks and bells. A chromic acid battery is very powerful, but rapidly grows weaker on a closed circuit, and the zinc must not be left in the liquid. It is very suitable for the induction coil, or for an electro-magnet, as it can be set in action by simply lowering the zincs into the liquid. The Daniell and gravity batteries are well adapted to giving constant currents for a long time, and, in fact, are the best for closed circuits. The resistance is, however, considerable. They are much used on telegraph lines, and for electro-plating. The Bunsen and Grove batteries are commonly used where a very powerful current is required, as for the electric light, large magnets and coils. The resistance is small, and the electromotive force large, but the fumes are very objectionable, and the zincs have to be re-amalgamated every day. They should always be dismounted and the parts washed after using. The advantages of the Léelanché cell are much like those of the Smee, and it is very well adapted to electric bells, and will keep indefinitely on an open circuit.

Two other sources of electricity are also sometimes employed, the thermo-battery and the magneto-electric machine. The first of these consists of a number of pair of strips of different metals soldered together at the ends, and heated by gas-burners at the inner terminals, the outer ends being kept cool by the air currents circulating around them. To use them, the gas is lighted, and when they are thoroughly heated, a current of great constancy is obtained. The magneto-electric machine consists in substance of a

magnet, in front of which an electro-magnetic armature revolves, and thus generates a current of electricity as long as the rotation is maintained. These instruments will be described more in detail hereafter, in connection with the methods of testing their efficiency.

Connections. To pass a current through any piece of apparatus, its ends must be connected with the two terminals of the battery by wires or other good conductors. The circuit is then said to be closed, and the current will flow from the positive, or carbon pole of the battery, through the connecting wires to the apparatus, and through the latter and the second connecting wire back to the negative pole of the battery. If two connecting wires cross each other so as to touch, or rest against the same metallic body, the current is liable to pass directly from one to the other, instead of going through the instrument to which they are attached. It is therefore common to cover them with some insulating material by winding them once, or better, twice, with cotton or silk. They are then called covered wires. To prevent unravelling the thread is sometimes braided, and to render it more flexible, several fine wires are sometimes used instead of a single coarse one. The wire is then often painted, or soaked in parafine to render it impervious to water, although it is safer to cover wires which are to be used in water with a layer of rubber. To connect two wires so that the current shall pass from one to the other, it is only necessary to scrape their ends clean and twist them together. Or, they may be cleaned by immersing them in acid and then washing in water and drying. If the junction is to be permanent, it is better to solder them. In this case resin should be used instead of soldering acid, for the latter being hygroscopic, is liable to absorb moisture, keeping the ends of the wire wet, rusting it, and spoiling the connection.

Where the connection has to be made whenever the apparatus is used, *binding screws*, or *screw cups* are more commonly employed. These consist of little pieces of brass in which a hole is bored, into which the end of the wire is inserted, and fixed in place by a set screw with a milled head, which presses against its side. Sometimes the screw cups are made with two holes, so that two wires may be attached at the same time. Generally the binding screw terminates below in a screw, by means of which it may be

attached to the wooden frame of the apparatus, a connection soldered on below, and connecting wires attached when desired by the screws. Wires are sometimes connected by double or triple binding screws, instead of soldering them.

Another method of connecting two wires is to dip both of them in a small cup containing mercury. The cup may be made by boring a hole partly through a board and putting a drop of mercury into it, or a metallic cup may be used, and one wire soldered to its exterior. Mercury cups are convenient from the ease with which the connections may be altered, but they are objectionable if they have to be moved, and in accurate experiments the resistance they interpose is found to be variable.

Keys. When a circuit must be broken and closed a great many times in the course of an experiment, a device called a key is employed. One of the simplest forms of key is made by fastening the two wires to a board, and screwing one end of an elastic piece of brass on to one of them, so that the other end shall be over the other wire. The circuit is closed by depressing this end with the finger, while the elasticity of the brass raises it, and breaks the circuit when the finger is removed. Sometimes a stiff piece of brass is used, placed between centre-screws, and raised either by a spring or by a counterpoise on the other end. The points of contact of the key should be tipped with platinum, or they will rust or burn away rapidly, especially if the current is strong.

Plugs. Two pieces of brass are attached to a base of hard rubber or wood, and separated by a short distance. A conical hole is then bored between them, so as to form a groove in each. Into this is fitted a brass conical plug and ground in, so that it shall fit tightly. When the two brass pieces are connected with the wires of the circuit, the latter may be closed at will by inserting the plug. It may therefore with advantage be substituted for a key when the circuit is to be closed for a considerable time. A plug is also used when we wish to throw the current out of a piece of apparatus without breaking the circuit. The current is here allowed to pass from one brass piece to the other through the apparatus. On inserting the plug the resistance of the latter is so small that all the electricity passes through it.

The great advantage of a plug is the excellence of the contact, the surfaces being ground together, and any dust or rust being rubbed off every time the plug is inserted. The surfaces are therefore kept bright, and the pressure renders the resistance exceedingly small. Sliding contacts are much to be preferred to simple pressures, as the latter are liable to introduce considerable resistances, even if the surfaces are protected from rust by platinum.

Switches. When the current, instead of being cut off, is merely to be diverted into another wire, a switch is used instead of a key. Let A be a wire connected with one pole of the battery, and B and C two similar wires connected with the other pole, and with the two instruments through which we wish the current to pass, and suppose we wish A connected sometimes with B, and sometimes with C; the wires B and C are attached side by side to a small board, their ends being held down by screws with rounded heads. A short distance from them A is similarly attached, the screw which fastens it passing through a flat strip of brass, which turns with friction, so that its end may rest on either B or C. Its shape is such that it always pushes against one of these screw heads, thus insuring contact, and the friction as it slides over them keeps the surfaces bright and clean. By merely moving it from side to side the current may be thrown into one wire or the other.

Commutators. It is frequently necessary to send a current through a given instrument, first in one direction and then in the other, and this is done by what is called a commutator. In Fig. 66 let A, B, C, D, represent four quadrants of brass, of which each is separated from the two adjacent to it by a short interval, but may be connected by plugs. Suppose A and D connected with the positive and negative poles of the battery, and B and C with the two terminals of the instrument through which the current is to be passed. Then if AB and CD are connected by plugs, as represented by the white circles in the figure, the current will pass from the battery to A, by the plug to B, through the instrument to C, and back through the second plug to D and the other pole of the battery. To reverse the currents, change the plugs so as to connect AC and BD, as shown by the black circles, when the current will pass through AC and the instrument to B,

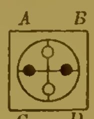


Fig. 66.

and back to the battery by D, in this case passing from C to B, and before from B to C. A commutator on the same principle is made by replacing the four quadrants by four mercury cups, and connecting them alternately by two bent wires which replace the plugs. One of the best forms of commutator is shown in plan in Fig. 67. F is a hard-rubber cylinder, which may be turned around a horizontal axis so that it shall rest against the two brass springs A and D. It is held in place by the supports B and C, and carries two strips of brass, one connected with its axle at the end B, the other at C, as shown by the dotted lines. If now, as in the previous case, A and D are connected with the battery, and B and C with the given instrument the current will pass from A through the cylinder to B, thence by the instrument to C, and back by D. When the cylinder is turned 180°, the current from A will pass to C instead of B, and thus traverse the instrument in the opposite direction. By turning the cylinder 90° the current is broken, and may thus be used as a key. Another commutator is made by connecting the terminals of the battery with two brass plates, one fastened to the table, the other held by a spring just above it. The wires attached to the instrument are fastened to two plates, separated by a piece of hard rubber, and forming a wedge. When the latter is inserted between the plates attached to the battery, the current passes and may be reversed by simply turning the wedge over. Another simple commutator is made by bringing the two pairs of wires together in a sort of swivel, so that on turning either around 180° the current is reversed.

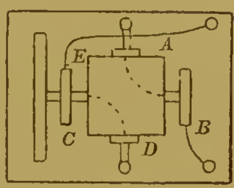


Fig. 67.

95. Galvanic Electricity.

Apparatus. A battery of three or four Bunsen cells, or of equivalent strength, an amalgamating trough, some examples of the screw-cups, plugs, keys, switches and commutators described above, some fine platinum wire, a coarse galvanometer, an electro-magnet, electric bells, an electro-magnetic engine, some U-tubes with platinum terminals, and some chemicals for decomposition.

Experiment. Mount the battery as described above, first amalgamating the zincs, and connect the cells for tension, that is, the

zinc of one cell to the carbon of the next. When the experiment is completed, dismount the battery, pour back the acids and soak the zincs, carbons and porous cells in water. The connections must all be made with care, the wires scraped bright, and the screws turned so that they press hard against the wires. On taking hold of the terminals, no shock will be felt unless the hands are moist and the battery powerful; but touching the wires to the tongue a slight metallic taste will be noticed, not present when the battery is not attached. On bringing the terminals slowly together, no effect is produced until they are in contact or the circuit closed, but on separating them so as to break the circuit, a small spark will be noticed. This effect is greatly increased by attaching one terminal to the end of a file and drawing the other over the roughened surface, when a series of sparks is produced, due to the combustion of the minute particles of metal thrown off.

If the terminals are connected with a short piece of fine platinum wire, the latter is heated, and by diminishing its length, its temperature increases, becoming red, yellow, white, and finally melting. Such a wire forms an excellent cautery, and would be much used in surgery but for the difficulty of procuring a sufficiently powerful and constant source of electricity.

To show the effect of a current on a magnetic needle, connect the two poles of the battery by a copper wire, so that the current shall pass through it; then holding the wire north and south, bring it down over the needle, when the latter will swing out to one side. See if the direction is that given by Ampère's law, and by the laws of currents given in Appendix *A*. Now place the wire below the needle, and the latter will turn in the opposite direction. These effects may be reversed by turning the wire over so that the current shall flow in the opposite direction. Next, connect the two battery terminals with two ends of a commutator, and the terminals of the galvanometer with the other two ends. By changing the direction of the current, the needle may be made to deviate to one side or the other. Try the other commutators in the same way. Place one of the keys in the circuit with the galvanometer, and notice that the needle deviates only when it is pressed down; do the same with a plug. Connect the latter also so that on inserting the plug in its hole the galvanometer is cut out

of the circuit. Connect two galvanometers, or a galvanometer and some other instrument described below, with a switch, and see how the current may be passed through either. Shunt the galvanometer by connecting its two terminals by double binding screws, both with the battery and with a wire whose length may be varied. Notice that in this case the current, and consequently the deflection, may be reduced as much as is desired.

Next, insert in the circuit a commutator and the electro-magnet, and notice that the latter has no effect on a piece of soft iron held near it. Now close the circuit, and the magnet becomes enormously powerful, capable, if large, of supporting several hundred pounds. It will also hold a heavy bar out horizontally by one end, or support many small pieces of iron by induction. As soon as the current is broken, the magnetism instantly ceases. Holding a basket of nails under the magnet they hang in long strings from it when the current is closed, and instantly drop when the circuit is broken. Next, see which is the north end of the magnet by noticing which end will attract the south pole of the compass-needle; the current, as shown by Ampère, will circulate around this in the opposite direction from the hands of a watch. Now reverse the current by the commutator, and the magnetism will be reversed.

An immense number of applications have been made of this power of producing a powerful attraction, and causing it to cease instantly. It is difficult to utilize it as a source of power, partly from the expense and inconvenience of the battery, and partly because the attraction diminishes very rapidly with the distance. One of the simplest forms of electro-magnetic engines is that of Page, in which the current passes through the coil of a small bar magnet placed between the poles of a permanent horse-shoe magnet. The bar magnet is free to turn end for end, and on its axle is placed a commutator, so that the direction of the current changes every 180°. Placing the bar magnet at right angles to the line connecting the poles of the horse-shoe magnet, and passing the current through it, its north end is attracted by, and approaches to, the south pole of the permanent magnet; as it revolves past, its magnetism is reversed, and consequently having the same polarity, it is now repelled, and attracted by the other pole. A rapid motion

is thus imparted to the bar magnet, which may be reversed by means of a commutator.

Another important application of electro-magnetism is to electric bells. These are of two forms, those in which there is a single stroke when the circuit is made or broken, and those in which the ringing is continuous as long as the current passes. The first class is very simply made by attaching a hammer directly to the armature of an electro-magnet, which is thus drawn up against the bell when the circuit is made, and pulled back by a spring on breaking the circuit. If the bell is large, so that the force of the magnet is insufficient, the hammer may be moved by clock-work, which the electicity serves simply to release. The arrangement for making a bell ring continuously is shown in Fig. 68. *A* is an electro-magnet through which the current passes and thence through the spring supporting the armature, and the screw *C* by a wire to the other pole of the battery. The first effect is to attract the armature *B*, and bring the hammer in contact with the bell *D*, striking it. But the current is thus broken, the spring supporting *B* having been drawn away from the screw *C*. consequently the magnet ceases to act, and the armature flies back until it makes contact again, and is again attracted. These effects succeed each other with great rapidity, producing a continuous ringing of the bell. *C* is a screw with a milled head, so that it is easily brought into exactly the right position. This arrangement is much used for all kinds of alarm bells, for hotel annunciators, on telegraph lines to announce that a message is to be expected, and in experimental work to denote that a looked-for event has taken place, since the bell will continue to sound until the attention of the observer is called, and the circuit broken. A similar arrangement is also employed to sustain the vibrations of a tuning-fork, and as an automatic break-piece, to make and break the current many times a second. This explanation, which is that usually given, does not seem to be adequate, since there would seem to be no power expended to overcome the various resistances. After contact the magnet tends to retard the armature until it comes to rest, as much as it accelerates it before

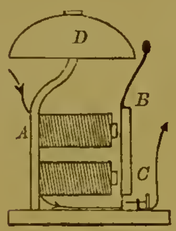


Fig. 68.

contact is broken, and hence it would follow that owing to the resistances, the vibrations should become less and less, and soon cease, while in reality they may increase until they attain a large amplitude, and overcome very considerable resistances. Probably the true explanation depends partly on the residual magnetism of A, owing to which the magnet does not begin to retard the armature until a short time after contact, and continues to accelerate it a little while after the circuit is broken. Another explanation is, that the current does not begin to pass until the instant of contact, while it continues passing a little while after the spring leaves C, as is shown by the spark. Therefore the period of acceleration exceeds that of retardation. It has been proposed to improve this device by attaching C also to a spring whose time of vibration is somewhat less than that of the armature, in which case, when approaching the magnet, C will follow B and keep the circuit closed, but on the return, as it vibrates more rapidly, it will break the circuit, so that the magnet accelerates half the time and does not retard the other half. Powerful vibrations may be sustained in this way, but the common method works sufficiently well in ordinary cases.

Connect the platinum electrodes and the poles of the battery with a commutator, fill the beaker with water and immerse the electrodes. On closing the circuit, little or no effect is produced unless the battery consists of a great many cells. Now add some sulphuric acid to the water, and immediately bubbles of gas will be given off from each electrode. By filling some test-tubes with water and inverting them over the platinum terminals, the gases may be collected. It will be found that the gas given off by the terminal attached to the negative pole has twice the volume of the other, and is hydrogen, as is easily seen by igniting it. The other gas is shown to be oxygen by holding in it a red-hot burnt match, when it will glow and be relighted. If both gases are collected in the same vessel, they will explode violently if ignited. This experiment should be tried only with minute quantities of gas, and is best performed by removing the mixed gases to another larger and deeper vessel, and allowing a bubble at a time to ascend from the bottom, and ignite it at the surface. The reason of the effect of the acid is that the resistance of pure water is enormous, so that the current with pure water is exceedingly minute, and most of

the gas dissolved as fast as formed. The acid acts merely by rendering it a better conductor. Reversing the current by the commutator, of course reverses the position of the gases disengaged.

When a solution of a salt is acted on by a powerful current of electricity, a decomposition takes place by which the base is carried to the negative or zinc pole, and the acid to the positive or carbon pole. This effect is shown by the following experiments. Fill a U-tube with a solution of sulphate of soda, and tinge it blue with a little litmus. Place a platinum electrode in each arm of the tube, and after some time the acid set free at the negative pole will redden the litmus. This effect is hastened by stirring up the liquid a little, so that the adhering layer of acid shall mix with the remainder. If the current is reversed, the blue color will return, the acid reuniting with the base. If a chloride, as common salt, is used, the chlorine set free will bleach the solution, removing the blue tint. By using a solution of iodide of potassium and starch, the characteristic blue color of iodide of starch is readily produced. To bring the starch into solution, it should be first soaked in cold water and then boiled. A similar effect is obtained with ferrocyanide of potassium, using iron wires as electrodes, instead of platinum. Prussian blue is then produced at the positive pole. The presence of the base at the negative pole is best shown by salts of the metals. Pour a solution of acetate of lead into a beaker and immerse the two electrodes. The lead will be at once deposited on the positive terminal, in beautiful crystalline leaf-like forms. A similar effect is obtained with nitrate of silver. With sulphate of copper, if the current is not too strong, a smooth coating of metallic copper is deposited, the electrode, in fact, being electro-plated. Copper may be deposited on other substances in a similar manner, but it is better to arrange the apparatus expressly for the purpose, as follows. A single large Daniell's cell is used as a source of electricity, and near it is a tank filled with a saturated solution of sulphate of copper, to which has been added some sulphuric acid in the proportion of one part to ten of water. To the positive terminal of the battery is attached a plate of copper to supply the metal to be deposited, and to the other terminal is fastened the object to be plated. This should be cleaned and brightened to remove the dirt or rust, and if not metallic, covered with a

coating of plumbago or black lead, to render it a good conductor. To make a copy of a coin, model or other object, two platings must be made, the first of the object, the second of the first plating. Or, a cast may be made in plaster or wax rubbed with plumbago and plated as above. A very constant current should be employed, and one that is not too powerful, or the metal is liable to be thrown down in lumps, in a fine powder, or to strip, that is not adhere. Silver and gold are best deposited from solutions of their cyanides, and in the case of gold the liquid should be heated to about 55° C. (130° F.).

96. Telegraph.

Apparatus. The best apparatus for this experiment is a real telegraph, or tables fitted up like real offices, two for terminals, and at least one way-station, with relay and local battery.

Experiment. A telegraph may be regarded as composed of four parts, the source of electricity, the line or conductor for connecting the two stations, the apparatus for sending the message, and that for receiving or reading. For a source, a common galvanic battery is employed, of a strength dependent on the distance and number of stations, or, more strictly, on the resistance of the circuit. For short distances, two or three Daniell or gravity cells are best. The line consists of a wire, which should be of iron, galvanized if the distance is considerable, suspended by glass or other non-conducting supports, if it passes out doors, but in-doors a copper wire may be simply tacked along the walls or floor, taking care that it does not touch any large metallic or other conducting body. Instead of a second wire to bring back the current, the two ends may be connected with a gas-pipe, or, much better, a water-pipe. If these are not available, two large metal plates may be buried in the ground, and wires connected with them, forming what is called an *earth.* The sending instrument is merely a form of key described above, so that the circuit may be closed for a longer or shorter time. The instrument for receiving the message, called a *register*, consists of an electro-magnet, whose armature is held back by a spring, and carries a point, which, when the circuit is closed, is held down on a long strip of paper drawn slowly under it by clock-work. If the circuit is closed for an instant, a dot will there-

fore be imprinted on the paper; if for a longer time, a line. It was soon found that the dots and lines thus formed could be read by the sounds produced by the armature, which clicks on being drawn down to the magnet, and gives a different sound when drawn back by the spring. Instead of a register, a *sounder* is therefore used, which consists of a single electro-magnet and armature, the latter being drawn back by a spring whose tension may be regulated by a screw. The object of this screw is to adjust the armature with the varying strength of the current, due either to changes of the battery, leakage along the line, or, in long lines, to currents of electricity from the air or earth. This is especially the case during displays of the aurora borealis, and during thunder storms, when in some cases the battery may be for a time dispensed with, and the line worked without. In wet weather the insulators become covered with moisture, and cause a great leakage. During violent thunder storms, the line should be attached directly to the earth wires, taking the instruments out of the circuit; otherwise there is danger of the lightning entering the building along the wire, burning up the magnets, and perhaps doing other injury. The instruments are so connected that the current passes from the battery, which may be placed at either end of the line, through the key and sounder, along the line to the other station, through its key and sounder to the earth, back through the ground to the earth of the first station, and thus to the other pole of the battery. When a message is to be sent from either station, the key at the other end must be held down or thrown out of the circuit by a plug or switch, as otherwise the circuit will be broken, and no current will pass. For the same reason, after sending messages, the circuit must be closed at both ends, as if left broken at either station, the operator at the other end could not give notice that he wished to send a message.

If the distances to be travelled are considerable, the circuit will not be powerful enough to work a register or sounder, and therefore a *relay* is used, which resembles a sounder, but contains a magnet wound with very fine wire, and is thus sensitive to a very feeble current. This is connected with a second battery, called a local battery, and an ordinary sounder, and is so connected that when the armature of the relay moves, it alternately makes and breaks

the local circuit. The sounder therefore acts with the full effect of the local battery independently of any leakage or other changes in the main line, whenever the main current is sufficient to work the relay. On very long lines *repeaters* are used, which consist of relays which throw a second main battery into circuit, and thus repeat the message automatically.

To send a message, therefore, it is necessary first to arrange a system of long and short currents or dots and lines to represent each letter of the alphabet; and then any message may be sent by spelling it out letter by letter with the key, when all the relays and sounders along the line will move in accord. The alphabet in use on the Morse telegraph in this country is given in Fig. 69.

A	- —	Q	- - — -	4	- - - - —
B	— - - -	R	- - -	5	— — —
C	- - -	S	- - -	6	- - - - - -
D	— - -	T	—	7	— — - -
E	-	U	- - —	8	— - - - -
F	- — -	V	- - - —	9	— - - —
G	— — -	W	- — —	Comma	- — - — -
H	- - - -	X	- — - -	Semicolon	- — - — -
I	- -	Y	- - - -	Period	- - — — - -
J	— - — -	Z	- - - -	Interrogation	— - - — -
K	— - —	&	- - - -	Exclamation	— — — -
L	——	0	———	Parenthesis	- — - - —
M	— —	1	- — — -	Italics	— - - - —
N	— -	2	- - — - -	Paragraph	— — — —
O	- -	3	- - - — -	Quotation	- — - — - — - —
P	- - - - -				

Fig. 69.

Thus the letter *A* is represented by a dot and line, or by making the circuit first for an instant, and then for a longer time; the letter *B* by a line and three dots, or one long and three short currents, and so on. The alphabet should not be memorized or practised in order as given above, but the following system adopted. Two students should work together at this experiment, and send and receive alternately. It is well at first to use the register, and record the letters on the paper, to see that they are correctly formed. The proper position for the hand, is to hold the button at the end of the

key by placing the fore and middle fingers on it, and the thumb under its edge, nearly closing the other two fingers. Keep the wrist limber, and rest the arm on the table at the elbow. The motion must be mainly from the wrist, which should be perfectly limber, but move up and down with the fingers, and *not* in the opposite direction.

Now begin by making a single dot, pressing the key down firmly and raising it instantly. The line thus made can scarcely be too short. Next make a series of these dots at regular intervals, gradually increasing the speed until it reaches five or six in a second, at perfectly equal intervals. Make the letters *E*, *I*, *S*, *H*, *P* and *6*, which consist of from one to six equidistant dots. In all cases begin slowly and make them very distinctly, gradually increasing the speed, until they can be made to follow each other rapidly in any order, and be read by the sound alone. Make the letters *O*, *R*, *&*, *C*, *Z* and *Y*, formed of dots unequally spaced. The short interval is called a *break*, the longer one a *space*. The former should have a length about equal to a dot, the latter twice this amount. The letters of a word should be separated by an interval of four dots, and the interval between words, six dots. The dashes of the Morse alphabet have a length about three times that of a dot, except in the case of *L* or *0*, which have double this length. Originally *0* was made longer still, or equal to nine dots, but it is now commonly made identical with *L*. The single dash representing *T* should now be practised, also the longer dash, *L*; both are liable to be made too short, especially the latter. Make a series of dashes succeeding each other, trying to bring them as close as possible, the hand jumping from each to the next. Practise together the characters *T*, *L*, *0*, *M*, *5* and *Paragraph*. The next combination is the dot followed by the dash, as in the letter *A*; this must be practised carefully, taking great care not to separate the two characters, and not to make the dot too long. Practise *T*, *O*, *M*, and *A*; when these four letters are well written, practise *A*, *U*, *V*, *4*, and *W*. The dash followed by a dot, as in *N*, is a still more difficult combination. There is the same difficulty in making the interval too great, and the dot and dash of the same length. Practise together *I*, *O*, *M*, *A* and *N*, until each is clearly distinguished. Then try *N*, *D*, *8*, *G*, *7* and *Exclamation*. A dash

between two dots and a dot between two dashes, as *F* and *K*, should next be practised, and after them *Q*, *X*, *2*, *3*, *J*, *Comma*, *Semicolon* and *Quotations*. The only remaining characters are *1*, *Period*, *Parenthesis*, *Interrogation* and *Italics*, which may be learnt next, although the only punctuation marks in common use on most lines are the comma and period. The alphabet may now be practised in order, words spelt and messages sent. Let each student take a book and send a line alternately until each has completed his page.

A convenient method of learning the alphabet is by a little instru ment formed of a steel spring, which makes a click when bent, or when allowed to snap back, thus imitating a sounder. A knowledge of the Morse alphabet will be found useful for signalling in many cases besides by a telegraph. Thus two persons may signal to each other at considerable distances by long and short notes on a whistle or horn. Again, messages may be sent by waving the hands or a piece of cloth, agreeing that one position shall represent a dot and another a line.

97. Induction Coils.

Apparatus. An induction coil, the battery of Experiment 95, a Leyden jar, some Geissler tubes, terminals of various metals, a spectroscope, and some fine uncovered copper wire for connections.

Experiment. The induction coil consists of two coils of wire, the inner or primary consisting of a few turns of stout copper wire, and the outer or secondary of a very long, fine, carefully insulated wire. The coils of the primary wire are connected with the battery, and the instant the circuit is made or broken a current is induced in the secondary coil, which on account of its great length, attains a high potential, each coil adding to the effect of the preceding. When the circuit is made, the secondary current has the same direction as the primary; when broken, an opposite direction. As a current is induced also in the primary circuit which diminishes the secondary current, a condenser is connected with the inner coil, formed of alternate sheets of tin foil and oiled silk, by means of which the current induced in the primary is absorbed. Inside the inner coil are placed a number of wires, or a bar of soft iron, by which the effect is greatly increased. In coils used for

medical purposes, the common method of reducing the current is by partially withdrawing this core. Its effect is due to the powerful magnetic action induced, which ceases when the primary circuit is broken. It acts, therefore, by magneto-electricity. Much depends on the rapidity with which the current is made and broken. In small coils this is usually done automatically by an arrangement like that described under *Electric Bells*, Experiment 95, the iron core being used as the electro-magnet. In larger instruments various devices are employed; sometimes the current is broken by withdrawing a point from a cup containing mercury, whose surface is covered with alcohol to protect it from oxidation, and to render the action more instantaneous. Sometimes a toothed wheel raises a spring hammer, which by its rapid descent suddenly breaks the circuit.

The two ends of the primary terminate in screw cups, to which the battery wires are to be attached. The secondary coils should be connected with brass points, whose distance apart is readily varied. Making and breaking the circuit a spark will pass between these points if their distance is not too great. Be careful not to take the shock, as its effects are very disagreeable, though not dangerous. All the connections of the secondary coil may be made with the fine wire, which is convenient from the ease with which it is bent, while the electro-motive force is so great that the resistance has little effect.

Separate the points and connect the terminals of the secondary coil with one of the Geissler tubes. The latter consist of glass tubes of various, often fantastic, forms, containing gases at extremely feeble pressures. Platinum wires are sealed in at each end of the tube, by which the electricity is conveyed to the interior, and thence passes through the rarefied gas. Now make and break the primary circuit rapidly, when the whole interior of the tube will be illuminated with a beautiful light, whose color depends on the kind of enclosed gas. Sometimes phosphide of calcium, or other phosphorescent substances are enclosed in the tubes, which then shine after the current has ceased. The phenomena of fluorescence are also well shown by placing sulphate of quinine in the tubes, washing them with uranium salts, or making them of uranium glass. The light of the spark being intermittent, if a moving body is

viewed by it, a large number of images are formed, as with the stroboscope. This effect is well shown by shaking the hand rapidly near it, or by moving the head from side to side. Sometimes the tubes are made to revolve, and beautiful colored stars are thus produced. If the wires connecting the coil and brass points are connected, one with the inner, the other with the outer coating of the Leyden jar, the spark at once changes its character; it becomes much more brilliant and dense, but shorter, producing also a much louder snap, since it now must first, so to speak, fill the jar or condenser before it can leap across, a much greater quantity therefore passing at a time.

Next, view the spectrum of the Geissler tube with the spectroscope, as described in Vol. I., Experiment 76, using by preference tubes contracted along the centre, so that the light is reduced to a narrow, bright line. The spectra thus obtained consist of several bright lines, characteristic of the contained gases. Measure the position of these lines, and determine their wave-length. Replace the tubes by terminals of various metals, and observe the spectra as before. The effect is here greatly improved by using the Leyden jar. Another method of obtaining the spectrum of a metal is to make a solution of its chloride or other salt, connect it with one terminal of the coil, and the other terminal with a platinum wire brought near its surface, and observe the sparks between them.

98. Law of Galvanometer.

Apparatus. A battery giving a nearly constant current of electricity, as a Daniell's cell or a thermal battery, a variable resistance, that is, a set of resistance coils or a rheostat, and the galvanometer to be tested.

Experiment. Make connections as in Fig. 70, so that the current shall pass from the battery, B, through the resistance, R, and galvanometer G.

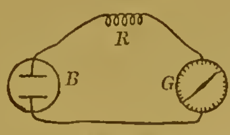


Fig. 70.

Give R various values, and record the reading of G in each case. If a tangent galvanometer is used, read from both ends of the needle and take the mean. If the galvanometer is very sensitive it should be shunted, that is, its two terminals connected by a German silver wire, which thus allows but a small part of the current to pass through it.

Next, construct a curve with abscissas equal to the various values of R and ordinates to those of the deflections a. Prolong this curve to the left of the axis of Y until the point is reached where $a=90°$. The abscissa of this point shows how much the resistance must be diminished to render the current infinite, or to reduce the total resistance to zero; it therefore gives the resistance of the battery and galvanometer. Next construct a second curve, having ordinates as before, equal to the angles of deflection, and abscissas to the strength of the current. The latter, since the electromotive force. E, is constant, by Ohm's law is inversely proportional to the total resistance, that is, equals the reciprocal of the sum of the resistances of the rheostat, galvanometer and battery, multiplied by a certain constant kH. Determine this constant from Experiment 99, and then construct the curve, which will give the absolute strength of the current in vebers, directly from the reading of the needle.

If we merely wish to see if the galvanometer follows the law of the tangents, construct a curve with abscissas equal to the resistance of the rheostat, and ordinates to the cotangent of the angle of deflection. If the law of the tangents holds, this construction will give a straight line which will meet the axis of X at a point whose distance from the origin will equal the resistance of the battery and galvanometer.

If the galvanometer is shunted and its resistance is R, and that of the shunt r, then the fraction $\frac{r}{R+r}$ only, will pass through the galvanometer, and the above results must be multiplied by this fraction to deduce the strength of current required to produce a given deflection when the shunt is used.

To measure a current with this instrument is then a very simple matter. Pass the current through it, note the deflection and find from the curve the absolute strength of current. If the galvanometer follows the law of tangents, multiply the natural tangent of the angle of deflection by kH, and the same result is attained.

99. Galvanometer Constant.

Apparatus. One or more galvanometers to be measured, a very constant battery, a German silver wire, and a beaker containing a solution of sulphate of copper, in which two copper electrodes may be placed.

Experiment. Several galvanometers may be measured simultaneously by this method almost as easily as a single one. The needles are brought carefully to the zero, and they are then all placed in the circuit as in Figure 71, taking care to place them at such a distance apart that their needles shall not affect each other. Connect the two terminals of the battery with the German silver wire S so that it may be lengthened or shortened, and thus by varying it, the strength of the current through the galvanometer kept constant. If either of the galvanometers as G'' is very delicate, it should be shunted so that its needle shall be deviated about as much as the others; this is easily effected by connecting its terminals with a wire of German silver, varying its length until the desired deflection is attained. Next, weigh the electrodes carefully, and connect one with one pole of the battery, the other with the terminal of the galvanometer.

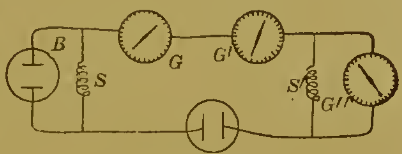


Fig. 71.

So much of the experiment is preliminary, and the remainder should, if possible, be performed at such a time that observations may be taken at intervals for several hours; for instance, starting early in the morning, and observing them every hour or two, during the day. Close the circuit by immersing the electrodes in the sulphate of copper, and bring them so near each other that the deviation of the needles shall be between 20° and 70°. Record the time and read carefully the deflection of each galvanometer. If the battery was perfectly constant, it might be left to itself for several hours, but as it is liable to vary, it should be watched, and any change in the needle corrected by shortening or lengthening the wire S, so that the current through the galvanometer shall be nearly constant. This should be continued for several hours, and the circuit then broken by raising the electrodes out of the liquid, washing them meanwhile with a stream of distilled water from a wash-bottle. Then wash again in distilled water, and finally in alcohol. Note the time when the circuit is broken, and see if all the needles return to zero. Having dried the electrodes weigh them carefully, when it will be found that one will have in-

creased, the other diminished, in weight, the copper being removed from one onto the other. The increase, w, is the most to be relied on, but it is well to measure the diminution of the other electrode also, as a check. A current of one veber will deposit .326 milligrammes of copper per second. Hence the current in the present case will be $\frac{w}{.326\, t}$ in which t is the time in seconds. If either of the galvanometers follows the law of the tangents, kH is determined directly from the equation $C = kH$ tang v, in which both C and v are given. In other cases, the observations of Experiment 98 must be employed. Determine from them what total resistance R' was required to produce the deflection observed in the present experiment. But the deflection being the same, the currents also must be equal; or since $E = CR'$, the electromotive force then employed may be computed. Substituting this value of E and the observed total resistance in the equation $C = \frac{E}{R'}$, we obtain a series of values of the current corresponding to various deflections. A curve should be constructed for each galvanometer with these quantities as coördinates, and will prove of the greatest value as it will show the absolute strength of the current in vebers, corresponding to any given deflection.

The quantity kH should be frequently determined to test its constancy, as it varies with the horizontal component of the earth's magnetism, with changes in the position of the needle in the coil, and of the distribution of its magnetism. The first of these causes will alter all the ordinates of the curve in the same ratio, while the last two will change its form. Variations in the intensity of the magnetism of the needle will not affect the curve, since it changes the component due to the earth and that due to the coil in the same ratio.

Having found the constant of one galvanometer, that of any others may be found directly from it. Thus, connect them as in Figure 71, except that the beaker of sulphate of copper may be thrown out of the circuit. Alter S until the deflection of the galvanometer previously measured is the same as before. Then the current is also the same, and hence from the deflections of the other galvanometers, their constants may be determined as above.

For a Thomson, or other very delicate galvanometer, a simple

shunt is not sufficient, unless the wire is so short that its resistance cannot be accurately determined. In this case, after shunting it, connect one terminal with a large resistance, and then connect the other terminals of the galvanometer and resistance with a second shunt. A third reduction may be made if necessary, and the deflection thus reduced indefinitely. If the galvanometer is astatic, or has a damping magnet, of course the slightest change in its magnetism, in that of the earth, or in the position of the magnet, will greatly alter its constant. Another method of finding the constant of a sensitive galvanometer will be given under Experiment 104, and the results should be compared to check each other.

100. Cosine Galvanometer.

Apparatus. A constant battery, a commutator, some German silver wire and a cosine galvanometer. The latter differs only from a tangent galvanometer in having the coils free to turn around a horizontal axis, the angle being measured by a graduated circle and index.

Experiment. Turn the galvanometer around horizontally, reading the two ends of the needle to see if they agree. If not, there is an error of eccentricity, and the mean should always be employed. Turn the instrument until the needle points to 0°, and connect with two of the terminals of the commutator, and the battery with the other two. The current may now be passed in either direction through the galvanometer, and should give its needle a deflection of 60° or 70°. If, as is probable, the deflection is greater, shunt the battery by connecting its terminals by the German silver wire, and reduce the length until the required deflection is obtained. This deflection may now be altered from its greatest value when the coils are vertical, to 0° when they are horizontal. Bring them into the latter position, or so that their index reads 90°, and see if the needle reads 0°. If not, the instrument is not levelled, and one side should be raised or lowered until the needle is brought to 0°. Turn the coils 180° and see if the needle again points to 0°. If the needle is hung by a fibre of silk this correction may alter its eccentricity. Next, make the coils vertical, or at 0°, and take a series of readings of both ends of the needle, turning the coils 10° at a time. Calling v the mean

angle of the needle, and w that of the coils, compute tang v sec w, which should be a constant in each case.

Turn the coil until the deflection is somewhat less than 45°, and bring the needle to 0°, by turning the whole instrument horizontally. Break the circuit, and let the needle come to rest. Its reading will show the amount the galvanometer has been turned, and its sine multiplied by the secant of the angle of the coils should give the same result as before.

It will be seen that the current may be measured in a variety of ways by this instrument. First, with the coils vertical, as by a common tangent galvanometer. Secondly, inclining the coils a series of readings may be taken whose mean gives the strength of the current with great accuracy. And thirdly, bringing the needle to 0° by turning the whole instrument, and determining the deflection by breaking the current. The instrument is then used like a sine galvanometer.

Comparing the three methods, the tangent galvanometer gives good results for angles less than about 60° or 70°, but above this point the tangents increase so rapidly that a considerable change in the current corresponds to but a small alteration in the position of the needle. The sine galvanometer is more troublesome to read, and cannot be used for strong currents except by inclining the coils, as when the deflection exceeds 45° the needle cannot be brought to coincide with them. In the neighborhood of this point, however, it is very sensitive, and might be used with advantage when, as in Experiments 99 and 108, we wish to detect a slight variation in the strength of a current. The advantage of the cosine galvanometer is that several independent readings may be taken; especially in the case of strong currents, when by turning the coils, the needle may be brought to that part of its scale where it is most sensitive. It is open to the objection, however, that if the coils are very much inclined they tend to make the needle dip, owing to the large vertical component. It is therefore generally better with very strong currents to partially reduce the effect by shunting the instrument.

101. Differential Galvanometer.

Apparatus. A battery of one Daniell cell, a differential galvanometer, a rheocord, an ohm, an accurate sheet-metal gauge or

wire gauge, and some copper wire whose resistance is to be determined.

Experiment. A differential galvanometer differs from the ordinary form in having two equal coils, through either or both of which the current may be passed. It may be used as a common galvanometer by employing only one coil, or connecting them together so that the current shall pass in the same direction through both. Thus calling the coils *AB* and *CD*, we may use *AB* or *CD* alone, or we may connect *B* and *C* and pass the current through *ABCD*. In this case we have double the number of coils of either separately, but double the resistance. Again we may connect *A* and *C*, *B* and *D*, and thus have a galvanometer of only one-half the resistance of either coil separately. The current in this case divides, a part going through *AC* and part through *BD*. The differential galvanometer is ordinarily used to test the equality of two currents by passing them through the coils so that they shall tend to turn the needle equally in opposite directions, or leave it at rest. The deflection in any case will equal the difference of the two currents.

This will only be the case when the two coils have the same resistance, and have the same relative position with regard to the needle and this must therefore be tested first. Connect the terminals *B* and *D* together, and *A* and *C* with the battery, so that the current shall traverse the path *ABDC*, or the same current go through both coils in opposite directions. Then if the coils are rightly placed they will have no effect on the needle, which will remain at zero. If not, one or both of them must be moved until this condition is fulfilled. Next connect *A* and *D*, *B* and *C*, and pass the current through, when it will divide, equal parts going through each coil if the resistances are equal, and not deflecting the needle. When both these tests are satisfied the instrument is ready for use.

The last connection is that commonly employed when using the galvanometer, two circuits being formed, one for each coil. Insert the ohm in one circuit, and the rheocord in the other. The latter consists of two platinum wires stretched side by side over a millimeter scale, and connected together by a slide formed of two thimbles joined together and containing mercury, through which

the wires pass. By varying the position of the slide, the length of the wire in the circuit may be altered, and its amount determined by the scale. Move the slide until the deflection of the needle is reduced to zero. Then remove the ohm and connect the wires attached to it directly together. Bring the needle again to zero, and the difference in reading gives the scale-reading corresponding to 1 ohm. Repeat two or three times, and it will be found that the results are not wholly concordant, owing to the imperfect connection made by the mercury. Now interchange the rheocord and ohm, and if the galvanometer is correct the same value of the ohm should be obtained; if not, the true value will be the mean proportional of these two.

Place the rheocord in one circuit, and a measured length of the wire in the other, and bring the needle to zero. This by a simple proportion gives the resistance in ohms. Repeat with another piece of different length. Now find the diameter of the wire by the sheet-metal gauge. For this purpose close the gauge by turning the milled head and see if the reading is zero; if not, this reading must be added to, or subtracted from, the observed reading, according to its sign. Next turn the milled head, insert the wire, and close the gauge on it. The reading is then taken as with any micrometer screw. In Brown and Sharpe's gauges one turn of the screw equals $\frac{1}{40}$ of an inch, or .025, and the head is divided into 25 equal parts, each of which accordingly equals one thousandth of an inch. The reading should be repeated several times on different parts of the wire. The resistance of a wire of pure copper 1 metre long and 1 mm. in diameter at 0° C. equals .0127 ohms, and for any other wire is proportional to its length, and inversely as the square of its diameter. This quantity must be multiplied by $(1 + .0038\, t)$ in which t is the temperature of the room. Compute from this what would be the resistance if the wire consisted of pure copper, and dividing the observed resistance by this quantity gives the relative conductibility compared with pure copper.

Various devices have been proposed to remedy the defects of the rheocord. It may be constructed like a sonometer, the connection being made through the movable bridge. Formerly resistances were generally measured by the rheostat, which consists of

two cylinders, one of wood the other of metal, so connected that a wire may be wound from one on to the other, and the resistance thus varied at will. In another form the wire is wound on a wooden cylinder, and the connection made at any desired point by a sliding elastic strip of brass. None of these instruments, however, give very satisfactory results.

102. Wheatstone's Bridge.

Apparatus. A Wheatstone's Bridge and set of resistance coils, a Thomson's galvanometer, shunt, lamp and scale, a battery of two Daniell's cells, some coils of wire whose resistance is to be determined and some copper wire.

Experiment. These various instruments must be described in detail before showing how to use them. The Wheatstone bridge, though in principle the same as that given in Appendix A, in its actual construction bears no resemblance to the figure there given. It is represented in Fig. 72, and consists of a number of resistance coils connected end to end, with stout brass pieces between them, which may be connected together by plugs, so as to form three continuous lines, CB, BO and OD, whose total resistance is extremely small. At A, B, C, and D, are placed screw cups with which wires may be connected, and between A and C are three resistances of 10, 100, and 1000 ohms, either of which may be thrown into the circuit by merely drawing its plug. Three similar resistances are interposed between A and B, while between O and B are coils of 1, 2, 2, 5, 10, 10, 20, 50 ohms, and between O and D coils of 100, 100, 200, 500, 1000, 1000, 2000 and 5000 ohms. The battery is now connected with A and D, the galvanometer with B and C, and the resistance to be measured P, with C and D. The current accordingly divides at A, part passing through M and O, and the remainder through N and P, the galvanometer remaining at rest only when $M : N = O : P$. M and N may evidently have values of 10, 100 or 1000 ohms, and O anything from 1 to 10,000 ohms.

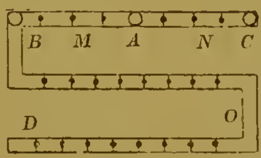


Fig. 72.

The Thomson's galvanometer consists of a long coil of very fine wire, at the centre of which a minute mirror is hung by a single filament of silk. To the back of the mirror is attached a little magnet whose motions are greatly magnified by placing in front of it a scale of equal parts and a lamp, in such a position that the image of the flame shall be reflected on to the scale by the mirror. To render the light as bright as possible, the flame which is formed by a flat wick, is placed edgewise, and the mirror is slightly concave, or a convex lens is interposed between it and the scale. Sometimes a narrow slit is interposed in front of the lamp, so as to form a bright line on the scale, but it is generally better to use a broader slit with a horse-hair stretched down its centre. A bright rectangle is therefore projected on the scale with a black line in the centre, whose position can be read with great precision. To bring the spot of light to the center of the scale, and to neutralize in part the magnetism of the earth, a magnet is placed above the coil with its north pole turned towards the north. Gradually lowering this magnet, the effect of the earth will be neutraltralized more and more, as is shown by the increased time of vibration of the mirror and spot of light. Finally, a point is reached where the needle turns and assumes any position at will. The earth's magnetism is here neutralized, and if the magnet is still further lowered the suspended needle will point with its north pole to the south. Since, with a given current the tangent of the deflection is always inversely proportional to the horizontal component of the earth's magnetism, if the latter is rendered very small, the former may be increased indefinitely, so as to produce a large deviation with even a very feeble current. The instrument as thus adjusted is very sensitive, so that a slight current will throw the spot completely off the screen, and an ordinary current might injure the instrument. To reduce its sensibility a shunt is employed in which its terminals may be connected by a wire having a resistance of $\frac{1}{9}$, $\frac{1}{99}$ or $\frac{1}{999}$ of that of the coil. In the first of these cases, for one part of the current passing through the galvanometer, nine will pass through the shunt, hence the galvanometer will receive only one tenth of the whole. The others, in the same way, cut off all but one hundredth and one thousandth. These three coils are attached to one terminal of the

galvanometer, and are connected with the other when desired, by a plug. By inserting the latter in another hole, the two terminals are connected directly, so that no current can pass through the instrument.

To set up the apparatus, place the galvanometer on the table, or preferably, for greater steadiness, on a bracket attached to the wall, or on a stone pier, and facing east or west; this direction is to be preferred as a little more convenient, but it is not indispensable. Level it by the screws in its base and raise the magnet and mirror by a little pin just above the coil, so that they shall hang freely nearly in the centre of the coil. Light the lamp and place it with its flame edgewise to the galvanometer, and place the scale in front so that the light shall shine through the slit on to the mirror. Next, to bring the spot of light to the centre of the scale, raise the magnet to its highest position and turn its north end to the north. See now if the mirror swings freely from side to side on turning the magnet. A bright spot should appear on the scale and the distance of the latter should be altered until a distinct image of the slit and vertical hair is formed. Now moving the magnet slightly, this image will swing from side to side, and may be brought to any point of the scale. Next, lower the magnet, turning it, if necessary, so as to keep the spot on the scale, until a point is reached where the spot goes off to one end of the scale, and the mirror tends to turn completely round. The earth's magnetism has now been a little more than neutralized by that of the magnet. The latter should next be raised a little, so that the earth's magnetism shall be a little in excess, when the spot will vibrate slowly over the scale, and on turning the magnet may be brought back to the centre. As it is difficult to turn the magnet slowly enough by hand, a tangent screw is attached, by which the spot may be brought exactly to any desired point. If the mirror cannot be brought parallel to the coils by raising the magnet to its highest position and turning it around, the magnet should be lowered and turned, if necessary, wholly around until the mirror is parallel to the coil; it may then be raised gradually, and finally left a little below the position of equilibrium. The above adjustment once made, it should be kept undisturbed except to bring the spot of light to the centre of the scale, which may be

done either by moving the lamp or scale, or turning the tangent screw slightly. The position of the spot often changes from day to day, owing either to changes in the torsion of the silk suspending fibre or in the magnetism of the earth or compensating magnet.

A convenient arrangement is to mount the galvanometer so that the mirror shall be four feet from the floor, and place a table about three feet in front of it; on this is placed a strip of ground glass at the level of the eye, on which the spot of light is received, and a scale is placed just below to show its exact position. The battery is placed below the galvanometer, and the other apparatus on the table; readings can thus be taken with great convenience and the galvanometer is protected from injury or disarrangement.

To show the extreme delicacy of the galvanometer, remove the shunt and connect two pieces of copper wire with its terminals. Place the two ends in the mouth, one above, and the other below the tongue, and a current will be at once formed, often sufficient to throw the spot off the scale. The cause is the different chemical action of the saliva, from different parts of the mouth. In the same way a current is produced by holding the ends with moist fingers, or dipping both into the same vessel of water, owing to slight differences in the two surfaces. Holding one terminal in the teeth and compressing the other with a pair of pincers, produces a similar effect. These experiments may be varied almost indefinitely.

The shunt and resistance coils may be placed in any convenient position, the former being connected by two of its terminals with the coils at B and C. The battery is also connected with the coils at A and D. Two keys must be interposed between the coils and the battery and galvanometer. These should be placed side by side so that they can be closed by the first and middle fingers of the right hand. These keys are best made of single strips of brass screwed down on to the table and insulated at the ends by rubber buttons. The objection to using a single key connected with the battery is, that currents may be induced in the coils which will disturb the galvanometer unless the battery circuit is closed first, and the galvanometer circuit afterwards. This is easily done with the two keys, after a little practice. Some-

times a single key is employed, formed of two flexible pieces of brass, so arranged that on pressing down the upper one, contact is made between it and the second strip, which closes the battery circuit, and pressing it still further, closes the galvanometer circuit by bringing two brass pieces in contact, of which one is attached to the table, the other to the lower surface of the middle brass strip.

When the instrument is not in use the plug should always be inserted between the terminals of the galvanometer, so that if the circuit is accidentally closed no current shall pass through it. Care should be taken not to send too powerful a current through the galvanometer, as the needle is then thrown violently to one side, and its magnetism may be weakened; for this reason it is generally best to keep the galvanometer shunted, and pass through it only $\frac{1}{100}$ or $\frac{1}{1000}$ of the current. A strong current should never be passed through the resistance coils for a long time, as it would heat them, injure the insulation and alter the resistance temporarily, if not permanently.

To measure the resistance of a coil of wire, connect its ends with C and D and see that the other connections are made as described above. Shunt the galvanometer so that only $\frac{1}{1000}$ of the current shall pass through it and insert two equal resistances of 1000 ohms between A and C, A and B. Now close the circuit for an instant, first pressing down the key under the middle finger, or that connected with the battery, and then the other, and instantly raising them. The spot of light will probably dart to one side so rapidly that it is hard to follow it, because the resistance interposed between B and D, which is very small, is less than the resistance to be measured. Now draw the plug next D, which inserts a resistance of 5000 ohms, and again depress the key when the spot will probably move the other way. If not, the resistance is either over 5000 ohms, or there is something wrong in the connections; to test this, connect the battery with C instead of D, and if the spot moves in the same direction as at first, there is something wrong in the connections, otherwise the required resistance lies between 5000 ohms and infinity. In the latter case, replace the battery connection and draw the other plugs, and if the spot still moves in the same direction the resis-

ance is over 10,000 ohms, and the method described below for very great resistances must be employed. If now the 5000 resistance is too great, replace its plug and draw the 2000 plug; if this is too small, draw in addition the 1000 plug, if too large replace the 2000 and draw the 1000. Proceed in this way precisely as in the method of weighing described in Vol. I, p. 47, always taking care to introduce the resistances in order. When a near approach to the correct resistance is obtained the deviations of the spot will be small, and they may then be increased by altering the shunt so that $\frac{1}{100}$ or $\frac{1}{10}$ of the current passes through the galvanometer. Finally, removing the shunt plug, employ the galvanometer with its full sensibility. When the resistance is determined within a single ohm, a still closer approximation may be obtained by interpolation. Thus suppose that with the smaller resistance the spot comes to rest m divisions to one side of the zero, and when the resistance is increased one ohm, to n divisions on the other side. Then the true resistance will equal the smaller resistance, plus the fraction m divided by $m + n$. Thus, if with 2815 ohms the deflection is 15 divisions to the right, and with 2816 ohms, 10 to the left, the true resistance equals $2815\frac{15}{25} = 2815.6$ ohms. Much time is commonly lost in waiting for the needle to come to rest, and a great saving may be effected in this respect by closing the circuit for an instant so as to check the swing. Thus if the current tends to send the spot to the right, wait till it swings to the left, and when passing the centre point, close the circuit for an instant. The magnet receives an impulse in the opposite direction which may be made to stop it almost entirely. This can be well done only with practice. It is not generally necessary to wait till the spot comes to rest, but merely to note the reading of each end of its swings and take the mean.

If the resistance lies between 1000 and 100 ohms one more place of decimals may be obtained as follows. Introduce a resistance of 100 ohms between A and C and leave that between A and B equal to 1000; then we have as $1000 : 100 = O :$ the required resistance, or each resistance coil of O is virtually reduced to one tenth its previous value. Accordingly 1 ohm will now equal .1 ohm and by interpolation, resistances may be measured to .01 ohm. If the resistance is less than 100 ohms, by making N

equal to 10 ohms, resistances of .01 ohm may be measured, and by interpolation .001 ohm. If the resistance is over 10,000 ohms make $N = 1000$, and $M = 100$ or 10. In this way resistances up to 1,000,000 ohms or a megohm may be measured.

Resistances greater than a megohm may be measured approximately as follows. Make $N = 1000$, $M = 10$ and read the position of the spot of light, giving O various values as 9000, 8000, 7000, etc., until the spot passes off the scale. Construct a curve which should be very nearly a straight line with abscissas equal to the deflections, and ordinates to the reciprocal of the resistances, and prolong it until it meets the axis of Y. At this point the deflection is zero, and the reciprocal of its ordinate multiplied by 100 gives the required resistance. Another point on this curve is obtained by making O equal to infinity, or connecting the end of P with the battery terminal, and disconnecting it from D Another method of measuring a very large resistance, if we have a coil of large and known resistance as a megohm, is to place them in turn in circuit with the battery and galvanometer, when the deflections will be nearly inversely as their resistances. For these measurements a battery of small cells, connected for tension, may be employed, as the great resistance prevents injury to the coils.

The resemblance of this instrument to the chemical balance is very marked, only it is a balance of prodigious range and possessing many most important advantages. The coils M and N correspond to the arms, O to the weights, the spot of light to the index, and the keys to the supports of the beam and scale-pans. It can measure from 1,000,000 to .001, a range as great as from 14 tons to 1 grain. Either arm may be made 10 or 100 times as long as the other, and the index is without weight, and moves over a long scale. Moreover the shunt enables us to diminish the delicacy of the balance to $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$, as if by merely inserting a plug we could convert a delicate chemical balance into a rough grocer's scale. It is well to measure several resistances, as described above, some of them large, and others small, and finally to find the conductibility of copper as described in Experiment 101. Another excellent experiment is to measure the resistances of two coils of wire, and then connecting their ends measure their combined resistance.

It should, by the law of divided circuits or shunts, equal their product divided by their sum. The law for three or more combined resistances may be found in a similar manner.

103. Resistance Coils.

Apparatus. A delicate galvanometer, a British Association divided-metre bridge, a battery, a standard ohm, some German silver wire, some small pieces of copper with holes bored through them, some solder, resin and a Bunsen burner.

Experiment. The B. A. bridge, Fig. 73, consists of a carefully drawn wire one metre in length, any point of which may be touched by a sliding key so as to divide it electrically into two parts. The current from the battery B then passes into A, and when the latter is pressed down it divides, part going through AC and the resistance to be measured R', and the remainder passing through AD and the known resistance coil R, to the other pole of the battery. The four resistances AC, AD, R' and R correspond to the four resistances of the common bridge. To measure any resistance R', connect as in the figure and press down A; the needle will in general deviate, but by sliding A along the wire, a point is easily found where there is no deviation. In this case the resistance is found from the proportion $AD : AC = R : R'$, AD and AC being given directly in millimeters from the divided scale.

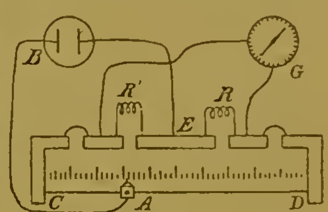


Fig. 73.

Now make a coil whose resistance is 1 ohm. Cut off a piece of the wire and measure its length and resistance. The resistance being proportional to the length, a simple proportion gives the length of the required ohm. To allow for variation of diameter of the wire and other causes, cut off a piece somewhat too long (3 or 4 per ct.). Measure its resistance and repeat until the wire is about an inch too long. Then pass the ends into two of the copper terminals, put on a little resin, heat them in the burner, touch the ends with a little solder and they are soon fastened firmly in place. Soldering acid (chloride of zinc) should not be used, as

being hygroscopic it attracts moisture and is likely to cause errors. When the ends are perfectly cool, measure the resistance which should now be very nearly the required amount. To make it exact, heat one of the terminals and slide it on or off by the desired amount. When cool, measure again, and repeat until an exact copy of the standard ohm is attained. To do this with greater certainty reverse the position of R' and R and see if the position of A is unchanged when no current passes through the galvanometer. Any inequality in the two halves of the wire CD is thus eliminated. An error of .004 will now correspond to a motion of A of about 1 millimetre. To increase the delicacy interpose between C and R', and D and R, two equal coils of large resistance. Suppose each has a hundred times the resistance of CD, then the latter is virtually extended to a length of two hundred metres, and an error of 1 millimetre is reduced in the same proportion. To make sure that the two coils have the same resistance, reverse them and adjust the length of R' until the mean position of A, when these two coils are reversed, is the same when R' and R are reversed. If the wire is very long it should be wound on a bobbin, and to eliminate the induced current we should begin at the middle and wind the two ends side by side. Instead of a bobbin, a common spool may be employed. The whole is then dipped in melted paraffine, when the air rushes out and is replaced by paraffine, which is an excellent insulator. A large rubber tube may then be stretched over the whole, and on it the resistance marked in ink. It is also well to mark it with a certain number to refer to a book giving the date and name of maker, in which may afterwards be entered the error in the resistance. As the soaking in paraffine may alter the resistance, this should be done before the final adjustment of the copper terminals is made. For short coils the simple wire may be used, and a short piece of fine rubber tubing slipped over it on which to mark its resistance and number.

104. Capacity of Condensers.

Apparatus. Two condensers whose capacities are to be compared, two switches, a differential galvanometer, a Thomson's galvanometer, two resistances, one of which may be varied at will, and a battery which need never be closed, but must have a large and nearly

constant electromotive force. Some Daniell or Leclanché cells may be used, but the best results will be obtained with a battery of the form proposed by Latimer Clark, and made with pure mercury for one element, and pure zinc for the other, the liquid employed being a paste formed by boiling sulphate of mercury in a saturated solution of sulphate of zinc. The cells may be very small, as an increased resistance makes but little difference, and their circuit should never be closed.

Experiment. Connect the inner coatings of the condensers with the two circuits of the differential galvanometer, and the other terminals of the later, with one pole of the battery. Connect the outer coatings together, and with a switch, so that they can be put in contact with either pole of the battery. By turning the switch in one direction the condensers are charged, and in the other the two coatings are brought electrically in contact, and hence discharge takes place. If the two condensers have the same capacities, no effect is produced when the switch is moved, as equal quantities of electricity in that case pass through each coil of the galvanometer. If the condensers do not have equal capacities the needle will swing to one side or the other, according as they are charged or discharged. In this case the coil connected with the larger condenser must be shunted, or its terminals connected with a variable resistance so that a part only of the current shall pass through it. By altering this resistance a value may be found for which there will be no deviation of the needle. In this case, calling G the resistance of the galvanometer coil, and S that of the shunt, the current through G will be to the whole current as $S : G + S$. But this current equals that in the other coil, so that, calling the two capacities C and C', we have $C : C' = S : G + S$.

Another method of comparing the capacities of two condensers is by a modification of the Wheatstone's bridge. In Fig. 74, let O and P represent the two condensers whose outer coatings are connected, and whose inner coatings are attached to the resistances M and N, and to the terminals of the Thomson's galvanometer G. A switch S is intro-

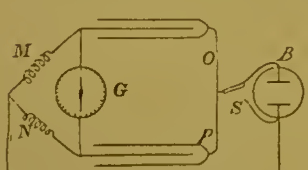


Fig. 74.

duced, as in the other arrangement, so that the outer coating of the condensers can be connected with the battery, or with their inner coatings. Now alter one of the resistances M, so that there will be no deflection of the needle when the circuit is made or broken. Then the two condensers will be proportional to the resistances M and N. A convenient arrangement is to make the two resistances M and N the two parts of the wire of the bridge employed in Experiment 103, or a set of resistance coils may be used for M, and a coil of wire for N.

When the condensers have small capacities the following method is preferable. Connect the Thomson's galvanometer by a switch with the two inner coatings and their outer coatings with a second switch, so that they can be connected either with the battery or with their inner coatings, through the galvanometer. By moving the first switch either condenser may be thrown into the circuit, by moving the second it may be either charged or discharged. Now charge and discharge the first condenser, and notice the swing of the needle in each case. Do the same with the second condenser, and the ratio of the mean of their swings equals their comparative capacity.

If the constant of the Thomson's galvanometer is accurately known, the capacity of the condenser is readily determined. It is only necessary to charge it with a battery of known electromotive force E, and discharge it through the galvanometer. Then calling t the time of the swing of the needle, v the amount of swing caused by the discharge, and c the capacity, we have $cE = \frac{2kHtv}{\pi}$ or $c = \frac{2kHtv}{\pi E}$. This value of c is correct only if the resistance of the air is so slight that the needle vibrates for a long time before coming to rest. If this is not the case, set it swinging, and measure the extreme amplitude attained during each vibration. The ratio of two consecutive deflections, or the difference of their logarithms will be nearly constant. Calling $l' = \log v' - \log v''$ and employing the mean value of l, we must in the above value of c substitute $v\ (1 + \frac{1}{2}l)$ for v. The value of kH should be determined at the time, by comparison with a tangent galvanometer as

described in Experiment 99, or if the capacity of the condenser is known this method furnishes an easy means of determining kH.

105. Electromotive Force and Resistance of a Battery.

Apparatus. The battery to be tested, a tangent galvanometer, a plug key, and a resistance coil.

Experiment. If the battery to be tested gives a current which is nearly constant, the problem becomes a very simple one. Connect the galvanometer G, Fig. 75, with the battery B and read the deflection of the needle; then interpose the resistance R, and repeat. From the curve accompanying the galvanometer determine the absolute currents C and C' in the two cases, and since $C = \frac{E}{B+G}$ and $C' = \frac{E}{B+G+R}$, we readily reduce B and E. If the galvanometer follows the law of the tangents, $C = kH$ tang v, from which C is readily determined. To insure accuracy, the readings of the needle should lie between 20° and 70°, and C should be about double C'. If the galvanometer is too delicate, it may be shunted, and G and kH changed to correspond.

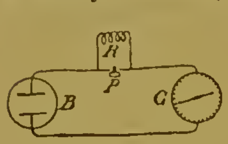


Fig. 75.

In general, when the circuit of a battery is closed, the current, at first strong, becomes rapidly weaker and weaker. To determine the law of this diminution, the current is allowed to pass alternately through G alone, and through G and R together, for intervals of one minute, and the reading of the needle taken in each case. This is most conveniently done by a switch, plug key, or other arrangement by which the two terminals of R are connected, or short-circuited, as it is called.

Next, construct curves with the times as abscissas, and currents as ordinates. Two curves are thus obtained, corresponding to the two positions of the key. From them take values of C and C', corresponding to various values of t, and compute the corresponding values of E and B by the formulas $C = \frac{E}{B+G}$ and $C' = \frac{E}{B+G+R}$. Finally, construct two curves in which the abscissas

represent the times, and the ordinates the values of E and B respectively. From these curves we shall see how much the diminution of the current is due to the increase of the resistance, and how much to the diminution of the electromotive force.

106. Resistance of Batteries.

Apparatus. The battery to be measured, a permanent magnet, a delicate galvanometer, two resistances, one of which may be varied, and a plug by which three wires may be connected. For the resistances, two German silver wires may be used, of which the length of one may be altered by drawing it through a screw cup.

Experiment. Connect the instruments as in Fig. 76, in which the battery B, and galvanometer G, are connected with one of the resistances, R', and with the plug. The third terminal of the latter being connected with the resistance R, and through it with the other terminals of the battery and galvanometer. When the plug is out, the current passes from B through G and R', but not through R which is then connected only at one end. When the plug is in, R' is thrown out of the circuit, its two terminals being connected by the plug, and R acting as a shunt to G. Evidently the deflection of G is reduced in the first case by the increased resistance R', and in the second case by the shunt R. A certain value of these resistances will therefore produce the same deflection whether the plug is out or in. This will be the case when $B = \frac{RR'}{G}$. On making the connections, the needle will commonly be deflected entirely to one side, and should then be brought back by the permanent magnet to the zero. If R is the variable resistance make this adjustment with the plug out, and then inserting the plug, alter R until the spot is brought back to the zero. If R' is variable, make the adjustment with the plug in, then remove it and alter R'. If the Thomson galvanometer and set of resistance coils are employed, the same rules must be followed as in Experiment 102. If German silver wire only is used, first insert a very short and then a very long piece, when the

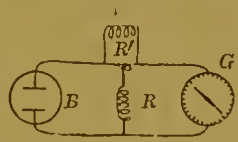


Fig. 76.

needle should deflect first to one side, and then to the other. By varying the length, the spot is soon brought to zero. The resistance is then found either by direct measurement or by measuring the length, and comparing the resistance with that of a known length of the same wire. Instead of bringing the needle to zero by the permanent magnet, the method described in Experiment 100 may be employed, using a cosine galvanometer with its coils placed at right angles to the meridian, and turning the coils until the needle is brought to the zero.

The formula $B = \frac{RR'}{G}$ may be proved either graphically or analytically. First, lay off in Fig. 77, $mn = B$, $no = R$, $op = G$, and erect perpendiculars equal to their potentials, when the plug is out. These are found by making mm' equal to the electromotive force of the battery. Then oo' will equal the difference of potential of the terminals of the galvanometer, and is proportional to the current passing through it.

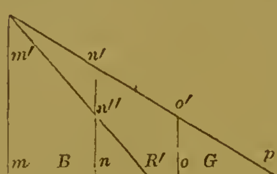


Fig. 77.

When the plug is in, the total resistance is much less, being composed of $mn = B$ and nR' the combined resistance of G and R, or $\frac{GR}{G+R}$. The difference of potential of the two ends of the battery will now be nn'', and if this equals oo' the current through the galvanometer will be the same in the two cases. The geometrical condition that $nn'' = oo'$ follows, if $mn : nR' = mo : op$, but $mn = B$, $nR' = \frac{GR}{G+R}$, $mo = B + R'$ and $op = G$. Hence $B : \frac{GR}{G+R} = B + R' : G$. Multiplying out $BG(G + R) = GR(B + R')$, or $BG^2 + BGR = BGR + GRR'$ $\therefore BG = RR'$ and $B = \frac{RR'}{G}$.

The same formula may be proved analytically by Kirchhoff's laws as follows. Let C_B, C_G, C_R represent the currents in B, G and R, when the plug is in, and C the current, which is the same for all, when it is out. Then $C = C_G$ . . . (1), since this is the condition that the deflection of the galvanometer shall be unchanged. By Kirchhoff's first law, $C_B = C_G + C_R$ . . . (2), and by

his second law applied to the circuit GR, we have $GC_G - RC_R = 0$ . . . (3). Applying the same law to the circuit BGR' gives $E = BC + GC + R'C$, and to the circuit BG when the plug is in, $E = BC_B + GC_G$, and equating these two, $BC + GC + R'C = BC_B + GC_G$ . . . (4). We have thus four equations between the variables C, C_B, C_G, C_R, and substituting (1) and (2) in (4) gives $BC + GC + R'C = BC + BC_R + GC$, or reducing $R'C = BC_R$, dividing this by (3) gives $\frac{B}{R} = \frac{R'}{G}$, or $B = \frac{RR'}{G}$.

107. Resistance of Galvanometers.

Apparatus. The same apparatus as in Experiment 102, with the addition of a permanent magnet, and a key.

Experiment. Sometimes as in the last Experiment we wish to determine the resistance of a galvanometer, and cannot employ the usual method of measurement since it is needed in the Wheatstone's bridge. In Fig. 78, let M, N, G, P, represent the four resistances, B the battery, and S the galvanometer, in the usual arrangement of the bridge. If the resistances are balanced it will make no difference if the galvanometer is replaced by a short wire, and key S, and since no current passes through this wire, the current in the four coils will be the same whether the key is opened or closed. Therefore replace the resistance to be measured, G, by the galvanometer, and see if the deflection is unchanged when the key is closed, and if so $N : M = P : G$ or $G = \frac{MP}{N}$. If the deflection is not the same, alter P until it is the same whether the key is up or down. As in the last Experiment, the spot should be brought to zero by the permanent magnet. In the actual case the galvanometer should be connected with C and D, Fig. 72, the key connected with B and C, and the resistance altered as in Experiment 102 until no effect is produced on depressing the key.

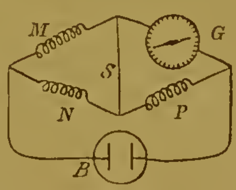


Fig. 78.

108. Manse's Method.

Apparatus. The battery to be measured, a delicate galvanometer, a magnet, a resistance coil, and a B. A. divided-metre bridge.

Experiment. Connect the terminals of the battery B, Fig. 79, with one end of the resistance coil R, and with the sliding key A of the bridge. Connect the other terminals of B and R with the ends of the metre and also with the galvanometer, G. The current will now pass through the galvanometer, deflecting its needle nearly 90°. Lay the magnet perpendicular to the coils of the galvanometer, and to the magnetic meridian, and move it up until the needle is brought to the zero. Press down the key A when the needle will, in general, be deflected. Move A, and find by trial the point at which it has no effect on the needle. Calling its distance from the two ends a and b, the resistance of the battery is given by the equation, $b : a = R : B$.

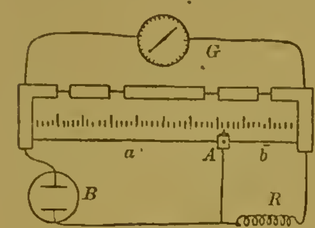


Fig 79.

This may readily be proved if we notice that the electrical conditions are precisely the same as in the last Experiment, except that the battery and galvanometer have changed places. The four resistances are a, b, B and R, and A replaces S.

109. Wiedemann's Method.

Apparatus. A standard constant battery, B, Fig. 80, a battery, B', to be compared with it, a tangent galvanometer G, and a commutator, C.

Experiment. The object of this experiment is to measure the electromotive force of B', in terms of that of the standard battery, B. Connect them, so that by turning the commutator their currents will pass through the galvanometer either in the same or in opposite directions as in Figure 80. Read the deflection of G in each case, and deduce the currents C and C', either from the curve accompanying the galvanometer, or from the formula, $C = kH \text{ tang } a$, $C' = kH \text{ tang } a'$. But by Ohm's law, the current, when the effects of the two batteries add, is $C = \frac{E+E'}{B+B'+G}$ and when reversed $C' = \frac{E-E'}{B+B'+G}$ hence $E' = E\frac{C-C'}{C+C'}$, from which E' is

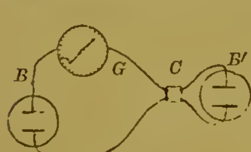


Fig. 80.

deduced in terms of the electromotive force of the standard battery. If the battery B' is not constant, the method given in Experiment 105, should be employed. Connect the batteries for one minute and read the galvanometer, reverse the commutator during the second minute and read again. Take in this way a series of readings until the deflections become sensibly constant. Now construct curves with ordinates equal to the currents in the two cases and abscissas equal to the times. Compute a number of values of E', using the ordinates of points of these curves having the same abscissas, or equal to the currents which would have passed had it been possible to make both observations at the same time. Construct a third curve with the same abscissas and ordinates equal to these computed values of E'.

110. Poggendorff's Method.

Apparatus. A constant battery to be taken as a standard, the battery to be tested, a delicate galvanometer, and a variable resistance.

Experiment. First measure the resistance of the standard battery, which should be the stronger of the two, as described in Experiment 108. Connect the apparatus as in Figure 81, in which the standard battery B, is connected with the other battery B' so that they shall tend to turn the needle of the galvanometer in opposite directions. Then connect the terminals of R with those of B. Vary R until the needle of G is brought to the zero, when we have the equation, $E' = E\dfrac{R}{B+R}$.

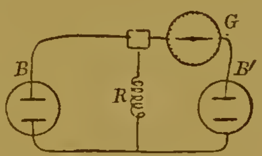


Fig. 81.

To prove this formula graphically, the construction of Fig. 82 may be employed. Make $mn = B$, the resistance of the standard battery, $mm' = E$, its electromotive force, and $no = R$, the variable resistance. Then drawing the straight line mo, mn' will be the difference of potential of the ends of R, or the tendency of the current to pass through B and G. If now $E' = pp'$, the

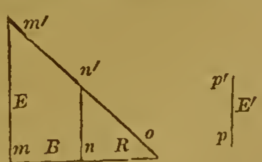


Fig. 82.

electromotive force of the second battery, equals this, or $pp' = nn'$, no current will pass through the galvanometer. But in this case $mm' : nn' = mo : no$ or $B + R : R = E : E'$ hence $E' = E\frac{R}{B + R}$.

The same formula may be proved by Kirchhoff's laws as follows. Call C_G, C_B and C_R the currents in G, B and R, then by the first law, $C_G + C_R = C_B$, or since $C_G = 0$, $C_R = C_B$. But in the circuit $B'RG$, we have by the second law $E' = (B + G)C_G + C_R R$, or $E' = C_R R$, since $C_G = 0$; in the circuit BR, $E = BC_B + RC_R = (B + R)C_R$, hence $E' = E\frac{R}{B + R}$.

111. Electrometers.

Apparatus. A Thomson's quadrant electrometer, which should be mounted like the galvanometer, with a lamp and scale in front of it. Two or three cells of a Clark's battery of constant electromotive force (Experiment 104), a Daniell's cell, some resistances, and several cells of a water-battery. The latter consists of small glass vials containing salt water, in which are strips of copper and zinc soldered together, so that the zinc of one cell is joined to the copper of the next.

Experiment. The principle of the Thomson electrometer is described in Appendix *A*. If the more complex form is employed, it should be carefully adjusted, as described in the pamphlet accompanying it, and will then, with care, require but little attention. When charged by exactly the right amount, the little aluminum balance in the upper part of the instrument is in equilibrium, so that on looking through the lens the horizontal hair is midway between the two dots. If too high, the handle of the replenisher must be turned in the direction of the hands of a watch until the hair rises to its proper position. If too low, it must be turned in the opposite direction. As the balance is liable to adhere to the stops limiting its movement, the glass above it should be gently tapped with the finger. Next, light the lamp and see if the image of the slit and vertical hair falls on the zero of the scale. If not they must be brought there by turning the screw, moving the fourth quadrant, and, if necessary, the other quadrants. To measure a slight difference of potential, as that of two metals

immersed in water, connect them with the studs of the key, the two springs being attached to the terminals of the galvanometer. On pressing down the key, the spot of light will be deflected, and on reversing the key an equal deflection to the other side will be attained. The instrument is intended to be so adjusted that a deflection of 100 scale divisions will correspond to a difference in electromotive force of 1 volt. To measure large differences of potential, one of the electrodes should be drawn up from the quadrant beneath it, and remain in connection only with the induction plate of the instrument. If this alters the position of the spot it shows that a charge has been thereby induced, which must be got rid of by connecting the quadrant with the earth. For this purpose the milled head of the disinsulator behind the instrument should be turned until the attached pin points to the letter "*C*" (connect). The spot will thus be brought back, and the quadrant is again insulated by turning the pin to "*D*" (disconnect). Differences of potential of 100 volts will thus be kept within the limits of the scale.

The simpler form of electrometer requires to be recharged every day it is used, and it will not give the same deflection on different days for the same differences of potential. Like the other electrometer, a lamp and scale is placed in front of it, so that the spot of light shall fall at the zero of the scale. It must next be charged by removing the glass cover and connecting the brass knob projecting from the interior of the little Leyden jar with an electrophorus, plate machine, Holtz' machine, or other source of positive electricity. Care must be taken to make a connection between the ground and the outside of the jar or the electrometer, as otherwise if the latter stands on a hard wood table it may not receive a proper charge. If charged too strongly the needle will swing out so as to touch one of the quadrants and discharge itself. When properly charged replace the cover and see if the needle remains at zero. If not, the movable quadrant should be drawn in or out until this condition is attained. Its terminals are then connected with the studs of the key, and potentials measured as with the other instrument. As it is impossible to charge it twice alike, and as there is no easy means of altering its charge, the deflections, as stated above, are not

comparable with one another, and for a given difference of potential will gradually become less and less.

The instrument having been adjusted, so that the spot stands at zero, and deviates equally to either side, when the current is reversed by the key, a number of measurements of differences of potential should then be taken. First, connect the terminals with a Clark's cell which has a electromotive force of 1.457 volts. With the absolute electrometer this should give a deflection of 145.7 divisions, and with the other instrument it should give a deflection from which the constant, or deflection per volt, is readily determined. Do the same with a second Clark's cell and then connect them, and see if together they give a double deflection. Next, measure the electromotive force of the Daniell, and other batteries, first, when they have been left on an open circuit and then when the circuit has been closed for some time. The polarization of a single fluid battery is thus well shown. Many simple, but instructive, experiments may be performed with this instrument. For instance, it may be shown that a zinc and copper plate when immersed in water assume a difference of potential before they are connected together, and that on connecting the terminals of a battery by a long wire, the potential of the various parts will vary by an amount proportional to the change in resistance, or that the curve formed by the potentials and resistances is a straight line. Again, the electrometer may be used like a galvanometer, except that the circuit through it is always open, instead of closed, and we may thus approximately measure resistances with a Wheatstone's bridge, or determine battery resistance. It forms, in fact, a galvanometer of infinite resistance.

Another interesting application of the electrometer is to the examination of condensers. The relative capacity of two condensers may be found by charging one, measuring its potential, and then connecting it with the other so that the charge will be divided between them, when the potential will be reduced in the same proportion that the capacity is increased. Again, if a condenser is charged and connected with the electrometer, as the electricity gradually escapes the deflection will diminish. The flow being always proportional to the electromotive force, by Ohm's law, if a curve is constructed with abscissas equal to the time and ordi-

nates to the logarithm of the deflection, it will give a straight line. The tangent of the angle, which this line makes with the axis of X or the change in the logarithm per second, gives the logarithm of the rate of diminution of the current per second. Calling this quantity a, and c the capacity of the condenser, the leakage current through the condenser will evidently be Eac, or since by Ohm's law $E = CR$, $C = CRac$, or $R = \frac{1}{ac}$, R is here the insulation of the condenser, and if the latter is in good condition will be a very large quantity. This is one of the best methods of measuring a very large resistance. It is only necessary to measure R and then connect the terminals by the unknown resistance r and measure again, when the combined resistance R' will equal $\frac{Rr}{R+r}$, from which r is readily deduced. Instead of the electrometer, a Thomson's galvanometer may be used, first charging the condenser for 10 seconds, then disconnecting it for one minute, and finally discharging it through the galvanometer. In this case the following formula is more convenient for determining the resistance. Let d be the deflection when the condenser is discharged directly through the galvanometer, and d' the deflection when an interval of one minute is allowed to elapse. Then $R = \frac{1563.6}{c(\log d - \log d')}$, which gives R in megohms.

112. Testing Telegraphs.

Apparatus. A telegraph line, the longer the better, but at least passing to another building and returning through the ground instead of by a second wire. If no telegraph is available, any long circuit may be employed, as that of an electric clock, or electric bell. With this is needed the apparatus described above for measuring currents, resistances and potentials.

Experiment. This, and the following Experiment, are intended principally as examples of the previous work, and practical applications of the methods of measurement there detailed.

First, remove the battery and connect the wires attached to it, and then determine the resistance of the line, magnets, and other parts of the apparatus, by the methods given above. For this

purpose, run wires from the ends of the line to the apparatus for measurement, find their resistance alone, and when connected with the line; the difference equals the resistance of the latter. Measure in like manner the resistance of each magnet by taking the difference of the resistances when it is in, and when out of, the circuit, the last condition being obtained by bringing its two terminals together. Next, measure the insulation of the line by breaking the circuit at the further end and measuring the resistance between the nearer end and the ground. This resistance should be enormously great, unless the line is very long, and should be measured by the methods given for determining very great resistances, Experiment 102.

The next question is, what kind of battery must be used to give the best effect. To test at any time the condition of the battery and line, a galvanometer should be employed, which may be interposed in the circuit and the deflection noted. The galvanometer used in Experiment 98 may be employed, but it is better to use a less accurate instrument, with the needle on a pivot, instead of suspended by a filament of silk, as it is then less likely to be injured in moving. It is not necessary that it should follow the law of the tangents, but the current corresponding to various deflections should be determined by placing it in the same circuit with a galvanometer, for which the curve of Experiment 98 has been constructed, and the current altered by varying the resistance. A curve may then be drawn, in which ordinates shall equal its deflection and abscissas the absolute current, as determined by the curve of the other galvanometer.

Connect the galvanometer with the line, and attach a battery somewhat more powerful than that which is to be used permanently. Now reduce the current by introducing additional resistances, or by shunting the battery, until it is just sufficient to make the magnets act properly. Then read the galvanometer, and from the curve determine the strength of current. This gives a minimum, below which the current must not fall. Next, alter the resistance so that the current shall have such a strength as to give the best effect. We must now see what battery will best give this current. In the equation $C = \frac{E}{B + R}$, or $E = C(B + R)$,

substitute this value for C, and make R equal to the total resistance of the line and magnets. Then the battery must have such an electromotive force and resistance that it will satisfy this equation. If, as is generally the case, we are to use several cells of resistance B and electromotive force E, we must use the formula for several cells explained in Appendix A, $C = \frac{mpF}{pB + mPC}$. Since the best effect is produced when the inside and outside resistances are equal, we must have $\frac{pB}{m} = P$. Combining these two equations, we deduce m and p. It should be remembered that while the first cost of a battery is proportional to the number of cells, or to mp, the current expenses or consumption of zinc or copper is proportional only to p. Other considerations also enter in the selection of a battery, according as it is to be used on an open or closed circuit, as detailed on page 5.

Having thus tested the circuit in its normal condition, if at any time it will not work properly, the nature of the trouble may be detected by similar measurements. First, test the battery, and see if this gives a good current when disconnected from the line, or better, measure its electromotive force and resistance. If this is what it should be, measure the resistance of the line and its insulation. If the line is broken, it is shown by the resistance becoming infinite. Imperfect connections are also shown by a great increase of the line resistance. If there is a ground, that is, any part of the wire in contact with the earth, the insulation and line resistance will become equal, and both less than the normal line resistance. The position of a fault may also be approximately found in this case. If the connection with the ground is only partial, these resistances will be unequal. A defect in any magnet is shown by first throwing it out of, and then into, the circuit, a great increase of resistance being produced if the wire is broken, while if there is a defect in the insulation, so that the current passes across, instead of through, the whole coil, the resistance will be less than when the magnet is uninjured. During damp weather the supports insulating the line become covered with moisture, and greatly diminish the insulation. If the wire comes in contact with the wire of another line, the messages of the latter will be received on it, though generally feebly. This fault is

shown by deflections of the galvanometer when no battery is attached, and by a diminished resistance when the other line is not in operation. On long lines trouble is sometimes experienced from earth currents, in which the two terminals assume different potentials, the earth acting precisely like a battery. This is especially the case during displays of the aurora borealis. It is shown by deflections of the needle when no battery is attached, the currents coming without the regularity of those produced by a cross with another line.

113. Testing Submarine Cables.

Apparatus. Since a real submarine cable is rarely available for experimental purposes, an artificial cable may be prepared as follows. Two points are selected for the two terminals, and between them is placed a vessel of salt water to represent the ocean. The cable is represented by two coils of known length of fine German silver covered wire, of two or three thousand ohms resistance each. A long coil or rubber covered wire is needed, and three of four shorter pieces, prepared to show the effect of various faults, in one the wire being broken, in a second the rubber scraped off at a single point, and in a third the wire being broken, but the rubber left intact. Two very large condensers should be provided, and this is the greatest difficulty in imitating a cable. For a battery, several Daniell cells are needed, and a water battery of one or two hundred cells.

Experiment. To represent the cable when in good condition, the resistances are connected together, and to the coil of covered wire which is then immersed in the vessel of salt water. One end is then attached to each terminal station, and copper wires, to represent the ground connections, also pass from the vessel of water to the same terminals. The condensers are connected with the junctions of the resistance coils, and also with wires passing into the vessel of salt water. Measure the total resistance of the line by closing the circuit at the further station, that is, joining the wire from the vessel of water to the end of the resistance coils, and determine the resistance by the bridge, as in Experiment 104, by connecting the wires of the nearer station with the bridge. It is especially important in this case to close the circuit through the galvanometer, as otherwise the current from the condenser will throw a violent current through the galvanometer. An error is

liable to be introduced from the polarization of the wires in the salt water, and therefore it is best to use a battery with large electromotive force, or to measure the resistance at short intervals, reversing the current each time.

Next, to determine the insulation, break the circuit at the further end and measure the current, which will now pass through the rubber covered wire. The resistance will, in this case, be too great to be determined by the methods of Experiment 102, unless the insulation is very poor, or the length of wire very great. Instead, therefore, the method given at the close of Experiment 111 should be tried. Another method employed to measure very great resistances, as that of the junction of two cables, is to allow the leakage current to flow into a condenser for one minute, and then discharge it through a galvanometer.

The capacity of the cable is determined precisely as if it was a condenser, the inner wire and outer covering, or the sea, replacing the two conducting coatings of tin-foil, and the rubber insulator replacing the insulating film of paper or mica. In the present case, determine the capacity as in Experiment 104. These three tests should be frequently applied to every cable, and as long as they give nearly the same results we may infer that the cable is in good condition.

Now suppose an accident occurs, by which the cable is injured, and that we wish to determine the kind and position of the fault, as it is called. This is imitated by disconnecting the resistance coils and inserting between them one of the rubber covered wires. When the broken cable is inserted, the resistance is diminished, and is the same, whether the circuit is made or broken at the further end. The position of the fault is found from the ratio of the resistances; the actual distance in miles is thus determined by a simple proportion. See how nearly this compares with the true length of the resistance coils. Precisely the same effect is produced with the second kind of fault in which the wire is exposed, but not broken, the resistance of the water being inconsiderable compared with that of the remainder of the cable. It is therefore easy to determine the position of a fault if the wire is in contact with the water. But it very frequently happens that this is partially protected by the covering, by salts deposited by the

current, or other causes, so that the current, in passing from the wire to the water, encounters a certain resistance called the resistance of the fault. This may be represented by leaving the two coils connected and interposing another coil, which may have any resistance from zero to infinity, between the broken wire just used and the junction of the two coils. If the position is now measured, as described above, too great a result will be obtained, but if intelligible signals can be sent to the further end, directing those in charge to first break, and then close their circuit, two measures may be obtained from which the true distance and resistance of the fault may be approximately determined. Call R, R' the resistances when the circuit is open and closed, f the resistance of the fault, l the resistance of the whole cable, and x that of the portion this side of the fault. Then when the circuit at the farther station is broken, $R = x + f$, while when closed the current divides between the two circuits, f and $l - x$, hence $R' = x + \frac{f(l-x)}{f+l-x}$. These two resistances give f and x, and from the latter the distance of the fault is at once determined. The polarization interferes seriously with this measurement, and therefore, if possible, a second cable should be used instead of the return circuit through the water. In all these cases the insulation resistance is supposed to be infinitely great as compared with that of the fault, otherwise other corrections are necessary.

A fault due to the breaking of the copper wire without injuring the insulating cover, is comparatively rare, and is illustrated by the third piece of rubber covered wire. Its effect is to introduce a very great resistance, which is unchanged, whether the circuit is open or closed. The position of such a fault cannot be very accurately determined. It may be roughly estimated from the insulation resistance, which is as much greater as the length is less. The method actually employed, however, is to compare the capacity of the unbroken portion with that of the whole, regarding them as condensers.

114. Frictional Electricity.

Apparatus. A plate electrical machine, a Leyden jar, some sealing wax, a glass lamp chimney, pithballs, a gold-leaf electroscope, a torsion electrometer, and the usual lecture-room apparatus for frictional electricity described below.

Experiment. Rub the glass chimney on a piece of silk, when some of the electricity will pass from the silk into the glass. The latter therefore becomes positively, the silk negatively, electrified. Now hang a pithball by a thread of silk, and bring the glass near it. The pithball has appreciable size, and has the same potential as the air, the glass a higher potential; therefore attraction takes place, until the ball strikes the glass, when it receives part of the excess of electricity, and both now being positively electrified repulsion takes place. Next, rub the wax with a piece of woollen and the electricity will pass from the wax to the woollen. If, then, the wax, which is negative, is brought near the pithball which is positive, they will attract. If, however, the pithball touches the wax it gives up its excess to the wax, and both being then negatively electrified, will repel each other. If a piece of metal is used instead of the wax, no effect is apparent if the metal is held in the hand. But this is because, being a conductor, the surplus electricity passes through it to the hand, and thus escapes. If the metal is insulated by a glass handle the electricity can no longer escape, and the above effects are easily obtained.

To determine whether an electrified body is charged positively or negatively, a gold-leaf electroscope may be employed. This is easily made of a wide-mouthed bottle, closed by a cork, through which passes a brass rod, terminating above in a ball or knob, and from whose lower end two strips of gold-leaf are hung. When the brass rod is electrified, these strips repel each other, and separate at their lower ends. Two strips of tin foil are attached to the bottle, so that if the gold strips are too strongly charged, instead of adhering to the glass they will strike the foil and discharge themselves. To use this instrument, bring the body to be tested near the upper knob, and the gold strips will diverge, the electricity of the knob passing into the strips, if the body is positive, and from the strips to the knob, if it is negative. Touch the knob for an instant, when the strips will come together; then remove the electrified body, when they will again diverge. Now approaching an electrified glass rod, if the body was positively electrified the divergence will be increased, if negative, diminished. Test in this way various substances, rubbing them together and determining which is positive, and which negative.

A far more exact instrument than this, is Coulomb's torsion electrometer, which consists of a cylindrical glass case, in which a straw with a disk of tin foil at one end is hung horizontally by a long, fine wire. The upper end of the wire is attached to an index passing over a graduated circle, which shows the angle through which it has been twisted. An insulated rod passes into the interior, so that on turning the index the tin foil may be brought in contact with it. A graduation outside the glass shows the angle through which the straw has been deviated. Turn the index so that the tin foil and ball shall be just in contact. Electrify a glass rod and touch it to them, when they will at once repel each other, and the straw will swing off through an angle which we will call a. Bring them nearer by turning the index through an angle u and call v, the deflection of the straw. Give u various values, and determine v in each case. If v is small the distance will be proportional to it, and the force of repulsion to the torsion, $u + v$. Assuming that the latter is inversely proportional to some power of the former, we must have $(u + v)v^n = m$. To see if this is the case, construct a curve with coördinates equal to log $(u + v)$ and log v, and it should be a straight line, since $\log (u + v) + n \log v = \log m$. Again, the tangent of the angle, or n, should equal 2, since the force is inversely as the square of the distance. If v is not small, the distance must be taken proportional to sin $\frac{1}{2} v$, or to the chord instead of the arc.

To show the unequal distribution of the electricity on different parts of a conductor, a proof plane is required. This consists of a small piece of silvered paper, at the end of a fine glass rod covered with shellac. To use it, the electrometer is discharged, the straw brought to zero, the proof plane touched to the points to be tested, and the electricity thus removed, transferred to the knob of the electrometer. A deflection is then obtained, which will be proportional to the cube of the amount of electricity of the given point. Charge several conductors, as an ellipsoid, an elongated cylinder and a circular disk, by rubbing a glass rod and touching it to them, and measure the amount of electricity of several points of each.

The electricity resides entirely on the surface of a body. This may be shown by a hollow sphere with a hole in it. Passing the

proof plane in, touching the interior and then withdrawing it, taking care not to touch the edge, it will be found that no electricity is withdrawn, however highly the sphere is charged. If a second sphere of the same size, but solid, is allowed to touch the first, it will also be found that the electricity will be divided equally between them each taking one half of that on the first sphere before contact.

The quantity of electricity obtained as described above, is exceedingly small; it may be greatly increased by the use of the plate electrical machine. This consists of a circular plate of glass, which may be turned between two pieces of felt covered with an amalgam of mercury, zinc and tin. An excess of electricity then passes into the glass, which thus becomes positively electrified, while the felt or rubber is negatively electrified. A comb of metallic points is placed opposite the glass, and draws off its surplus electricity into a large brass cylinder, called a prime conductor. The latter is supported on a glass pillar to prevent the escape of the electricity to the ground. On turning the plate the action soon ceases, because the rubber gives up so much of its electricity that no further supply can be taken from it. It should therefore be connected by a chain with the earth, from which an indefinite amount of electricity is readily drawn. To use the machine, it is only necessary to connect the rubber with a gas or water pipe by a chain, and turn the plate by a crank attached to it. Electricity will then appear on the prime conductor, which will soon attain so high a potential that if the finger, or other conductor, is brought near, the electricity will at once overcome the resistance and leap across in the form of a spark.

When the machine has not been used for some time, or if the air is moist, it is often difficult at first to obtain electrical effects. In this case the machine should be carefully dusted and warmed, as if very cold, dew may be deposited on it, which will form a conducting surface, over which the electricity will escape rapidly. Again, the amalgam may not be in good condition, and in this case the rubber should be removed, the surface roughened by scraping it with a knife, and, if necessary, fresh amalgam mixed with lard applied.

When the machine is in good condition the sparks should follow each other rapidly, and if there is no outlet for the electricity, a peculiar hissing sound should be produced, due to the escape of the electricity into the air. In a darkened room pale brushes of purple light should appear on various parts of the machine.

The phenomena of attraction and repulsion are much better shown by the electrical machine than by the simple means described above. Pieces of paper or pith are violently attracted and then repelled. Various electrical toys have been devised to show these effects, for instance, bells, dancing dolls, the spider, head of hair, etc. A curious effect, known as philosopher's wool, is obtained by attaching a little sealing wax to a rod projecting from the prime conductor and melting it with a candle. As soon as the machine is charged the mutual repulsion causes the wax to throw out fine filaments, which may be collected on a sheet of paper held near it. By electrifying the water contained in a vessel pierced with a number of fine holes, it will escape in fine streams instead of in drops. A similar effect is obtained with a syphon formed of a capillary glass tube. This instrument has a most important practical application in Thomson's syphon-recorder for registering messages received on submarine cables.

Owing to the force of repulsion, the excess of electricity in a body instantly passes to the surface. For the same reason it collects in greatest quantity on the more curved portions. In electrical apparatus sharp edges or points are therefore particularly objectionable, since the electricity collects on them and escapes more rapidly into the air. The adjacent particles of air becoming electrified are repelled, and form a current from the point. This is shown by attaching a pointed wire to the prime conductor when the current may be perceived by the hand, or by holding the flame of a candle near it. The electrical flier consists of a wire with the two ends bent in opposite directions, like an S, and balanced like a compass-needle on a pivot. When electrified, it will revolve rapidly, owing to the reaction of the air on the points, like a Barker's mill. On viewing a point strongly electrified in a darkened room, the escape of electricity is readily seen by the production of a purplish brush of light. If the point is electrified

negatively, the brush is reduced to a simple bright point, although the escape of electricity is considerably increased.

If the electricity is allowed to pass through a tube from which the air is partially exhausted, the spark lengthens, and finally forms a long purple brush-like discharge, resembling the aurora borealis. A certain amount of gas, however, seems essential, as with the highest attainable exhaustion no electricity will pass.

The uses of the electrical machine are greatly extended by the instrument known as the Leyden jar. This is a condenser formed of a glass bottle coated inside and out with tin-foil and closed by a wooden stopper, through which passes a brass rod from which hangs a chain touching the interior of the jar. To charge it, hold the brass rod, which commonly terminates in a ball, near the prime conductor, and connect the outer coating with the earth, or with the rubber. On turning the machine, the positive electricity will collect on the interior of the jar, and repelling that on the outer coating will cause it to pass off into the earth. This will go on until a considerable quantity of electricity is thus stored up in the jar. Then connecting the inner and outer coatings, or the latter with the brass ball, the whole of the electricity thus accumulated instantly passes out with a bright spark and loud snap. If the discharge is through the body, a violent shock will be felt.

To show that it is indispensible that an outlet shall be afforded to the electricity on the outer coating of the jar, place the latter on a plate of glass and try to charge it. In this case the outer coating will become charged and give sparks, like the prime conductor, while but little electricity will enter the jar, as is proved by connecting the outer and inner coatings. The electricity does not reside in the coatings but on the surface of the glass, as may be shown by means of a jar with movable coatings. A cylindrical or conical vessel is used for this purpose, the tin-foil being replaced by closely fitting tin cups. Charge the jar in the usual manner, then remove the outer coating, place it on the table, or better on a sheet of glass, and remove the inner coating. Now place another inner coating in it, and finally replace it in a second outer coating, taking care during the last operation not to touch the jar. The latter will be found to be still quite strongly

charged. Another evidence that the charge is in the glass, and not in the metal is, that a few minutes after the jar is discharged a second feeble spark may be drawn from it, due to the electricity which has penetrated a little way into the glass. This is known as the residual charge.

The powerful sparks of a Leyden jar are capable of producing many effects not readily obtained directly from a machine. This is especially the case with a battery composed of several jars having their inner and outer coatings connected, equivalent in fact to a single, very large jar. A much longer time is required to charge such a battery than a single jar and the spark although no longer, will be much brighter and more intense. It resembles, in fact, a galvanic battery connected for quantity. Remarkable effects may be obtained by connecting the outer coating of one jar with the inner coating of the next, like a galvanic battery connected for tension. Very long sparks are thus obtained, but the jars should be disconnected and charged separately.

The simplest way to discharge a Leyden jar or battery is by a wire bent in the form of a semicircle, and terminating in brass balls. To avoid receiving any portion of the discharge the wire should be held by a glass insulating handle. Sometimes the wire of the discharger is jointed, so as to vary the distance between the balls. The best instrument for studying the effects of the spark is the universal discharger, which consists of a small insulated table and two brass insulated rods mounted on universal joints, so that they may be brought into any position with regard to one another. The body to be submitted to the spark is placed between them on the table, and they are then brought in contact with it, one being connected with the outer coating of the jar or battery, and the other with a wire which is connected with the inner coating when the discharge is to be effected. If a spark is passed through a thick piece of paper or cardboard, a hole is made with a burr on each side, which was formerly considered an evidence of two electric fluids, but is probably due to the sudden generation of steam, or other explosive action, inside the paper. A plate of glass is readily penetrated by the spark, if the action is concentrated by surrounding the wire with some non-conductor, except just at the end. The best way is to fill a bottle with oil and pass a wire into

it so that it shall touch the glass; bringing a second wire near it on the outside, the spark will pass, producing a hole often too small to let the oil escape. With a powerful charge, however, the bottle may be broken. Alcohol, cotton covered with resin, ether and gas, are readily ignited by an electric spark. The spark generally scatters gunpowder without firing it, but the latter may be effected by lengthening the time of the discharge by introducing into the circuit a large resistance, as a wetted string. To show the magnetizing power of the current, wind a wire in the form of a helix, place a steel needle in the interior, discharge a powerful battery through it, and the needle will be rendered magnetic.

In the above description, we have assumed that the interior of the jar is electrified positively, the exterior, negatively. It is then said to be charged positively. The same effects may, however, be produced by reversing these electrical conditions, or charging the jar negatively. For this purpose it is insulated, and the exterior connected with the prime conductor, and the interior with the rubber. The difference in the two cases is well shown by the experiment known as Lichtenberg's figures. Charge two jars, one positively, the other negatively, and draw a series of lines with the knob of each, on a flat surface of resin or vulcanite. Then mix some red lead and sulphur, and sift them over it. The sulphur in mixing becomes negative, and adheres to the positive lines in tufts with spreading branches, while the lead, which is positive, collects in small round spots on the negative lines.

To measure the amount of electricity generated by a machine the unit jar is sometimes used. This consists of a small Leyden jar, which is connected with the prime conductor, and a wire attached to the outer coating so bent that it nearly touches the rod connected with the inner coating. If, now a continuous stream of electricity is allowed to pass into the jar, it will discharge itself at regular intervals whenever the potential of the interior becomes sufficient to enable the electricity to leap across the interval to the outer coating. To measure by the unit jar the amount of electricity generated by the machine, connect the inner coating with the prime conductor, and the outer with the rubber. The number of discharges per hundred turns serves to compare the efficiency of the machine at various times. To deter-

mine how much electricity has passed into a battery, insulate the unit jar and connect its inner coating with the prime conductor, and the outer coating with the battery. The outer coating of the latter is, of course, connected with the ground, or rubber. The number of discharges, as before, measures the quantity of electricity.

If pieces of tin foil are attached to a sheet of glass at short distances apart, a spark will pass from each to the next over a long series, and by a suitable arrangement of the foil, letters or figures of light may be thus formed. By scattering iron filings on a glass plate wet with gum, and when dry discharging a jar over the surface, the electricity passes from point to point in irregular branching lines, somewhat resembling lightning.

115. Induction Machines.

Apparatus. An electrophorus, a piece of fur, a Holtz machine, and a piece of vulcanite.

Experiment. The electrophorus consists of a thin disc of some insulating material, generally resin or vulcanite, resting on a metallic disc connected with the earth. A second metallic disk with a glass handle may be laid on it, and removed at will. To use the electrophorus, rub the upper surface of the resin with the fur, by which the latter is charged positively, the former negatively, or some electricity is transferred from the disk to the fur. Replacing the metal disk, its electricity rushes down towards the resin, but cannot enter, owing to the slight conductivity of the latter. The disk now becomes positively electrified on the lower surface, and negatively electrified on the upper surface. Therefore on touching it with the finger, a spark will be formed, by the electricity entering it from the hand. But now the upper surface is in its normal condition, and the lower surface still positively electrified. If, therefore, the disk is raised by the insulating handle it will be found to contain more than its normal amount of electricity, or to be positively electrified, and on touching it a spark will be obtained. By this operation the electrical condition of the resin has been in no way altered; it may therefore be repeated indefinitely without recharging, laying the disk on the resin, touching it with

the finger, lifting the disk, and approaching it to the object to be charged. This instrument is often very convenient as a source of electricity, from its simplicity and the ease with which it is used.

The Holtz machine consists of two plates of glass, one of which is very thin and may be made to revolve rapidly, by a system of belts and wheels driven by a crank. The second plate is somewhat larger than the first, and is placed as near it as possible. Two apertures are cut in the second plate, and pieces of paper, called armatures, glued to the further side. These terminate in points which project over the apertures, so that when electrified they will act by induction on the revolving plate. On the other side of the latter, but opposite the points, are combs of points, like those of a frictional machine, connected with brass rods and balls, whose distance may be varied at will, and between which the spark is to pass. A Leyden jar is hung on each of these rods so that its inner coating is connected with the rod, and the two outer coatings are united by a metallic conductor. To charge the machine, the two brass balls are brought in contact, the movable plate turned rapidly, and a small electric charge given to one of the armatures. This is readily done by rubbing a piece of vulcanite with fur, and touching it to the armature, or by an electrophorus. Soon an increased resistance will be felt to the motion of the crank, accompanied by a sort of hissing noise, and on separating the balls a volley of sparks will pass, of a length which may reach a foot or over. The machine, as thus constructed, is liable to stop working suddenly, and requires recharging each time it is used. These difficulties are remedied by a second pair of combs connected together by a brass rod, placed just opposite the edge of the armatures to which the points are not attached. The amount of electricity generated by the Holtz machine is about the same per turn as that of the frictional machine of the same size, but since the speed is much greater, much longer sparks, and more electricity per second is obtained, and the labor of turning it is much less. It has, accordingly, almost superseded the plate machine as a source of frictional electricity. Most of the experiments described in connection with the plate machine may be shown much more satisfactorily with the Holtz machine.

If the condensers are removed the sparks are more frequent but less brilliant, and are accompanied by a sort of brush discharge. By increasing the size of the jars a shorter, but much more intense spark is produced, giving a snap, in some cases almost as loud as the report of a pistol. The best effect is obtained with the condenser attached to the negative pole double the size of the other, and the ball forming the negative terminal also larger than that attached to the positive terminal.

116. Magnetism.

Apparatus. Some magnets and needles, a stand to which a fine thread with a wire stirrup may be attached, soft iron armatures, a piece of cardboard, some iron filings, and two cylinders of wood or cardboard on which two arrows are painted, to represent Ampère's currents.

Experiment. According to the theory of Ampère, magnetic phenomena are due to electric currents circulating around the particles of iron, and the attractions and repulsions are caused by the effect of these currents on each other. Hold the two wooden cylinders end to end, and notice that if the *N* or *S* poles are brought together, the currents move in opposite directions, and hence repel, while if turned so that an *N* and *S* pole are brought together, the currents move in the same direction, and attract; this is sometimes expressed by saying that like poles repel, and unlike, attract. To prove that this is the case with real magnets, place a bar magnet in the stirrup and hang it from the stand; bring the other bar magnet near it and see if the above law holds in all four cases. The earth also acts like a large magnet with its south pole to the north, and hence the suspended magnet will come to rest, only when its north pole is turned to the north. This is the principle of the mariner's compass. When a piece of soft iron is brought near a magnet, induction takes place, and the iron becomes temporarily a magnet with all its currents flowing in the same direction, but as soon as the magnet is withdrawn the currents turn back, and the magnetism ceases. To show this, bring a magnet near a piece of soft iron, when it at once becomes magnetic, and will attract a second piece of soft iron, and sustain

its weight, if the magnet is strong. On removing the magnet the second piece of iron at once falls. The same effect is still better shown by letting the soft iron deflect a compass needle.

Lay a bar magnet on the table, and the sheet of cardboard over it, supporting the sides so that the card shall be level. Then sprinkle over it some iron filings, and tap gently on the edge of the card. The particles will arrange themselves along certain lines, called magnetic curves, extending from one pole of the magnet to the other. The reason is, that each particle is rendered magnetic by induction, and the direction of the curves is that which a magnetic needle would assume at that point under the influence of the two poles of the magnet. By placing a second magnet on a piece of soft iron near the first, other magnetic curves may be formed. The object of tapping the card is to neutralize the friction and enable the particles to assume the positions they would take if perfectly free to move. The curves may be rendered permanent by using waxed paper instead of cardboard, forming them as before, and holding a hot piece of metal just above them, when the wax will melt and hold the filings in place.

117. Making Magnets.

Apparatus. Some good permanent magnets, and some short bars of hardened steel, such as pieces of stout knitting needles about two inches long. They should be hardened by heating to redness, and letting them cool quickly, then drawing the temper by heat till they acquire a violet straw color. A stand is needed from which the magnet may be suspended by a filament of silk to test its strength, and a glass shade to cut off currents of air.

Experiment. The larger the piece of steel the more difficult is it to magnetize it to saturation. Common needles, or the small pieces of watch-spring used in galvanometers are easily charged by merely rubbing the end that is to be north, on the south pole of a permanent magnet about a dozen times, and the other end the same number of times on the north pole. For larger bars much more care must be taken, several methods of rubbing the bar having been proposed, some of which will be described below. To test the magnetism imparted, the magnet must be suspended freely, as described in Vol. I, Experiment 3. If too heavy to be

5

supported by a single filament of silk, a bundle of several must be employed, taking care that they are not twisted. To determine whether the bar is already magnetized, suspend it, cover it with the glass shade, and see if either end points to the north, and if when disturbed, it vibrates, and finally returns to its original position. If so, measure the time of a number of these vibrations, find by division the time of a single vibration, and take the reciprocal of its square. This gives a measure of the strength of the magnetism or more strictly of the magnetic moment. Remove the magnetism by rubbing the north end once or twice on the north pole of a magnet, and the south end on the south pole. Suspend it again, and see if the time is increased. If rubbed too much, the polarity will be reversed, and the other end will now point north. Repeat until the magnetism is nearly removed, and the time of vibration is very great. Then magnetize by one of the following methods, and again take the time of vibration. Remove the magnetism, by turning the bar end for end, and repeating, see if the time can again be rendered very great. Do the same with the other methods of magnetizing. Finally, compare the results, and see in which way the strongest magnetism can be induced.

The first method to be described is known as that of *single touch.* The bar to be magnetized is fastened to the table, which is best done by placing its ends on the opposite poles of two permanent magnets, the end which is to be north against a south pole, and *vice versa*; it is well to mark one end of the bar, to show which is north. Now bring two permanent magnets down over the centre of the bar, not quite touching each other, with unlike poles together and inclined outwards so that each shall be inclined about 15° to the horizontal. To prevent their touching, it is well to lay a piece of wood on the centre of the bar. Now draw them apart, letting them slide over the bar until they reach the ends, then raise them and bring them back through the air to a point over the centre and then down into their former position; repeat several times, then turn the bar over, and stroke the other side in the same way. Of course, the north end of the bar must be stroked by the south pole of the magnet, and the south end by the north pole.

By the method of double touch, the two magnets are held vertically, separated by a bit of wood, and brought down onto the centre of the bar. They are then drawn together to one end of the bar, and back to the other end, and thus backwards and forwards taking care to stop in the middle after stroking each end an equal number of times. A horse-shoe magnet is particularly convenient for this purpose.

A third method of making horse-shoe magnets, proposed by Jacobi, consists in laying its poles against those of two permanet magnets, and drawing a piece of soft iron over it from end to end.

A still more effective method is to place pieces of soft iron against its ends, and enclose the whole in a helix of insulated copper wire through which a powerful current of electricity is circulating, making the whole in fact an electro-magnet. In the other methods the effect is much improved by using electro-magnets instead of permanent magnets.

118. Force of Magnets.

Apparatus. In Fig. 83, ABC is a small steelyard with a rider of such a weight that each division of the arm shall correspond to one tenth of a gramme. Two pins limit the motion so that it shall only rise or fall by a small amount. D is a soft iron bar, hung a short distance above the magnet to be tested, E. The latter rests on a board hinged at G, and which may be raised or lowered by the micrometer screw F. The pitch of the latter should be somewhat over a millimetre, as, for instance, a twentieth of an inch.

Experiment. In the practical application of magnets it is often important to know the amount of attraction at various distances. This is determined with precision by the following method. Remove the magnet, and set the rider so that the piece of soft iron shall be exactly balanced. Then replace the magnet and set the board FG under D, in such a position that the distance of the point under D from G shall be to the distance FG, in the proportion of

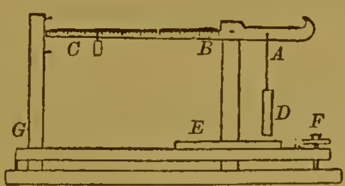


Fig. 83.

1 millimetre to the pitch of the screw. Thus, if the latter is $\frac{1}{20}''$, make $DG = .7874\ FG$. One turn of F will then produce a motion of the point of the board under D, of one millimetre. Turn F so as to raise the board until the magnet is in contact with D, and its weight removed from the steelyard, so as to bring C to the lower pin. Read the number of turns and fraction of a turn, then move F until the bar touches the upper stop, and read again. If the magnet is very powerful, a plate of glass may be placed over it, and the thickness added in measuring the distance of D, or D may be removed, the magnet raised, and then, after replacing D, lowered into the required position. An undue strain on the steelyard is thus avoided. Next, lower the board and move the rider towards the end of the arm one division, or more, if this is not sufficient to bring it against the lower pin. An attraction of the magnet of .1 gramme will then be required to bring the beam again into equilibrium. Turn F until the bar rises from the lower pin, and read F. Then turn it back, until the beam returns to the lower pin. Subtracting the readings just taken from these, gives two values (one for each pin) of the distance at which the force of attraction is .1 gramme. Take a series of readings with various positions of the rider, and read the position of F for each. Subtracting the first readings from them, gives the comparative values of the distances x, and forces of attraction y. To see if these quantities are connected by the relation $y = m\,x^n$, or if the force is proportional to any power of the distance, construct a curve with coördinates log y and log x, and if it forms a straight line, the tangent of the angle it makes with the axis of X gives the power n. This experiment may be used to study the best form of magnet for electro-magnetic engines, or for various other purposes.

119. Law of Magnets.

Apparatus. A compass resting on a scale divided into centimetres, and placed at right angles to the magnetic meridian, and a bar magnet.

Experiment. Remove the bar magnet to a considerable distance so that on turning it end for end, the position of the compass needle will not alter perceptibly. Place the compass over

the zero of the scale and turn it so that the needle shall point to zero. Now place the magnet at the further end of the scale with its centre an exact number of decimetres from the centre of the compass. Read the change in position of the compass needle, taking the mean of the two ends. Take a series of readings for various positions of the magnet, first with one pole and then the other, turned towards the compass. The tangent of the angle of deflection equals the ratio of the deflecting force of the magnet to the horizontal component of the earth's magnetism. Construct a curve with these tangents as ordinates, and the distances measured on the scale as abscissas.

If we assume that the effect of a magnet is the same as if its whole mass were concentrated at the two poles, the theoretical form of this curve is readily deduced. Let x equal the distance of the centre of the magnet, and d the distance between its two poles, which is somewhat less than its length, and y the corresponding force of attraction. This may be regarded as composed of two forces, one acting at a distance $x—d$, and the other, which is weaker, due to the further pole, at the distance $x+d$. These forces being inversely as the square of the distance, $y = \frac{a}{(x-d)^2} - \frac{a}{(x+d)^2} = \frac{4axd}{(x^2-d^2)^2}$. To see if any values of a and d will satisfy the observations, this equation must be reduced to a linear form. Solving with regard to (x^2-d^2), we have $(x^2-d^2) = \sqrt{4ad\frac{x}{y}}$, or calling $d^2 = m$ and $\sqrt{4ad} = n$, $x^2 = w$, and $\sqrt{\frac{x}{y}} = z$, $w - m = nz$, which is linear, or represents a straight line. Compute, therefore, for each observation $w = x^2$, and $z = \sqrt{\frac{x}{y}}$, and construct the curve. If the above assumption is correct it will become a straight line, and the point at which it cuts the axis of w will give $m = d^2$, or the square of the distance between the poles.

120. Distribution of Magnetism.

Apparatus. A long iron bar which can be rendered magnetic at will by two coils of coarse wire, C and D, Fig. 84, placed near its ends. A current is passed through the coils by a constant bat-

tery B, and may be sent through them in either direction by the commutator E. Soft iron cores are inserted in the coils, which thus render them powerful bar electromagnets. A thin coil of fine wire, A, slides over the long bar, and has its ends connected with a reflecting galvanometer G. Its position is measured by a millimetre scale.

Experiment. Remove one of the coils to a short distance from the bar, and draw its core out so that it shall have no effect on the coil A. Then make the circuit by the commutator, when a current will be suddenly induced in the long bar, and by the latter in A, thus deflecting the galvanometer. Read the extreme deviation of the spot of light, and after a few minutes break the circuit and read again.

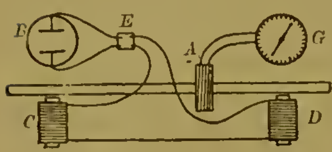


Fig. 84.

A second current will be induced, this time in the opposite direction. The magnitude of this deflection affords an excellent measure of the strength of the induced magnetism. Repeat the experiment, giving A various positions, and recording the deflection in each case. Construct a curve with ordinates equal to the galvanometer readings, and abscissas to the distance of the coil A from C. To make sure that the coils have no effect by their direct action, substitute for the long bar a glass tube, when the galvanometer needle should remain at rest. The theoretical form of the curve in the above experiment calling y the deflection and x the distance, is $y = ab^x$ or $\log y = \log a - x \log b$, so that constructing a second curve with ordinates equal to $\log y$ instead of y, we should obtain a straight line. Now replace the coil D, and passing the current through C and D in the same direction observe the deflection for various positions of A. Do the same with the current passing in opposite directions through the coils. Construct curves for both cases, also the curve midway between them. The latter is found by taking the mean of the ordinates of points having the same abscissa. The last curve will be found to be coincident with that obtained with a single coil. Moreover, if a curve is constructed with ordinates equal to the deviation of the two curves from their mean, and abscissas equal to the distances of the coil A from D, instead of C, we shall again obtain

the same result as with a single coil. We may therefore conclude that each coil will produce the same effect as if the other was not there. This method of studying the distribution of magnetism is widely applicable; the coils C and D may be placed directly on the bar if we repeat the experiment, using a glass tube instead of the iron bar, and subtract the deflections thus obtained, to eliminate the direct action of the coils. Again, if the coil A is placed on a permanent bar magnet and a soft iron armature withdrawn, a deflection is obtained, whose amount will vary with the position of A.

121. Magnetic Field.

Apparatus. A constant battery B, Fig. 85, a circular coil of wire, C, about half a metre in diameter, and a compass G. The needle of the latter is suspended by a filament of silk, and by an index is read to tenths of a degree. The coil is mounted so that its position with regard to the compass may be varied, by moving it either parallel or perpendicular to its own plane by an amount which may be measured by a millimetre scale.

Experiment. Set the compass so that the reading shall be zero, then place the coil so that the needle shall lie in its plane, and their centres coincide, and connect the terminals of C with the battery. A tangent galvanometer is thus formed, and the needle will be deviated by an angle whose tangent gives the strength of the magnetic field produced by the coil compared with that due to the earth's magnetism. Now move the coil in its own plane half a decimetre to one side, and repeat the reading. Take observations in this way at intervals of half a decimetre until the coil touches the compass, and then continue the readings with the compass outside the coil. Construct a curve with abscissas proportional to the distance of the compass from the centre of the coil, and ordinates to the tangent of the angle of deflection, or to the strength of the magnetic field.

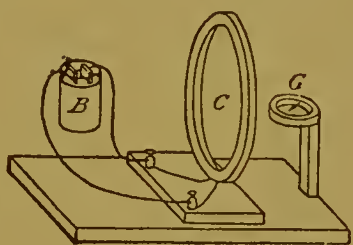


Fig. 85.

Replacing the compass at the centre, take a series of readings

of the deflection of the needle when the coil is moved perpendicular to its plane, a decimetre at a time. Construct a curve as before, and compare it with the result of the following theoretical considerations. Let y equal the radius of the coil, and x the perpendicular distance of the needle from its plane. Then the distance of any point of the coil from the compass, will be $(x^2+y^2)^{\frac{1}{2}}$ and the line connecting them will be inclined to the line connecting the needle and centre of the coil by an angle which we will call v. Then the effect of each element of the coil will be inversely proportional to the square of its distance, or to (x^2+y^2), and its component perpendicular to the coil, is the only one which will act, since the component in the plane of the coil is exactly neutralized by an equal and opposite component from the element of the coil distant from it 180°. Since, moreover, the total effect will be proportional to the number of elements, to $2\pi y$, or to y, we may write the strength of field $f = \frac{ay \sin v}{x^2+y^2} = \frac{ay^2}{(x^2+y^2)^{\frac{3}{2}}}$, substituting for sin v its value in terms of x and y, and calling a the strength of magnetic field at the centre of the coil where $x = 0$. Give proper values to a and y, compute f for various values of x, and construct a curve with f and x as coördinates. It should give the same result as that obtained by experiment.

If, in the above formula we make f a constant, we obtain $x^2+y^2 = by^{\frac{4}{3}}$, in which b is also a constant and equal to $a^{\frac{2}{3}}f^{-\frac{2}{3}}$. Suppose now a galvanometer constructed with a series or shell of coils of diameters and distances from the needle equal to the values of y and x taken from this equation. Then evidently all the coils will produce equal effects, all greater than that of any coil wound outside of this shell, and less than that of any coil inside of it. Accordingly, if the whole interior is filled with coils the greatest effect on the needle will be produced, or we shall have the greatest deflection for a given current, and the galvanometer constant will be reduced to a mininum. This equation therefore is important as giving the best shape for the coils of a delicate galvanometer.

HEAT.

---

122. Testing Thermometers.

Apparatus. The thermometer to be tested, and two tin vessels, one to contain melting snow, the other boiling water. The first of these, *AB*, Fig. 86, is cylindrical, terminating below in an inverted cone, with an orifice by which the water may escape. The second vessel, *ABC*, Fig. 87, is also cylindrical, and high enough for the bulb and stem of the thermometer to hang in the steam. The upper part should be double, so that the steam may pass up in the centre and down on the outside, otherwise the upper portion will cool off, and the thermometer reading be too low. If the tube is to be calibrated, reading microscopes, or the Dividing Engine, Vol. I, Experiment 21, are also needed.

Experiment. First to determine the error of the zero point, place the bulb of the thermometer in the first vessel and surround it with snow, or if this cannot be obtained, with pounded ice, as in Fig. 86. If the snow is very dry, wait until it begins to melt, when the reading of the thermometer should be 0° C., or 32° F.; the deviation will be the required error, and should be read to tenths of a degree. If the position of the zero is accurately determined, it will be found to alter continually, especially if the bulb has been recently blown and has not been well annealed.

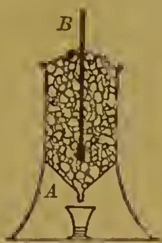


Fig. 86.

On this account the best makers keep their tubes months or even years, before using. The change goes on increasing for years and may amount, in extreme cases to 1° or 2°. Beside this change there is a temporary change produced whenever the thermometer is suddenly heated even to the temperature of boiling water. This effect does not pass off for several days.

Secondly, to determine the error of the boiling point. Place a little distilled water in AB, Fig. 87, and heat it to boiling. Pass the thermometer through the cork closing the top, and push it down until the boiling point is just outside, but not so low that the bulb shall touch the water, since the temperature of the water changes, while that of the steam is nearly constant. The outlet for the steam, C, should be large enough for its free escape, otherwise a pressure will be produced inside, which will affect the temperature. Observe carefully the reading of the thermometer, and the height of the barometer, H. The true temperature will equal $100° + \frac{3}{80}(H - 760)$. The difference of the observed and calculated readings equals the error for this point also. For ordinary work these observations are sufficient, and assuming that the tube has a uniform diameter throughout, we may determine the errors for any temperature as follows. Construct the points with abscissas equal to the zero and computed boiling points, and ordinates equal to the differences between the observed and computed temperatures enlarged. Connect them by a straight line and it will give the error for any intermediate point of the scale.

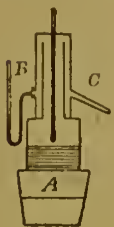


Fig. 87.

If greater accuracy is desired, the tube must be calibrated, to see if it is cylindrical. For this purpose a short column of mercury must be separated from the rest, and its length measured in different parts of the tube. To separate the mercury, invert the thermometer, and if the column does not at once descend, tap the tube on the table. If the mercury descends without breaking, so as to fill the tube, a small air bubble will be seen in the bulb. In this case turn the tube back, when with a little patience the bubble can always be made to ascend to the end of the tube, and then the mercury will separate at that point. The point of separation is usually determined by a minute air bubble adhering to the glass, which expands when the column separates. If the thread is too long by an amount equal to n degrees, warm the bulb by this amount after separation has taken place, and the expanding mercury will push the air bubble forward with it. Let the mercury reunite and cool, when it will contract past the bubble. Now, make it separate again, and the column will have the desired

length. If too short, a longer column is obtained by heating the mercury by the desired amount, then causing the separation to take place. In this way a thread of any given length is readily obtained. It is then brought to any required position by inclining the tube.

Separate a column about 20° in length, and take a series of readings of the position of each end, to tenths of a degree, as it is successively moved to various points of the tube. Call l the reading of the lower end of the column, or that next the bulb, and u the reading of the upper end, and construct a curve with abscissas proportional to l, and ordinates to $u - l$. Since the latter quantity will vary but slightly it is better to subtract a constant quantity from all the readings, and construct the differences on an enlarged scale. Determine from this curve the value of u, when $l = 0$, and call it u'. Then make $l = u'$ and find the corresponding value of u, or u''; make $l = u' + u''$ and find u''', then $l = u' + u'' + u'''$, and thus proceed until we obtain a series of values of points of the scale separated by spaces whose volume will precisely equal that of the mercury column. If now we construct the points with abscissas equal to u', u'', u''', etc., and ordinates to 1, 2, 3, etc., and draw a smooth curve through them, it will show the true volumes of various portions of the tube, in terms of the volume of the mercury column taken as a unit. But as this curve will nearly coincide with a straight line, it is better to draw a residual curve at once, with abscissas as before equal to u', u'', u''', etc., and ordinates $1 - nu'$, $2 - nu''$, $3 - nu'''$, etc., n being so chosen that these quantities shall be as small as possible. Determine from this curve the volume corresponding to values of u equal to 10°, 20°, 30°, etc., and divide each by the volume when $u = 100°$. The results will give the volume in fractions of the volume between 0° and 100°. Multiplying by 100 and subtracting the products from 10°, 20°, 30°, etc., gives the error at these points. Call e the error thus found, due to the shape of the tube, and E that determined above from the freezing and boiling points. Then the entire error will equal $e + E$ and a curve should next be constructed with temperatures as abscissas, and the values $e + E$ as ordinates. It will be noticed that the values of e, unlike those of E, need only be determined once for all.

The above method of determining the errors, e, due to the shape of the tube, may be divided into two parts. First, to find the points separated by equal volumes, and secondly the computation from them of the errors. Other methods may be substituted for either of these; thus the required points may be found directly by moving the mercury column along by an amount exactly equal to its length. This is the method commonly employed, but it is both more troublesome and less accurate than that given above, and is open also to the serious objection that an error in any one reading is communicated to all. Instead of the second portion also, we may assume that the tube is nearly cylindrical for a length equal to the mercury column, and find the volume of the intermediate portions by a simple division. But this assumption is correct only when the column is short, and in that case we cannot measure the changes in its length with precision.

123. Weight Thermometer.

Apparatus. Some test tubes, a piece of the solid and some of the liquid to be tested, some mercury, some ice, a balance and weights.

Experiment. Draw out one of the test tubes in the flame of a Bunsen burner to a fine point, and bend the end into a hook. Weigh it and call the weight W. Fill it with mercury, by heating it and dipping the end into mercury, which will pass into the tube as the enclosed air cools. Heat again, and repeat until the tube is full. It may be necessary to boil the mercury, but this must be done with great care, as it is very liable to break the glass. A quicker but less exact method is to introduce a drop or two of ether, which boils much more easily, and can be in a great measure expelled by heat.

Cool the tube with its point in mercury, by immersing it in ice water, and call its weight W'. Then heat to a temperature t, when a portion of the mercury will be driven out of the point of the tube by its expansion. Collect this overflow, and call its weight w. The amount of mercury remaining equals $W' - W - w$, and if this were heated to t, it would expand by an amount equal to w. Hence calling m the coefficient of expansion of mercury in

glass, we have $w = mt(W' - W - w)$, from which m is readily determined. But this expansion equals the difference between the absolute expansion of the mercury and that of the glass, or $m = .00018 - g$, calling g the expansion of the glass. A temperature is now measured by this thermometer by filling it with mercury at 0°, exposing it to the temperature to be tested, and weighing the amount of mercury expelled. Evidently the maximum temperature attained is always given. Instead of the overflow it is sometimes more convenient, but less accurate, to observe the weight of the tube and contents after exposure, and determine the overflow by subtraction.

To measure the expansion of a solid, its weight and volume, or specific gravity, must first be determined. It is then placed in a test tube, the latter drawn out to a point, filled with mercury at 0°, and weighed; heating to t, the overflow is determined precisely as before. The coefficient of expansion of the solid, e, is given by the equation,

$$\frac{w}{13.6} = \frac{W' - W}{13.6} mt + \frac{s}{p} te - \left(\frac{s}{p} + \frac{W' - W}{13.6}\right) gt.$$

in which s is the weight of the solid, p its specific gravity, W the weight of the tube and solid, and the other quantities the same as in the last paragraph. It will be noticed that the three terms of the second member of this equation represent the expansions of the mercury, solid and glass, respectively, and that each is equal to the product of the volume by the coefficient of expansion by the change in temperature. The volumes, moreover, of the mercury and solid equal their weights divided by their specific gravities, and for the glass equals the sum of the volumes of the other two.

The expansion of a liquid may be determined precisely like that of a solid, except that the tube must be inverted so that the liquid shall not escape. A simpler method, however, is to employ the above method of determining the expansion of the glass, replacing the mercury by the liquid. This gives, however, only the average expansion, and will vary according to the value of t employed.

The expansion of air, or other gases, may also be determined by this apparatus, and by carefully drying the gas and taking many other precautions, very accurate results may be obtained. Heat the tube when filled with air or gas, to a temperature t'', by im-

mersing it in a bath of water or oil carefully stirred. Then seal the end of the tube in the flame of a Bunsen burner, remove it and let it cool. Observe also the height of the barometer, P. Dip the sealed point into a vessel of mercury and break it beneath the surface. The mercury will immediately rush into the tube and stand in it at a height p. Observe the temperature, or better, surround it with a bath of cold water at temperature t'. Invert the tube, taking care that no mercury shall escape, and weigh it; find also the weight when entirely full of mercury, and when empty. Call these three weights, w', w'' and w. Then since the volumes are proportional to the weights, we see that the volume $w' - w$ at temperature t' and pressure $P - p$, will expand to a volume $w'' - w$, at a temperature t'', and pressure P. Calling W the weight it would have at the standard pressure $H = 760$ mms., and temperature 0°, we have, as shown, Vol. I, p. 51, $w' - w = W(1 + at')\frac{H}{P - p}$, and $w'' - w = W(1 + at'')\frac{H}{P}$, whence eliminating W and H by dividing and solving with regard to a, we deduce $a = \frac{(w' - w)(P - p) - (w'' - w)P}{(w'' - w)Pt' - (w' - w)(P - p)t''}$.

124. Expansion of Solids.

Apparatus. A long straight bar or wire of brass, or other metal to be tested, about a quarter of an inch in diameter, and three or four feet long. A fine line is drawn near each end of the bar, and it is enclosed in a glass tube through which either water or steam may be passed continually. A thermometer is inserted at each end to show the temperature of the interior. Instead of a glass tube, a rubber or metallic tube may be used with the ends of the wire and the thermometers projecting. Two reading microscopes with eyepiece micrometers, which may be fastened firmly to the table, are also required.

Experiment. Place one reading microscope over each end of the bar, and determine their distance apart, and the magnitude of one division of each micrometer, as described in Vol. I, Experiment 20. Then pass a stream of cold water through the tube, measure the temperature of each end, and read the position of the lines marked on the wire by the micrometers. Call t the mean of these temperatures, and l the distance between the two

marks, which is readily determined from the distance of the microscopes, and the magnitude of the micrometer divisions. Now pass a current of hot water or steam through the tube and measure again the mean temperature t'' and length l''. Then if l is the length the bar would have at 0°, and a the coefficient of expansion, we have by the law of expansion, $l' = l + at'l$ and $l'' = l + at''l$. Dividing, to eliminate l, and solving with regard to a, we deduce $a = \frac{l'' - l'}{t''l' - t'l''}$. Repeat, and then measure the expansion of some other metal, or of the same metal between other limits of temperature.

125. Expansion of Liquids.

Apparatus. A graduated tube closed at one end by a cylindrical bulb, whose volume is dependent on the liquid to be used. If this is water, the volume of the bulb should be about twenty times that of the tube. A thermometer, some mercury, a balance and weights are also needed.

Experiment. If the bulb is empty, it should be filled with the liquid to a point near the bottom of the tube, either by pouring it down the side of the interior, or if this is too small, by the following method. Warm the bulb and dip it into the liquid, when, on letting it cool, some of the latter will rise into the bulb. Then invert it and heat carefully until the liquid boils. Dip again into the liquid, when on cooling, the latter will fill the tube. A portion of the liquid must now be removed, so that its surface shall be near the lower part of the graduation, either by shaking the tube, or by inserting a wire. Next, take a series of readings of the position of the liquid for various temperatures, extending over as wide a range as possible, but not approaching too near the boiling point. Construct a curve with temperatures, t, as abscissas, and the positions of the liquid as ordinates, which we may call l.

To determine the expansion, two quantities must now be known, the volume of the bulb B, in terms of the divisions of its tube, and the expansion of the glass, g. These quantities may be determined once for all, or they may be found as follows. Weigh the bulb empty, and when filled with mercury to a point l', near the bottom of the tube, and again to a point l'', near the

top of the tube. Call the three weights w, w' and w''. Then if m is the weight of mercury required to fill one division of the tube, evidently $w' - w = m\ (B + l')$ and $w'' - w = m\ (B + l'')$, eliminating m by division and solving with regard to B, we deduce $B = \frac{l''(w' - w) - l'(w'' - w)}{w'' - w'}$. Instead of weighing w' directly, it is better to first obtain w'', then remove part of the mercury and weigh it; w'' minus this quantity gives w' more accurately.

To find g, partly fill the tube with mercury, measure the reading of the surface L' and L'' at two temperatures t' and t'' which should by preference be near the freezing and boiling points of water. Then if L represents what the reading would be at 0°, we have by the law of expansion, $L' + B = (L + B)\ (1 + mt')$ and $(L'' + B) = (L + B)(1 + mt'')$, or eliminating, $m = \frac{t''(L' + B) - t'(L'' + B)}{L'' - L'}$; m is here the apparent expansion of the mercury, or its true expansion minus the expansion of the glass. Since the first of these quantities equals .00018, we have $g = .00018 - m$.

Returning now to the original curve for the apparent expansion of the liquid, prolong it, if necessary, to the point where $t = 0°$, and call the corresponding value of l, l_0. Find the value of l for values of $t = 10°, 20°, 30°$, etc., and the total apparent expansion from 0° to these points will equal $l - l_0$ divided by the volume at 0°, or $G + l_0$. But this is the true expansion minus the expansion of the glass, hence the true expansion $E = \frac{l - l_0}{(G + l_0)} + tg$. To find the rate of expansion of the liquid at various temperatures, draw lines tangent to the curve at the points employed above, and find the increase in volume per degree. Dividing this quantity by the volume at 0°, $G + l_0$ gives the apparent expansion, and adding g to each, gives the true expansion. Finally, draw curves with temperatures as abscissas, and expansions as ordinates, and compare the results with those given in the Tables.

126. Expansion of Gases.

Apparatus. A Florence flask, A, Fig. 88, immersed in a vessel which may be heated, and whose temperature may be measured

by a thermometer, B. The flask is filled with dry air and closed by a cork through which passes a bent glass tube, C, serving as a gauge. The lower part of the tube is filled with mercury whose height is measured by a scale attached to each arm.

Experiment. Read the height of the barometer, the temperature of the water by B, and the difference in level of the mercury in the arms of C. Then the pressure of the air in A will equal the height of the barometer, plus the height of the mercury in the right hand arm minus that in the left hand arm; that is, the difference in the two arms, supposing the pressure of the air replaced by a column of mercury of height equal to that of the barometer, added directly to the mercury in the right arm of the gauge. Heat the water twenty or thirty degrees, and withdraw the lamp. Stir briskly with B when it will be seen that the temperature at first rises, attains a maximum, and then begins to fall. Read the thermometer B and gauge C, and repeat. Four or five readings should be taken in this way between the freezing and boiling points.

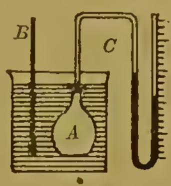


Fig. 88.

We have thus a number of readings of the corresponding temperature and pressure of a given quantity of gas under nearly constant volume, since the volume of C is very small compared with that of A. Call t the temperature, P the pressure, and P_0 the pressure it would have at temperature 0°. Then $P = P_0\,(1 + at)$ in which a is the required coefficient of expansion. Apply the method given Vol. I, p. 5, to this case, and determine the most probable values of a and P_0. The above equation may be written $0 = 1 - P_0\frac{1}{P} - aP_0\frac{t}{P}$, in which $-P_0$ corresponds to a, $\frac{1}{P}$ to x, $-aP_0$ to b, and $\frac{t}{P}$ to y. Apply the rule by substituting proper values of t and P from the observations taken above, and thus form as many equations of condition as there are observations. Then multiply each equation by the values of $\frac{1}{P}$ and equate their sum to zero. This gives one normal equation, and the second is found similarly by multiplying by the various values of $\frac{t}{P}$. Solving the two normal equations gives P_0 and aP_0, and hence a.

127. Change of Volume by Fusion.

Apparatus. A test-tube *A*, Fig. 89, closed by a cork through which passes a graduated bent glass tube *B*. A thermometer, some mercury, ice and a balance and weights, are also required.

Experiment. Put some dry ice in the test tube, and fill the remaining space nearly to the top of the graduation with mercury cooled to 0° C. Read the position of the top of the mercury, and let the ice melt. As the water occupies less space than the ice, the mercury will fall until the fusion is complete. Read the level at this instant. If the water is allowed to grow warmer, it will continue to contract, until a temperature of 4° C. is attained, and then it will expand again. Care must therefore be taken to read the level as soon as all the ice has disappeared, or else the tube should be immersed in ice water to prevent its becoming warmed. Dry the outside of the tube and weigh it. Weigh the mercury now in the tube, and weigh the latter, when empty, when filled to the top, and when filled to the bottom of the graduation. From the last two weights the volume of each division of the tube is readily obtained by dividing their difference by 13.6 times the number of divisions. The volume of the water is also readily deduced from the weight of mercury, of the tube when full, and the magnitude of the divisions. The change in volume by fusion is then found from the comparative volumes of the ice and water.

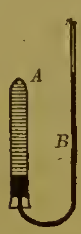


Fig. 89.

The change in volume of any other substance may be similarly determined, except that a correction must be applied for temperature. In the case of fusible metal, or other alloys, water or oil should be used instead of mercury, to avoid amalgamation, and as the solid is then usually heavier than the liquid, the tube *B* should be straight, instead of bent.

128. Conduction of Solids.

Apparatus. In Fig. 90, *BC* is a bar of the metal to be tested, with the bulbs of several themometers inserted in it at regular intervals. *A* is a vessel which may be filled with boiling water, and *D* is a short piece of the metal, with a thermometer in it to show what the temperature would be if the bar was not heated.

This experiment may also be performed with a thermo-pile (Experiment 131) sliding along the bar.

Experiment. Read the thermometers in *BC* and *D*, which should mark the same temperature. Fill *A* with boiling water, and at the end of a minute read again. Repeat, at intervals of a minute, always beginning with the thermometer next *B*, and reading them in order, until the temperature has become constant, and the readings do not alter. Then construct a series of curves, one for each minute, in which abscissas will represent the intervals between the thermometers, and ordinates the increase of reading of each thermometer, or its reading minus that of the thermometer *D*. The final curve should be such that the logarithms of the ordinates will be proportional to the abscissas, or the latter being taken in arithmetical progression, the former will vary geometrically. See if this is the case, by using as ordinates the logarithms of the excesses of temperature, and abscissas as before, when the result should be a straight line. The other curves will show the gradual progress of the heat along the bar. By using several bars of various metals, but having the same dimensions and covered with the same varnish, the comparative conductibility may be determined.

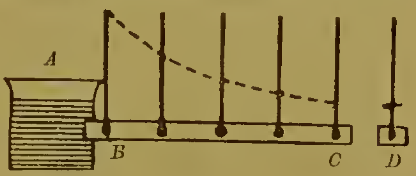


Fig. 90.

If the thermo-pile is used, the temperature of any point of the bar may be determined, as described in Experiment 131, and the law of the distribution of its heat tested, as described below.

129. Conduction of Crystals.

Apparatus. A thin plate of quartz cut parallel to the axis, with a minute conical hole cut in its centre. Some stout silver wire ground to a point, some wax and the Dividing Engine, Vol. I, Experiment 22, are also required.

Experiment. Warm the crystal and touch the wax to it, so as to form a thin uniform layer over the surface. Let it cool, and then heat the wire by a lamp after inserting one end in the hole. The heat transmitted to the quartz will melt the wax, until the

loss by radiation will equal that received from the wire. Allowing it to cool, the edge of the fused portion will be marked by a line whose position is easily observed. To ensure contact of the wire and crystal, the latter should be turned. Measure the curve thus obtained with the Dividing Engine, and construct it on an enlarged scale on paper divided into squares. If quartz conducted heat equally in all directions, this curve would be a circle; but as the conductibility is greatest in the direction of the principal axis, the curve is found to be an ellipse, with its transverse axis parallel to the principal axis of crystallization. Construct an ellipse which shall coincide as nearly as possible with the curve, and measure the ratio of its axes. This curve may be obtained in a more marked manner by using Meusel's double iodide of copper and mercury, which changes from red to black on being heated to 70° C., but the curve thus obtained is not permanent, the color returning as the crystal cools.

130. Contact Thermometer.

Apparatus. A thermometer with its bulb in a small funnel, the stem passing through the neck, and the larger end being covered with sheet rubber after filling the funnel with mercury. Some pieces of cloth, silk, woollen and other fabrics, and a large surface heated to a constant temperature by boiling water or steam, are required. The slab of a radiator is well adapted to this purpose, and to avoid air currents it should be vertical, rather than horizontal.

Experiment. Measure the temperature of the room as given by the thermometer, and then hold it against the heated surface until its temperature becomes stationary. Next, interpose in turn the various fabrics to be tested, when the maximum temperatures attained will depend on their relative conductibilities. This method is not one of precision, and comparative results only can be expected; but by interposing successively one, two, three or more pieces of the same material, the law of variation may be approximately determined.

131. Radiant Heat.

Apparatus. A thermo-pile and a delicate short-coil galvanometer, with a mirror and scale, rendered astatic either by a second

needle, or by a damping magnet. Various sources of heat are required, as the flame of a lamp, a platinum wire heated to redness, a sheet of hot metal, and a cube containing boiling water, with one face polished, a second varnished, a third painted white, and the fourth black. Melloni's thermo-bank may be used to hold the various portions of the apparatus, but this is not indispensable, as they may be placed in their proper positions on the table. Plates of glass and of other materials are needed to study the absorption of heat, and to prove the laws of reflection and refraction of heat a horizontal graduated circle is required, with a movable arm and index, to which the thermo-pile may be attached. The mirror may be placed at the centre of the circle, and its position marked by a second index. For the polarization of heat a number of plates of clear mica, thin glass or collodion, are fastened together and set at an angle of 55°, like the bundle of thin plates in a refracting polariscope. Two sets of such plates are required, as polarizer and analyzer, and they should be free to turn by a measured amount around their axes.

Experiment. Light the burner and place it a short distance from the thermo-pile, whose ends should be covered to protect it from the heat. Attach the terminals of the pile to those of the galvanometer, and light the burner connected with the latter, so as to form a distinct spot of light at the centre of the scale. The galvanometer must be adjusted, as described in Experiment 102. Remove the cover of the pile so that the heat of the lamp shall fall on it, when the spot should at once move nearly to the end of the scale. It is generally better to note the maximum deflection rather than wait for the spot to cease vibrating, and much time will be saved by using a galvanometer of such a form that the needle will soon come to rest.

Take a series of readings, placing the pile at various distances from the flame, and see if the deflection is inversely as the square of the distance. Otherwise a curve may be constructed for the galvanometer by using as ordinates the deflection, and as abscissas the reciprocal of the square of the distance. This should give a straight line; and if not, all later observations should be reduced by means of it.

Next, place the thermo-pile at the centre of the graduated circle, and read the deflection when its face is inclined at various angles to the incident rays of heat. The total amount of heat which will fall on the pile will evidently be proportional to the cosine of

the angle of incidence, and of this the amount absorbed by the pile will also be proportional to the cosine of the same angle. Hence the deflection should be proportional to the square of the cosine of this angle.

To prove the law for the emission of heat, expose the thermo-pile to the tin cube of boiling water, and note the deflection, as the cube is inclined to it at various angles. The deflection should be proportional to the cosine of the angle of emission. This law is more simply proved by interposing a screen with a hole in it, when the deflection will remain unchanged when the tin is turned, as long as the angle of emission is not so great that a line from the thermo-pile may fall off the heated surface. But the radiating surface which acts on the pile, will in this case be inversely proportional to the cosine of the angle, hence the total amount of heat remaining unchanged; the radiation per square unit must be proportional to the cosine of the angle. The law of the distance may be proved in a similar manner.

The transparency of bodies to heat, or their diathermancy, is measured by placing the pile at such a distance from the flame that the spot of light will move nearly to the end of the scale. Now interpose a plate of glass, or other substance to be tested, when the deflection will be much less, a portion of the heat being absorbed. The ratio of the two deflections gives approximately the amount of heat transmitted. Of this, however, a portion is reflected specularly, in amount depending on the index of refraction. A loss of about eight or ten per cent may be ascribed to this cause. After allowing for this error, the absorption by different transparent bodies will be found to vary very greatly, especially when various sources of heat are employed. To show that this is the case, measure the transmitted heat from the incandescent wire, heated metal and hot water vessel, when it will be found that while rock salt is almost perfectly transparent or diathermanous to all heat rays, that glass cuts off a large portion, especially in the case of the heated water, where a plate of glass is found to be nearly opaque, or athermanous. The reason is, that each source of heat consists of a bundle of rays of various wave-lengths, hence accurate quantitative results can be attained only by separating these rays by a prism, or otherwise, and testing each

separately. The absorption of two plates of glass is not double that of a single plate, but follows a more complex law, each additional plate cutting off less and less, as if it acted like a sieve, and removed the portions more easily absorbed. The absorption of liquids is measured by two tanks of unequal thickness, the difference in the transmitted rays in the two cases serving to determine the absorption. The absorption of gases and vapors may be similarly determined by a long tube whose absorption is measured when empty, and when filled with the gas or vapor to be tested. In this case, as the absorption is generally small, it is well to use a second cube as a source of heat opposite the other face of the pile, and measure the deflection before and after the vapor is interposed. The galvanometer then shows the difference of the two bundles of radiant heat.

The amount of heat radiated by a given body will depend greatly on the condition of its surface. Expose the thermo-pile to the four surfaces of the cube in turn, when it will be found that the least heat will be received from the polished side, and the most from that covered with the lampblack. The more heat a surface radiates the more it will absorb; but this is not easily shown, except by covering the face of the pile with various varnishes. Commonly the pile is covered with lampblack, since this is one of the best of radiators and absorbers.

When a ray of heat is allowed to fall on a polished surface, the greater part of it is reflected, as in the case of light, so that the angle of reflection will equal the angle of incidence. To prove this, place a mirror of glass or metal at the centre of the graduated circle, and the thermo-pile on the movable arm. On turning the latter, little effect is produced, except in a particular position, and then a marked deflection is obtained. Note the position of greatest deflection, and read the angles of incidence and reflection, when they will be found to be equal. Repeat, giving the angle of incidence various values. For this, and for some of the following parts of the experiment, the pile should be constructed in the form of a narrow strip, or line. If care is exercised, it will be found that some heat will be reflected at other angles than that given by the law of reflection. This is what is known as diffuse reflection. It is best seen by using a very intense source of heat.

Replace the mirror by a prism of rock salt, when it will be found that heat is refracted like light, the deflection of the galvanometer attaining a marked maximum, in a position nearly corresponding to that of the red end of the spectrum. Measure the angles of incidence and refraction and compute from them the index of refraction of the heat rays, as in Vol. I, Experiment 77.

Rays of heat may be polarized also, like rays of light. For this purpose interpose the two bundles of plates of mica between the source of heat and the thermo-pile, when it will be found on turning one of the bundles, that the deflection will be much greater when they are parallel, than when at right angles to each other. Call m and n the deflections in these two cases, and call A and B the portion transmitted of the heat polarized in the plane of incidence, and in the plane at right angles to it, when the incident beams are equal to unity. Then when the plates are parallel we shall have A transmitted of one ray by one bundle of plates, and A^2 by both. Of the other ray, B^2 will be transmitted by both, or of the whole light, $A^2 + B^2$. When the plates are crossed, we shall have of one ray AB, and of the other BA, or $2AB$ in all. Therefore $m = A^2 + B^2$, and $n = 2AB$. But the polarization effected by one bundle of plates is $\frac{A - B}{A + B}$, or substituting values of m and n, $\frac{A - B}{A + B} = \sqrt{\frac{m - n}{m + n}}$.

132. Law of Cooling.

Apparatus. A large thermometer with a bulb about an inch in diameter, enclosed in a flask from which the air may be withdrawn if desired. The whole is immersed up to the neck in a vessel of water, whose temperature may be kept constant by stirring.

Experiment. Heat the thermometer very carefully and slowly over a Bunsen burner, until the reading is about 300° C., then insert it in the flask and immerse the latter in the vessel of water. Now take a series of readings as the temperature falls, for every 10°, until a temperature of 100° C. is attained, and below this at the end of every minute. It is well to stir the water occasionally, and see that its temperature t does not alter. The experiment may be varied by exhausting the air or replacing it by another gas,

or by altering the temperature of the water in the containing vessel. To establish the relation between the temperature y and the time x, the simplest hypothesis that we can make is that the radiation, or rate of cooling, $\frac{dy}{dx}$, is proportional to the temperature. But the surrounding medium radiates back an amount proportional to its temperature t. Hence we may write, $\frac{dy}{dx} = ay - at$, or integrating, $ax = M \log(y - t)$, in which $M = .434$, the modulus of the common system of logarithms. This is Newton's law of cooling, and may be expressed by saying that if the times are taken in arithmetical progression, the excesses of temperature will vary geometrically. This law may be tested by constructing a curve with times as abscissas, and logarithms of the excesses of temperature as ordinates. If the law is correct, the result should be a straight line; but this will seldom be the case, except for small differences of temperatures.

Dulong and Petit showed that the rate of cooling could be more correctly represented by the formula: —

$$\frac{dy}{dx} = m\,1.0077^y - m\,1.0077^t + np^b(y - t)^{1.233}$$

in which the last term represents the cooling effect of the air. In this formula m and n are constants, dependent on the volume and extent of surface of the cooling body, m depending also on the material of the surface, and n on the nature of the gas; b also depends on the kind of gas present, and p equals its pressure.

133. Pressure of Steam.

Apparatus. In Fig. 91, A is a flask half full of water, closed by a rubber cork, through which pass a thermometer, B, and a bent tube, CD, serving for a gauge. The water is first boiled for some time to expel the air, and mercury then poured into the open end of CD. As the flask cools the mercury will rise, until when cold the difference of level will be equal to the height of the barometer within about an inch.

Experiment. Read the height of the barometer, the temperature of the water, and the difference in level of the mercury in the two arms of the tube. Heat A carefully, and take a series of readings of the thermometer B, and difference of level of the mercury in the two arms of CD. Subtracting the latter from the

height of the barometer, gives the pressure of the vapor corresponding to these various temperatures. Construct a curve with

Fig. 91.

coördinates equal to these pressures and temperatures, and draw a second curve from the results of Regnault's experiments, as given in the Table of the pressure of steam. If the observed pressure at low temperatures is much greater than that given in the Table, there is probably some air in *A*, in which case the mercury should be emptied out of *C*, and the air expelled by boiling. If time permits, it is well to observe the pressure as the water cools, and compare the curve thus obtained with that given during heating. If the water is heated too rapidly it is liable to boil irregularly, and endanger the flask. This may be avoided by applying the heat more gradually, or by placing some sand or scraps of platinum in the flask before sealing. If water collects above the mercury in *C*, it should be allowed for by adding an equivalent column of mercury, which, since the specific gravity of the latter is 13.6, is found by multiplying its height by .0735.

Instead of bending the tube *CD* into a U it may terminate at *D*, and dip into a vessel containing mercury. It is then very easily freed from air at any time by simply boiling the water and removing the mercury vessel. When taking readings, the position of the vessel should, however, be constantly altered, so as to keep the surface of the mercury always at the same height.

134. Pressure of Vapors.

Apparatus. In Fig. 92, *ABCD* is a bent glass tube, closed at *A*, and filled with mercury, like a siphon barometer. A drop of water, or other liquid to be tested, is passed through the mercury into the vacuum, and evaporating, depresses the mercury column by the pressure of its vapor. The arm *AB* is enclosed in a large thin glass tube, which may be filled with hot or cold water, and the temperature measured by a thermometer, and rendered uniform by stirring.

If preferred, a straight barometer tube may be used, instead of the bent tube, as in the last Experiment. For low temperatures it is better to use a tube bent at the top, and the end immersed in a freezing mixture, as the pressure will be that due to the coldest part of the space occupied by the vapor.

Experiment. Read the temperature by the thermometer, the height of the barometer, and the difference in level of the mercury in the two arms of the tube. Heat the water gradually, when, owing to the increased pressure of the vapor the mercury column in AB will descend. Take a series of readings of the temperatures and corresponding vapor pressures, found by subtracting the differences in height of the two columns of mercury from the height of the barometer. Construct a curve with these quantities as coördinates, and a second curve with the numbers given in the Table of the pressure of steam.

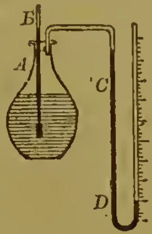


Fig. 92.

135. Specific Gravity of Vapors.

Apparatus. In Fig. 93, AB is a graduated glass tube about 2 cms. in diameter and 30 cms. long. C is a larger tube surrounding the upper end, which may be filled with hot water, or heated by steam. The temperature is marked by an immersed thermometer. The liquid to be examined may be enclosed in minute glass stoppered bottles made for the purpose, or in fragments of glass tubes drawn out to a point. Some mercury, and a balance and weights, are also required.

Experiment. Close a thin glass tube at one end, and draw out a small piece, so as to form a minute bulb terminating in a fine point. Weigh it and fill with the liquid whose vapor is to be measured, by warming and dipping the end in the liquid, and as it cools a drop will be driven inside by the pressure of the outer air. Heat the glass carefully, so as to boil the liquid, and immerse the end again, when, on cooling, it will be completely filled. Close the end by holding it for an instant in a gas flame, and then weigh it. The increase of weight gives the amount of enclosed liquid. If the small glass stoppered bottles are used, it is easy to weigh them empty and full, taking care, in the second case, that the exterior is dry and clean. Now fill the graduated glass tube with mercury, and invert it over the vessel D, taking care that no air bubbles remain inside. Pass the bulb containing the liquid under its edge, when it will rise to the top and float on the mercury. Warm the tube

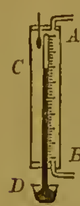


Fig. 93.

by steam or warm water, when the liquid will expand, break the bulb, and being converted into vapor, will displace the mercury. The temperature maintained must be sufficient to evaporate all the liquid, which is known by the surface of the mercury appearing dry. Observe the height of the barometer, and the height of the mercury inside the tube, above that outside. The temperature is then read by the thermometer, and the volume by the graduation. This must be reduced to the standard temperature and pressure by the formula, $V_{0H} = V_{tP}\frac{273\,P}{(273 + t)760}$, as explained in Vol. I, p. 51, in which P equals the height of the barometer minus the difference of level of the mercury inside and outside of the graduated glass tube. The specific gravity compared with water will then equal the weight of liquid employed, divided by the volume computed as above. Its specific gravity compared with air is found by dividing this quantity by .001293, the specific gravity of air. If the liquid has a known chemical composition, its two specific gravities are found by dividing its atomic weight by 28.88 and .0373, respectively.

136. Density of Gases.

Apparatus. A delicate balance and weights, a thin glass globe closed by a stopcock, an air pump, drying tubes, and a supply of the gas to be examined.

Experiment. Exhaust the globe as completely as possible, and measure the pressure of the air remaining. Then weigh it, or rather place a somewhat heavier weight in the other scale pan, and counterpoise very exactly by weights in the pan over the globe. Read also the height of the barometer and the temperature of the air of the room. Connect the drying tubes with the globe and allow the air to enter very slowly. Weigh a second time by counterpoising again, and the change in weight equals the weight of the air required to fill the globe. Exhaust again, and fill with the gas to be tested. This is done by passing the gas through the drying tubes to expel the air they contain, and then allowing it to pass into the globe by partially opening the stopcock. To get rid of the small remaining amount of air, it is best to exhaust and refill a second time. Weigh, the flask as

before, and the increase compared with that when filled with air, gives approximately the specific gravity of the gas, as in Vol. I, Experiment 46. The absolute density of the gas may be found by reducing its volume to 0° and 760 mms. pressure, Vol. I, p. 51, and recollecting that 1 litre of dry air weighs 1.293 grammes. A much more accurate method, however, is to fill the globe with mercury or water, measure the increase of weight, and thus deduce the volume in centimetres.

The preceding method can also be applied to finding the density of a vapor; a few grammes of the liquid, very pure and carefully distilled, must be poured into the globe, and the latter then immersed in a bath of water or oil, and raised to a temperature considerably above the boiling point of the liquid. The stopcock is of course left open, and the vapor will rapidly escape, carrying the air with it. When all the liquid has been converted into vapor, which is known by the escape of vapor ceasing, the stopcock is closed, the temperature of the bath and the barometric pressure being first noticed. The globe is then removed from the bath, allowed to cool, and the exterior carefully dried and weighed. The computation is made precisely as in the last Experiment, except that the volume of a given weight is measured, instead of the weight of a given volume.

137. Mixture of Vapors.

Apparatus. In Fig. 94, *AB* is a glass tube closed above and below with stopcocks, a third stopcock *C* being added above, in which the hole passes only part way through the plug, thus allowing a liquid to be added, a drop at a time. A second tube, *D*, is connected with the first, and serves to measure the pressure of the enclosed gas.

Experiment. The tube must first be dried, which is best done by unscrewing *C*, opening *A* and *B* and blowing dry air through the tubes. Then close *A* and pour mercury into the open tube till it stands at a point marked on the tube *AB*. Read the height of the mercury in the open tube, and screw *C* in place. Pour some water into the end of *C* and turn its plug around once. When the aperture in the latter is up, it fills with water which escapes into the tube as the plug is turned

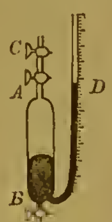


Fig. 94.

over. The water evaporating will increase the pressure and make the mercury fall in AB, and rise in D. Add more mercury, therefore, through the open tube, until it stands exactly at the mark in the closed tube. This is best done by adding an excess of mercury and letting it slowly escape through A. The increased height of mercury in D represents the pressure due to the water, and will be found to be nearly the same as that formed at the same temperature in a vacuum.

138. Specific Heat.

Apparatus. A cylindrical vessel of thin sheet copper silvered, supported in a second similar vessel of the same material, by resting it on three wooden points, or on two strings stretched inside the outer vessel, near the bottom. Two thermometers, one for measuring small changes of temperatures, the other graduated up to 100° C., a balance and weights, a vessel in which water may be heated, and some mercury and sand, are also required. Instead of the copper vessels, common glass beakers may be employed, if great accuracy is not required.

Experiment. Weigh the inner copper vessel, or calorimeter, as it is called, and then partially fill it with cold water, and weigh again. Heat some water and notice its precise temperature, also that of the cold water and of the room. Then pour part of the hot water into the calorimeter, stir briskly, and read the temperature of the mixture as soon as it has become uniform. The success of the experiment depends, in a great measure, on this operation, which requires much care. It is well first to take the temperature of the cold water, at the beginning of a minute read the thermometer in the hot water, then pour quickly and take a series of readings as the calorimeter cools. Now weigh the calorimeter with the mixture, and call its weight when empty, w, when containing cold water, w', and after the hot water is added, w''. Call T the temperature of the hot water, t that of the cold water, and t' that of their mixture; also call c the specific heat of the calorimeter, and S that of the hot water, which should equal unity if the experiment is correctly performed. Then the weight of hot water added is $w'' - w'$, and its fall in temperature $T - t'$; hence the amount of heat it gives up is $S(w'' - w')(T - t')$, since

the specific heat equals the amount of heat given out by a unit of weight of the substance in cooling 1° C. The cold water, on the other hand, gains in temperature $(t' - t)$, and in weight $(w' - w)$; to the latter must be added the water-equivalent of the calorimeter, or weight of water which would require the same amount of heat as the calorimeter to warm it 1°. But for every gramme of the calorimeter we must have c grammes of water; hence for w grammes we must have wc grammes of water. Accordingly the total amount of heat received will equal $(t' - t)(w' - w + wc)$, or since this must equal the heat given out by the hot water, $S(w'' - w')(T - t') = (t' - t)(w' - w + wc)$, or $S = \frac{(w' - w + wc)(t' - t)}{(w'' - w')(T - t')}$. If no errors were committed, S should equal unity, and it is well to repeat the experiment two or three times, or until a value closely approaching this, is attained.

One of the principal sources of error is the loss due to radiation from the hot water after its temperature is taken, and before that of the mixture is observed. The readings taken during the cooling of the mixture are designed to correct this error. Construct a curve with abscissas equal to the times, and ordinates to the logarithms of the excesses of temperature above that of the room. This, by Newton's law of cooling (Experiment 132), will be very nearly a straight line, and continuing it back to the point where the hot water was poured into the calorimeter, will give the temperature which would have been attained had there been no loss of radiation, or could we have mixed the liquids instantly and read the temperature at once. The value of t' thus obtained is that which should be used in the above formula. To still further reduce this source of error, the water in the calorimeter should be somewhat colder than the air of the room, and the amount of hot water added should be such as to bring the temperature of the mixture about as much above that of the surrounding air.

This same method may be used for such other liquids as do not undergo a chemical change on contact with water or with the calorimeter; solids in powder may be similarly treated. Find in this way the specific heat of sand, heating it for some time in a vessel surrounded by boiling water, to be sure that its temperature is uniform. Find also the specific heat of mercury, replacing the

copper calorimeter by one of glass. The mercury must not be heated over 100° C., or it will convert some of the water into steam, and create a great loss, due to its latent heat.

139. Latent Heat of Fusion.

Apparatus. The same as in the last Experiment, except that some fresh, dry snow is needed, instead of the mercury and sand.

Experiment. Latent heat is measured almost precisely like specific heat, and the same precautions are necessary in both cases. The calorimeter is weighed empty, and when partly filled with warm water; the temperature of the latter and of the room is then observed. Take the temperature of the snow, and put some of the dryest portions into the calorimeter; stir briskly, and as soon as all is melted, take a series of readings of the temperature every half minute. The correction for radiation is here much greater than in finding the specific heat of liquids, since a much longer time will elapse before all the snow is melted. Finally, weigh the calorimeter and contents, to determine the amount of snow added.

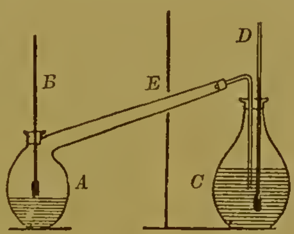
To compute from these observations the latent heat, call, as before, w, w' and w'', the three weights of the calorimeter, $-T$, which will always be negative, the temperature of the snow, and t and t' the temperature of the calorimeter, before and after adding the snow. Call L the latent heat, and S the specific heat of the snow, which is about .5. Then the weight of the snow will equal $w'' - w'$, and its gain in heat may be divided into three parts. First, heating the snow from $-T$ to 0°, its melting point; secondly, the latent heat L, and thirdly, after fusion, warming the water from 0° to t'. The sum of these three will be $(w'' - w)$ $(ST + L + t')$, or $(w'' - w')\,(.5\,T + L + t')$, since the specific heat of water is unity. The heat given out by the calorimeter will be $(t - t')(w' - w + cw)$, and equating these two, and solving, gives $L = \dfrac{(t - t')(w' - w + cw)}{(w'' - w')} - ST - t'$.

140. Latent Heat of Vaporization.

Apparatus. In Fig. 95, A is a tubulated retort, with a thermometer B passing into it to mark the temperature of the vapor,

and C is a Florence flask, into which passes a second thermometer, D, to mark the temperature of the enclosed water. A screen, E, serves to prevent the heat from passing directly to C by radiation. A balance and weights should be provided, and a Bunsen burner to boil the water in A.

Experiment. Fill A half full of water, and heat it by lighting the burner under it. Disconnect C and weigh it, first when empty. and then when partly full of water. Let the water in A boil for some minutes, and observe the temperature of the air of the room, of the water in C, and of the steam in A. Then connect A and C, so that the steam from the former shall pass over into the latter, condensing and giving up its latent heat. Observe the temperature of the water in C by the thermometer D, every minute for ten or fifteen minutes, then disconnect, and observe the temperature as C slowly cools. To keep the temperature of the water in C uniform throughout, it should be stirred continually with the thermometer, or by a metallic stirrer raised and lowered by a wire handle. Weigh C with its contents, and the increase of weight will equal the amount of steam received from A. Now construct a curve with abscissas equal to the times, and ordinates to the temperatures, as given by D, minus that of the air of the room. The curve thus drawn will consist of two parts, one representing the heating, the other the cooling of the water. Were there no loss by radiation, or other causes, the first of these would become sensibly a straight line, inclined to the axis by an amount proportional to the rate at which the heat is conveyed from A to C, and the second curve would become a horizontal straight line, since the temperature would remain unchanged. Owing to radiation, however, the water is continually losing heat, and a very considerable error is introduced if this loss is neglected. To apply a correction, we must know the rate at which the temperature would fall if C was heated 1° above the surrounding air. By Newton's law of cooling, which will be sufficiently exact in the present case, the rate at any temperature will be proportional to that temperature. Hence



Fig, 95.

if we draw tangents to the curve representing the cooling of C at two or three points, then determine how much the loss is per minute at these points, and divide this loss by the ordinate, or excess of temperature of the point, we obtain values of the required rate of cooling for an excess of 1°. A more accurate method of determining this quantity is the following. As shown on page 89, the rate of cooling is $\frac{dy}{dx} = ay - at$, in which a is the quantity we wish now to determine, and integrating, $ax = M \log (y - t)$. Accordingly, if we construct a curve with abscissas, as before, equal to the times, and ordinates to the logarithms of the excesses of temperature over that of the air, we obtain a straight line, and the tangent of the angle it makes with the axis of Y multiplied by M, gives a, the required rate of cooling. See if similar results are found by both methods.

The loss by cooling during any short time, dx, will evidently equal $a(y - t)dx$, since it is proportional to the rate of cooling, the excess of temperature, and the time. Hence the total loss while the water is being heated will be proportional to the total area included between the curve and the line $y = t$. This is commonly found with sufficient accuracy by multiplying the total time of heating by the average of the initial, and final temperatures. Multiplying this product by a, and adding the result to the final temperature gives the temperature which would have been attained had there been no loss. To make this correction as small as possible, it is well to begin with water in C as cold as possible, so that the gain of heat by radiation from the outer air may in part compensate for the loss, as C becomes heated.

To determine the latent heat from the quantities thus obtained, let T equal the temperature of the steam, t and t' the initial and final temperatures of the water after correcting for loss by radiation, w, w' and w'', the weights of C when empty, after the water is added, and at the end of the experiment, so that $w' - w$ will equal the weight of water in C, and $w'' - w'$ the weight of steam passed from A to C. Then the heat given out by the steam will consist of two parts, that due to its latent heat in converting it into water, and to its sensible heat given out as the water so produced cools from T to t', or $(w'' - w')[L + T - t']$ The heat

received by the water equals the water equivalent of C, and its contents, or $(w' - w) + sw$, calling s the specific heat of C, or .2, multiplied by the increase of temperature, $t' - t$. Equating these two quantities $(w'' - w')[L + T - t'] = (w' - w + sw)(t' - t)$, and solving with regard to L gives $L = \frac{(w' - w + sw)(t' - t)}{w'' - w'} - T + t'$.

141. Carré Machine.

Apparatus. A Carré ice machine, such as is represented in Fig. 96, in which AB is an iron boiler containing ammonia and water, and connected with a double cylindrical vessel, C, by an iron tube. In the upper part of A a tube is inserted, in which a thermometer is placed, and surrounded by oil so as to take the temperature of the tube. A cylindrical tin vessel is inserted in C, and contains the water to be frozen. A little alcohol is poured around it to prevent its adhering to C. A must be heated in a small furnace, and an abundance of cold water is needed to carry off the heat from C.

Experiment. Set the Carré machine on end for five or ten minutes, so that C shall be uppermost, and all the liquid in it driven into A. This is very essential to the success of the operation. Then place A on the furnace, and C in a tub of water at as low a temperature as is readily attained. The tin vessel is of course taken out, and C is placed entirely under the water. A moderate and constant heat is now applied, first pouring a little oil into the tube in the upper part of A, and inserting the thermometer. The temperature will gradually rise, the ammonia separate from the water and distil over into C, where it will condense in the liquid form. Its latent heat will thus be given up to the surrounding water, which must therefore be constantly changed, or it will soon become warm. The thermometer should be watched, as it gradually rises, until it attains 130° C., when the heat should be withdrawn and A allowed to cool. This concludes the first part of the operation, the ammonia being converted into a liquid form, and its latent heat carried off by the water. Now turn the Carré machine around,

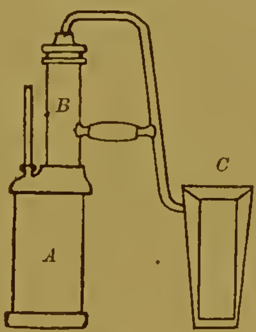


Fig. 96.

so that *A* shall be in the water instead of *C*, taking care not to cool *A* too suddenly. Fill the tin vessel with the water, or other substance to be frozen, close the hole in the bottom of *C* with a cork, insert the tin vessel and pour a little alcohol or brandy around it to prevent its freezing to *C*. Wrap a woollen cloth around *C* to protect it from the air, and renew the water around *A* as it grows warm. The liquid ammonia will now evaporate rapidly, pass over into *A*, and be absorbed by the water; the only source from which it can obtain the heat needed to vaporize it will be *C* and the water, which will consequently soon begin to freeze. The heat given up by the absorption in *A* will be carried off by the surrounding water, which must therefore be changed to keep it cool. After some time the water in the tin will be found to be completely frozen, and may then be extracted by simply dipping the tin in water.

142. Freezing Mixtures.

Apparatus. Some snow or ice, salt, nitrate of ammonia, sulphate of soda and chlorhydric acid, a beaker surrounded with wool or other non-conductor, and a thermometer.

Experiment. A great variety of freezing mixtures have been employed, all dependent on the formation of a liquid from the mixture of a solid and liquid, or of two solids, where the heat required to effect the change, being withdrawn from the substances themselves, lowers their temperature. In each of the following cases, measure the temperature of the substances employed before and after mixture. The most common freezing mixture is formed by adding one part of common salt to two of snow or pounded ice, when the temperature will fall nearly 20° C. The cold thus produced was supposed by Fahrenheit to be the absolute zero of temperature, and was hence selected by him as the starting point of the thermometer which bears his name. Mix equal parts of water and nitrate of ammonia, when the temperature will fall 26°; again, to five parts of chlorhydric acid add eight parts of sulphate of soda, when the temperature will fall 27°. By distillation the salt may be recovered in each of these cases.

Far lower temperatures than these may be obtained by the vaporization of liquified gases, as in Experiment 141, but the ap-

paratus required is generally not adapted to daily laboratory work. Liquid carbonic acid and protoxide of nitrogen are most commonly employed, and act both by their latent heat and by the heat absorbed on the enormous increase of volume when the gas is allowed to expand into the open air. If liquid carbonic acid is allowed to evaporate, the temperature will fall to —70° C., and a portion of the remainder will be frozen. If a jet of carbonic acid under high pressure is allowed to escape, a temperature of —93° may be attained. A portion of the gas is, in this case, frozen into flakes, like snow. Mixing some of this snow with liquid protoxide of nitrogen and ether, so as to form a paste, and placing the whole under the receiver of an air-pump, so as to accelerate the evaporation, gives a temperature of —110° C., the lowest yet obtained.

143. Pyrometers.

Apparatus. The various pyrometers described below, including a mercury thermometer, graduated to 360° C., and an air thermometer formed of a glass, or better, a porcelain, bulb, filled with dry air, and connected by a fine tube with a gauge containing mercury. A Wedgewood pyrometer and some clay cylinders, a piece of platinum, or of iron, if platinum is too expensive. A thermo-electric pile, formed of two wires of platinum and iridium welded together at the ends, and connected with a delicate galvanometer, also a Siemens' resistance pyrometer, consisting of a coil of fine platinum wire, forming one side of a Wheatstone's bridge. As sources of heat we may use boiling water, oil, sulphur, cadmium or zinc, baths of various alloys at their melting points, and for higher temperatures any form of furnace or gas-flame.

Experiment. The following are the more common methods of measuring very high temperatures. Try each in turn with those temperatures to which it is applicable, and compare the results. Measure the temperatures of the water, oil and alloys, with the thermometer, taking care that it is not heated above 360°. Do the same with the air-thermometer, immersing the bulb in the bath to be tested, and reading the difference in level of the mercury in the gauge. Read also the height of the barometer, and adding it to the level of the mercury in the outer arm of the gauge, the difference will give the true pressure of the enclosed air. This pressure will then be proportional to the absolute tem-

perature, or temperature above —273° C. Calling P the pressure, and P_0 the pressure at 0°, we may write $P = P_0(1 + at)$, in which t is the temperature, and a equals $\frac{1}{273}$ the coefficient of expansion of gas. A correction may be applied for the increased volume, as the mercury is driven down in the gauge, but this may be neglected if the tube is small and the bulb large. If a glass bulb is used, temperatures up to 800°, or nearly the softening point of glass, and with a porcelain bulb, much higher temperatures may be measured.

Wedgewood's pyrometer depends on the principle that dried clay contracts when exposed to heat, by an amount nearly proportional to the temperature. A number of short clay cylinders are accordingly made of precisely the same diameter, and this diameter is measured by placing them in a wedge-shaped cavity with graduated sides, formed of two graduated metallic rods slightly inclined to one another. The distance to which the clay may be inserted will mark, on an enlarged scale, its diameter. Expose a cylinder to the temperature to be measured, and after cooling insert it in the wedge-shaped cavity. The distance to which the cylinder will enter shows the temperature. The scale must be reduced to degrees empirically, and will vary with the kind of clay. It is found that on a long exposure to high temperature the clay continues to contract, and thus very accurate readings cannot be obtained with this pyrometer.

Another method of measuring temperatures is dependent on the specific heat of platinum. A piece of this metal is exposed to the temperature to be measured, and then dropped instantly into a calorimeter, as if measuring its specific heat, Experiment 138. The same formula is employed, except that instead of knowing the upper temperature and determining the specific heat, we now have the latter given as equal to .032, and therefore $T = \frac{(t' - t)(w' - w + wc)}{.032(w'' - w')} + t'$. If iron is used, .114 must be taken for the specific heat. The great difficulty with this method is the loss of heat in transferring the metal to the calorimeter, and also that a portion of the water is converted into vapor, causing a large loss, due to the latent heat absorbed by the steam.

The thermo-pile affords an easy means of measuring high tem-

peratures. It is only necessary to connect its terminals with the galvanometer and expose its junction to the temperature to be measured, which will be nearly proportional to the deflection of the galvanometer needle. It is better to immerse the other terminal of the thermo-pile in cold water, when the electromotive force, and consequently the current and the deflection, will be proportional to the difference in temperature.

Siemens' resistance pyrometer depends on the change in electrical resistance in a platinum wire when exposed to changes of temperature. It is merely necessary to make the coil one side of a Wheatstone's bridge, or connect it with one coil of a differential galvanometer, and measure its resistance when exposed to changes of temperature. It may also be used to measure ordinary temperatures of inaccessible places, as in deep sea-soundings, by interposing as the second arm of the bridge a similar coil immersed in water, which may be warmed or cooled at will. The temperature is altered until no current passes through the galvanometer, when the temperature will equal that of the other coil, and may be measured directly with a thermometer. When the point whose temperature is to be determined is very distant, the unknown temperature of the connecting wires is likely to introduce a large error. This may be avoided by inserting in the circuit of the other arm of the bridge a second wire running side by side with that connected with the platinum. The temperature is thus always the same for both, and the error thereby compensated.

144. Heat of Combustion.

Apparatus. A Dulong calorimeter, which consists of a vessel in which the combustion takes place, with four outlets. One is connected with a long spiral tube, like the worm of a still, through which the products of combustion are drawn; a second aperture serves to admit the substance to be burned, a third admits the air or oxygen, and the fourth, closed by a plate of glass, enables the observer to watch the combustion and see that it is complete. The whole is contained in a larger vessel containing water, whose temperature is rendered uniform by a stirrer, and is measured by a thermometer. A second thermometer serves to measure the temperature of the escaping gases. The latter should pass through a meter into an aspirator, and if the substance to be tested is a gas, a second meter should be inserted to measure its volume.

Experiment. Measure the temperature of the air of the room, of the water of the calorimeter, and the height of the barometer. Light the gas burner, place it inside, and regulate the flow from the aspirator, so that the combustion shall be complete. Read the temperature at regular intervals, keeping the water well stirred. Extinguish the light and letting the calorimeter cool, determine the correction for loss by radiation precisely as in Experiment 140. Call w the weight of gas burnt, w' the weight of air used to consume it, W the water equivalent of the calorimeter and contents, H the required heat of combustion, t the corrected increase of temperature, and t' the excess of temperature of the escaping gases above the air of the room. Then $wH = Wt' + (w + w')t$. The weights of the gases are determined from their volumes and specific gravities, correcting for temperatures and pressures. If a solid or liquid combustible is employed it must be weighed directly.

145. Efficiency of Gas Burners.

Apparatus. A Bunsen burner, whose consumption is measured to thousandths of a foot by a meter, or an alcohol lamp which may be weighed while burning, a tin vessel containing water to be heated, a thermometer and a balance and weights.

Experiment. Weigh the tin vessel empty, and when partly filled with water, and observe the temperature of the room and of the water. Light the gas, and take a series of readings of the temperature of the water at the beginning of every minute, and thirty seconds later of the meter, as described in Vol. I, Experiment 57. When the water begins to boil weigh again, then let it boil for ten minutes and make a final weighing. The average of each two consecutive readings of the meter may be taken as its true reading at the beginning of the minute. Construct a curve with these readings as abscissas, and temperatures as ordinates. The tangent of the angle this curve makes with the axis of X, gives the increase of temperature per cubic foot consumption of gas. Multiplying the number of degrees thus obtained by the water equivalent in kilogrammes of the tin vessel and contents, gives the number of units of heat evolved per cubic foot of gas burned. This same quantity divided by the consumption per minute, will give the number of units of heat per minute with the

particular burner employed. From the loss of weight of the water during boiling, the heat received may also be determined, calling the latent heat of vaporization 537. This method is much less delicate than the other, unless the source of heat is very powerful. Compare the effect of placing the tin vessel at different distances from the lamp, and also of using a luminous, instead of a non-luminous flame. Comparing the results with those obtained in Experiment 144, we see how small a portion of the whole heat of the flame is utilized.

146. Mechanical Equivalent of Heat.

Apparatus. In Fig. 97, *A*, *B*, are two hollow iron cones, of which the outer and lower one may be kept revolving with a constant velocity by a belt passing over a pulley *C*. Any small motor, as a steam or electric engine, clockwork, or even a crank, may be used to maintain this motion, which should be as uniform as possible. The upper cone is filled with mercury, and contains a thermometer to measure its temperature; a light arm, *ED*, is attached above, to whose ends cords are fastened over pulleys, and equal weights are hung at the ends to tend to turn it in the opposite direction from that in which *C* is turning. Stops should be placed on each side of *ED* to prevent its turning too far.

Experiment. Start the motor so that *C*, and with it the lower cone *B*, shall revolve at a uniform rate, and see what loads must be attached to the pulleys to hold the beam in equilibrium. It is well to use somewhat too small a load, and check the motor with the finger, so as to keep the beam balanced between its two stops. Everything being in readiness, read the temperature of the mercury by the thermometer *F*, start the motor, keep the beam between the two stops, and observe the speed. This may be done by a shaft-speeder, or better, by the arrangement described in Experiment 158. The work required to overcome the friction between *A* and *B* will now be converted into heat, and the thermometer will accordingly rise steadily. Read the temperature every minute for five or ten minutes, and then determine the correction for radiation by stopping the motor

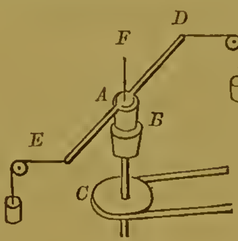


Fig. 97.

and taking readings as the cones cool. Determine the increase in temperature, t, correcting for radiation, as in Experiment 140. The water equivalent of the two cones will equal their weight multiplied by .114, the specific heat of the iron, and to this must be added the weight of mercury multiplied by .033, its specific heat, or $w's' + w''s''$; multiplying this quantity by t gives the amount of heat generated. To determine the amount of work expended, call W the weight on the strings over each pulley, $2l$ the length of ED, or perpendicular distance between the two horizontal strings, and n the number of turns per minute of the pulley C. Then the work done will be the same as if a force W on each end of ED, or $2 W$ on one end, turned it round n times, or traversed a distance of $2\pi ln$. The work done is accordingly $4\pi ln W$. If then M is the mechanical equivalent of heat, or work which may be done by one unit of heat, we must have $M(w's' + w''s'')t = 4\pi ln W$, or $M = \frac{4\pi ln W}{(w's' + w''s'')t}$, in which care must be taken to use as units the kilogramme and metre. This experiment should be repeated several times, and also varied by placing a load on the inner cone, A, thereby increasing the friction, and consequently the rate of heating.

147. Two Specific Heats of Gases.

Apparatus. In Fig. 98, A is a large flask closed by a cork, through which pass a tube with a large stopcock, B, and a bent tube forming a gauge, CD. A large rubber tube may be attached to B so as to partially exhaust the air.

Experiment. The theoretical determination of the mechanical equivalent of heat and of the velocity of sound in gases, depends on the accurate determination of the ratio of the specific heat of gases under constant pressure to that under constant volume. Evidently the former quantity must be the greatest, since when a gas is heated under constant pressure, besides warming the gas, a certain amount of energy must be expended in overcoming the pressure, so as to allow the expansion to take place. The ratio of these specific heats is best determined by the apparatus of Clément and Désormes, represented in Fig. 98. Connect the rubber tube with B, open the stopcock and partially

exhaust the air, either by the mouth or by an air pump, so that the water shall rise nearly to the top of CD. Close the stopcock and disconnect the rubber tube, when, even if there is no leak, the liquid will slowly descend, because the gas cooled by the rarefaction gradually recovers the temperature of the surrounding medium. Wait until it comes to rest, and read the exact level of the water. Open the stopcock for just one second, close it and take readings every five or ten seconds, as the water rises in the gauge, until it comes to rest. When the cock is open, air rushes in, heating the enclosed air, so that when the cock is closed and the air has time to give up its heat to the surrounding bodies, it is found that the exhaustion is still about a third of what it was at first. To determine this fraction with precision, or rather, what it would be were there no loss by radiation while the stopcock was open, construct a curve with abscissas equal to the times, and ordinates to the height of the water level in CD. This curve forms a nearly horizontal line before the stopcock is opened, then is nearly vertical until it reaches the axis, then a sinuous line, owing to the vibrations of the air at the orifice, and finally a smooth curve after the stopcock is closed. Only the first and last of these forms can be observed. Prolong the curved portion until it meets the vertical line, and repeat the experiment, if necessary, until the time during which the stopcock is left open is such as to bring this point near the surface of the water.

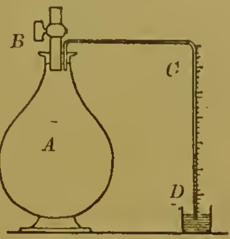


Fig. 98.

Call p' the height of the water before the stopcock is opened, and p'' the height it finally attains, so that it first descends through p', and then rises through p''. Then the ratio of the specific heat under constant pressure, to that under constant volume will equal $\frac{p'}{p'-p''}$.

To prove this formula, suppose a given mass of gas confined, so that its volume cannot alter, and placed in a medium, whose temperature is somewhat greater than its own. It will gradually be heated, and an amount of energy which we may call A will thereby be transferred from the medium to it. Its pressure also

will be increased. Now suppose that it is allowed to expand, until its pressure becomes the same as at first. The first effect would be to cool it, but soon it will absorb enough heat from the surrounding medium to render the temperature the same as that of the medium. Call B the additional amount of energy thus absorbed. Evidently the ratio of the specific heat under constant pressure, to that under constant volume will be as $A + B$ is to A. If now we compress the gas to its original volume, the energy B will be set free as heat, and will soon be lost by radiation to surrounding objects, while the energy A will remain and may be recovered if the gas is cooled down to its original temperature.

Now in the experiment just performed, when the gas is compressed by an amount which may be represented by p', corresponding to $A + B$, the quantity of heat set free will be represented by p'' corresponding to B, or since for these small changes, the energy may be taken as proportional to the change in pressure, we shall have $A + B : B = p' : p''$. This may also be written $A : B = p' - p'' : p''$ or $A + B : A = p' : p' - p''$, hence the ratio of the two specific heats, $\dfrac{A + B}{A} = \dfrac{p'}{p' - p''}$.

MECHANICAL ENGINEERING.

The number of experiments a Mechanical Engineer is called upon to perform, is generally small, but their importance can scarcely be overestimated, as no other branch of Physics has so great a value, both as a saving of money, and as a protection to life and limb. The following Experiments require little apparatus beyond that usually accompanying a furnace, a boiler and engine, and a dynamometer. The large original cost of the engine is, in part, compensated by its value in a technical school or college as a source of power, on which account alone it is considered a necessity in many such institutions. A knowledge of piping, or carrying steam in pipes, and of running a boiler or engine, is so requisite to the following work, that a special description of them is prefixed. The proper method of taking care of a boiler or engine can, however, be learned only by experience, and no one should be entrusted with either, for the first time, except in the presence of an experienced engineer. The instructions given below must therefore be regarded merely as aids to the pupil, and to simplify the work of the instructor.

Piping. To understand the proper method of conveying steam from one point to another, a short description is here given of piping, and applies, with slight changes, to the conveyance of any other fluid, as air, gas or water. Pipes from 15 to 20 ft. in length are used, and of diameters reckoned in eighths of an inch, as $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and 3 inches. The intermediate sizes are not in use. These distances denote interior diameters, but they are really too small, the actual diameter of a $\frac{1}{8}$ pipe being .28 in., of a $\frac{1}{2}$ pipe .62 in., and of a 1 inch pipe 1.05 ins. To connect two pipes of the same size, a screw thread is cut on the end of each with a die, and a *coupling,* or short piece of larger and thicker pipe with a thread inside of it, is screwed on one, first interposing a little red lead. The other pipe in then screwed into

it until a tight joint is obtained. The red lead serves to lubricate the joint, and at the same time renders it tighter; without the red lead the pipes could be turned only with great difficulty, and it would be almost impossible to take them apart. A rusted joint is made by wetting the ends and screwing them together, when the iron rusts, and it becomes almost impossible to separate them. When a screw thread is cut on the outside of a pipe it is called an *outside* or *male* screw; when cut in the interior, an *inside* or *female* screw. A die cuts a male, a tap a female screw. The latter are never cut on common steam pipes as they weaken them too much, and render them liable to split.

If the pipes are not of the same size, *reducing couplings* are used, or thick tubes with inside screws of different sizes cut in the two ends. If the difference in size is very great, a *bushing* must be inserted in the smaller end of the coupling. This consists of a short tube with an outside thread to fit into the coupling, and an inside thread to fit on to the pipe. If two couplings have to be connected, *nipples* are used, or short pipes with outside threads on both ends. To close a pipe a *cap* is employed, made like a coupling, except that it is closed at one end. A coupling may, in the same way, be closed by a *plug*, a piece of iron with an outside screw on one end, and cast square at the other, for convenience of turning it with a wrench. When two pipes are already laid, they cannot be connected as described above, since one must be turned around in order to screw it into the coupling. What is called a *right and left* is then used; that is, a coupling with a right handed screw cut in one end, and a left handed screw in the other. Right and left handed screws are cut on the pipes, and the coupling turned into place without disturbing them. Of course, if there is a space between the two pipes, an additional pipe and coupling must be inserted. Rights and lefts when larger than ½ inch are marked by a number of ridges on the outside, so that they may be recognized at a glance. Two pipes are connected at right angles by an *elbow*, or *L*, which looks like a coupling bent at right angles. If the pipes are to be inclined at any other angle, an *L* must be screwed on the end of each, and the two *Ls* connected by a nipple. In this case, the two pipes will not lie in the same plane, one being above the other.

When a branch is to be inserted in a pipe, a *T* is employed. This resembles a coupling with a short pipe on one side with an inside screw, forming, in fact, a combination of an *L* and coupling. The three ends may be either of the same, or of different sizes. For convenience of fastening pipes to woodwork, *Ls* and *Ts* are sometimes made with projections cast on the side, with holes through which screws may be passed; such fittings are called *drop Ls*, and *drop Ts*. Larger pipes are held in place, when necessary, by clips, or pieces of sheet metal bent around the pipe, and fastened down by screws. To insert two branches into a pipe at the same point, or to make a pipe divide into three, a *cross* is used, which is a *T* with two branches instead of one, that is, a short pipe on each side.

The above are the most common fittings, and with them almost all connections can be made; it will be noticed that couplings, caps, *Ls*, *Ts*, and crosses have only inside screws, pipes, plugs and nipples only outside, and bushings both. Evidently the inside and outside screws must always come alternately. Where there is any probability that additional connections will have to be made, it is best to put in *Ts* frequently, instead of couplings and *Ls*, plugging the extra holes. The additional expense is small, while the saving effected may be very great. In long pipes it is also often better to insert plugged *Ts* at short intervals, or rights and lefts. If this is not done, and a branch must be inserted, it is either necessary to take the fittings all to pieces at one end, so that the pipe will turn round, or else to cut the pipe in two, remove a piece, and insert a *T*, making the last joint by a right and left. If a right and left has been already inserted, it may be disconnected at once at this point, while if a plugged *T* had been used, it would only be necessary to remove the plug and screw the pipe in, in its place. If the pipe is used for gas, it is not even necessary, in this case, to shut the latter off.

To avoid turning the pipe, *unions* are sometimes used. In these, two planed surfaces are screwed on to the pipes to be joined, a washer interposed and then brought together by an outside screw cut on one end, and a loose nut fitting over the other. Screwing the nut in place fastens the pipes together, and they are easily separated or turned at any time.

To cut off communication through a pipe, either wholly or in part, a *cock* or *valve* is used. The former, of which we have examples in common gas and water fixtures, consists of a plug passing through the pipe at right angles, and with a hole through it, which may be turned either in the direction of the pipe or across it. If the pipe is large or the pressure great, valves are much better. They consist of cast iron boxes, in which a screw turned by a small wheel forces a conical plug against a partition in the box so as to close a hole bored in it, thus cutting off communication between the upper and lower parts, which open on opposite sides of the valve. The valve is connected with the pipes by two female screws, like a coupling. Other forms of valves are sometimes used, as for instance, one in which a screw forces a diaphragm at right angles to the pipe. This valve has the advantage, when open, of opposing much less resistance to the flow of the fluid, but it is much more liable to leak.

The tools used for piping are few in number. To divide a pipe at any required point, it is held in a stout vice, and cut by turning around it a cutter in which a sharp edged steel wheel is forced by a screw against the pipe. Care must be taken to turn the screw gradually, or the pipe will be flattened or bent, and to hold it at right angles to the axis, or a screw-like cut will be made. Outside screw-threads are cut on pipes by a die turned in the usual way, by two long handles. Inside screws are never cut on pipes, and connections always come with the screws cut. To screw the parts together, pipe-tongs are used, made somewhat like pliers, only so arranged that they wedge on the pipe, holding tighter the harder they are turned. For unscrewing they must be turned over. For different sized pipes different tongs must be used, or they are sometimes made adjustable with a screw to fit any size. To turn the *Ls*, *Ts*, plugs, etc., a monkey-wrench is most convenient.

Steam Boilers. Boilers are made in a great variety of forms, but are generally of sheet iron or boiler plate, held together with rivets. The tubular form is the most common, in which the hot air and gases from the fire are carried in tubes through the centre of the boiler. Being thus completely surrounded with water, the heat is rapidly transmitted to it, producing steam quickly and

preventing much of the heat from escaping with the products of combustion. Cast iron is sometimes employed, the earliest and best known form being the Harrison boiler, which consists of a series of cast iron spheres like bomb-shells. This is one of the safest forms of boilers, but is heavy and sometimes gives trouble from collecting scale, as described below.

A pipe is connected with the upper part of the boiler to carry off the steam as it is generated, and the space not filled with water, called the steam space, should be considerable, otherwise when the water is boiling violently, it will be carried off with the steam, which is called *foaming* or *priming*. Sometimes a projection, called a steam dome, is made in the boiler to avoid this difficulty. The water is admitted by a second pipe, which may be connected directly with the hydrant if the pressure is sufficient, or with a force pump driven by hand or by an engine. To show the height of the water in the boiler, one or more vertical glass tubes or water gauges are connected below with the water, and above with the steam in the boiler. Great care must be taken when cleaning them, which should be done only when necessary, and then by pushing a cloth through them with a stick, as metal is liable to produce a scratch which will eventually cause the tube to break. Three or four outlets closed by valves are commonly placed at different heights in the side of the boiler, and the height of the water determined by opening them in turn. Steam will come from those above, and water from those below the water line. Unfortunately, owing to the foaming of the boiler, it is sometimes very difficult to determine the true amount of water in the boiler when steam is made very rapidly, as the water in the glass gauge will be in constant motion, and both water and steam will come from all the valves. To measure the pressure in the boiler a gauge is connected with it by a pipe, showing the pressure in pounds by the motion of an index. To obtain the real pressure, 15 pounds must be added for the pressure of the atmosphere. To empty the boiler another pipe enters near the bottom through which the water may be drawn out. A large hole, called a manhole, is also commonly made in the top, so that a man can get inside for repairs or other purposes. Every boiler should also be provided with a safety valve, or a hole closed by a plate pressed

8

against it by a weight, such that if the pressure is too great it is lifted and the steam escapes.

Every boiler, after it is set, should be tested by what is called the cold water test, to see that it is strong enough, and that it does not leak. For this purpose it is completely filled with water and connected with a small force pump worked by hand. Working the pump gradually, the gauge at once rises and should be carried considerably above the pressure at which it is to be used. Communication being then cut off between the pump and boiler by a valve, if there is no leak the index should remain unchanged. With small boilers the plan has been tried of inserting a metallic plate inside the boiler, and connecting it with the positive pole of a powerful galvanic battery by an insulated wire. Connecting the other pole with the boiler, decomposition of the water will take place, and the gases thus set free will produce the required pressure. The conductibility of the water should be increased by adding a little salt.

In running the boiler, care should be taken that the water does not get too low. Of course the fire must never be made when the boiler is empty, or it would soon destroy it. After the boiler has been used for some time much trouble is often experienced from a stony sediment or coating of the interior of the boiler, called *scale*. The non-volatile salts remaining in the water as it is boiled away, collect, often in large quantities, especially when the water contains much lime. This prevents the heat from being transmitted freely to the water, and hence the iron is overheated and soon burnt out. To avoid this difficulty, the same water should be used over and over again if possible, as with a condensing engine, or in buildings heated with steam. Sometimes, also, the water should be partly blown out from the lower aperture by the steam pressure. The mechanical disturbance thus carries off much of the scale. Various other remedies are recommended, but if the scale still collects, a man should occasionally be sent inside to chip it off with a hammer and cold chisel.

The management of the fire is much the same as that of a common house furnace. There are two doors, one above through which the coal is thrown, and one below, for removing the ashes. In each door is a slide by which a small aperture may be closed to

a greater or less extent, as desired. When the lower door is open, the draft in the chimney draws air through it and through the coal, producing an intense combustion. When the upper door is open the cold air is drawn above the coal, cooling it and deadening the fire. The draft in the chimney may be regulated either by dampers which close it to a greater or less extent, or by a slide which admits cold air, cooling it and thus lessening the draft. There are therefore three ways of increasing the heat; closing the upper door, opening the lower door, and opening the damper, or closing the slide which admits cold air into the chimney. When the fire is low, and fresh coal has been put on, or when starting the fire, the upper door should be closed and the lower opened, but when under way it can generally be completely regulated by the slides. No definite rules can be given, as every thing depends on the particular conditions in each case, as draft, kind of fuel, size of furnace, heat required, etc. On leaving the furnace for the night, or when not wanted for some time, the slide in the upper door should be opened, the lower one nearly or quite closed, and the draft lessened.

When the pressure of steam is just equal to that due to its temperature, as is the case when it is in contact with the water of the boiler, it is said to be saturated, and it will begin to condense at once if cooled, or if the pressure is at all increased. If heated above this point it is said to be superheated. If much water is carried over in drops with the steam, the latter is said to be wet, while if no water is present, it is called dry steam. Dry, superheated steam is easily recognized by its bluish color, and the hand may be held in a jet of it with impunity.

Steam Engine. The most essential part of an engine is a cylindrical iron box called the *cylinder*, in which is a movable partition called the *piston*. Steam is admitted on one side of this, driving it to the other end, and then on the other side driving it back. This rectilinear motion is converted into a circular motion by means of a crank, and is rendered nearly uniform by a heavy cast iron wheel, called a *fly-wheel*. The steam is directed by means of a *slide valve*, so that it shall be admitted first on one side and then on the other of the piston, which is done automati-

cally by moving the valve by an eccentric on the axis of the fly-wheel.

To start the engine, it is merely necessary to turn on steam, when the engine will begin to move, unless the piston is at the dead point, that is, at the end of its stroke. In this case, the fly-wheel must be turned slightly, by hand, when the steam will carry it round. If the steam is turned on at once, it will rush into the cold cylinder, and condense, forming water, which being almost incompressible, and coming between the piston and the end of the cylinder, is likely to break off the cylinder-head. Accordingly an outlet is made in the cylinder which should always be opened before the steam is admitted, and closed when the engine has run for some time, and the cylinder heated so that the condensation is slight. To prevent the engine from going too fast, when it is doing no work but overcoming the friction of its parts, a governor is attached, which commonly consists of two balls turned by the flywheel, forming a conical pendulum, and which if the speed becomes too great, fly apart and cut off the steam. The pipe by which the steam is admitted into the cylinder is called the *supply pipe*, that by which it passes off, the *exhaust*. A great deal of power is lost in driving the piston back against the steam on the other side and forcing it through the exhaust. To diminish this loss, the exhaust is sometimes connected with a condenser, or cold space, by which the steam is reduced to water, and its pressure greatly diminished. This form of engine is called a condensing engine, but is not much used, on account of the expense and bulk of the condenser, except to avoid using salt-water at sea. Again, there is a great loss, since the steam is admitted at high pressure, and when the exhaust is opened, allowed to expand until its pressure is no greater then that of the atmosphere, without doing any useful work. A part of this loss is prevented by a cut-off, by which the steam is admitted into the cylinder until the piston has performed part only of its work, communication with the boiler is then interrupted and the piston is forced on by the expansion of the steam. The cut-off is accomplished in various ways, but generally by giving a proper motion to the slide-valve.

148. Efficiency of Boilers.

Apparatus. The furnace and boiler, a large graduated vessel to measure the water, and a platform balance to weigh the coal.

Experiment. The most important experiment that a mechanical engineer is called upon to perform, is to determine how much coal is required to evaporate a pound of water in a given boiler. Its pecuniary value often represents many thousands of dollars, and therefore too great care cannot be taken with it. The trial should last at least twelve hours, and better thirty-six, or even a longer time. A large number of students may participate in the trial, and watching by turns, render the work less laborious. It is best to combine this experiment with Nos. 153 and 154, as there will be ample time for all without interfering.

Different results are obtained with different pressures, and changes in the intensity of the fire. Accordingly, both must be kept as nearly constant as possible. The best effect is generally attained with a moderate fire, and less if the combustion is very rapid or very slow. The coal is weighed directly by shovelling it on to the platform scale, and thence into the furnace. To measure the amount of water converted into steam, is more difficult. It is best done by a condenser, as in a condensing engine. A temporary substitute is a steam coil, such as is used for heating buildings, immersed in water, and collecting the water as it condenses. Approximate results may be obtained by measuring the water admitted, but it is then essential that the water level shall be the same at the beginning and end of the measurement, a condition not easily attained. The amount of water admitted may be measured by a water meter, by counting the number of strokes of the force pump, or by connecting a strong, iron vessel with the boiler, by two pipes which may be closed by valves. A third pipe and valve serves to admit the water. The latter valve is then closed, and the other two opened when the steam displaces the water, and lets it run into the boiler.

The day before the experiment is to be performed the boiler should be filled until the water stands exactly at a height marked on the glass gauge tube, and the amount measured. The fuel should also be weighed and put in the furnace ready to be kin-

dled. Early the following morning the fire is lighted, the time noted, and if a thermometer is in the boiler, its rise in temperature per minute observed. The gradual rise in the pressure of the steam should also be recorded. When the desired pressure is reached, the steam is allowed to escape, and this pressure maintained. A careful record is then kept of the amount of coal and water used, and the time at which each is added. The level of the water in the boiler should be kept nearly constant, though, owing to the foaming, this cannot be done with any accuracy, when steam is generated very rapidly. The real commencement of the experiment is when the steam begins to escape, and at the end of the time everything should be brought as far as possible into the same condition as at the beginning, that is, the fire about equally intense, and the water level and steam pressure the same. Then draw the fire, measure the amount of steam generated, and the lowering of temperature as the water cools. If the water entering the boiler is measured it is better on drawing the fire to shut off steam, if this can be done without unduly increasing the pressure, and measuring the amount of water which must be added, or withdrawn, to bring the level to the same height as at first.

The observations made before the steam was allowed to escape from the boiler, serve to show how much time is required to fire up, and the amount of fuel used. The amount of fuel wasted in heating the furnace, boiler and chimney, and escaping up the latter, is then easily calculated, as follows. Let W be the weight of coal burnt, and h its heat of combustion, then Wh will be the total amount of heat generated, if the combustion is complete. Again, let w be the weight of water in the boiler, and t the difference in temperature of the water when admitted to the boiler and that due to the pressure at which the steam is blown off. Then wt is the amount of heat employed usefully in heating the water, and the remainder, or $Wh - wt$, the amount lost. The most important observations, however, are those taken while the steam is passing off, and hence often these only are taken, beginning and ending the trial with a good fire of equal brightness at each time. The weight of water evaporated during the whole trial divided by the amount of coal burnt, gives directly the evaporation per pound of

coal. It is very instructive to construct curves showing the relation of each two of the three quantities, time, weight of coal, and weight of water. All should be approximately straight lines, and the inclination of that showing the relation of the coal to the water, gives the weight of water to the pound of coal. The theoretical amount, calling W' the weight of the coal burnt, $= W'h$, and the amount usefully expended equals $w'(t + L)$, calling w' the amount of water evaporated and L its latent heat at the pressure of the steam. If T is the temperature of the steam, $L = 606. + .3\ T$ from which L is readily determined. The ratio of the heat received to that expended or $\frac{W'h}{w'(t + L)}$ equals the efficiency of the boiler. For good coal, h will equal abont 8000, or the same as pure carbon, the presence of foreign matters being compensated by the small amonnt of hydrogen, whose calorific power is much greater. Accordingly the maximum amount of water at 20° which could be evaporated, would be $\frac{8000}{716} = 11$ pounds, while in actual practice, 5 to 7 pounds are very high results.

One of the principal difficulties to be apprehended in this experiment is that some of the water will escape in the liquid form, being carried over mechanically by the stream. This of conrse greatly increases the apparent evaporation, while in reality it is a serious defect in a boiler, throwing much water into the engine. On this account a boiler which appears to give most excellent resnlts by this test, may be in fact, only one which foams very badly. Great care shonld therefore be taken at intervals during the test to let a little steam escape, and see that it is dry.

149. Covering Steam Pipes. I.

Apparatus. A number of 1″ steam pipes CD, $C'D'$, Fig. 99, about 8 or 10 ft. long, closed below by small stopcocks D, D', called pet-cocks, and above by steam valves, C, C', are connected with the boiler so that they shall be vertical. The best way is to bring a pipe A horizontally from the boiler, and then vertically to the required height B; on the end of this, put a T and hang the pipes from the branches. Two pipes only need be used at a time, but it is in some respects better to use four. All should be arranged symmetrically from the central pipe which should be well covered to prevent loss by radiation. The vertical pipes

are precisely alike, but are covered in various ways, one with plaster or cement, a second with felt, a third with felt covered with canvass and painted, and the fourth left in its ordinary condition. *E*, *E'*, are two similar graduated vessels to collect the condensed water.

Experiment. When steam is conveyed through a long uncovered steam pipe, the loss of heat by radiation and condensation of the water is far greater then is ordinarily supposed. A great saving may often be effected by covering the pipe with felt or other material, and the object of the following experiment is to determine the comparative efficiency of different coverings.

The fire during the experiment should be kept as nearly as possible the same so that the pressure may be unchanged, and the steam in the boiler should be very dry. Every thing being in readiness, open the pet-cock *D* slightly, to let the water escape as it condenses, and at the beginning of a minute open *C* wide. The steam rushing into the cold pipe will rapidly condense so that a considerable amount of water will be forced into the graduated vessel. Open the pet-cock so as to allow the water to escape freely, but not so wide as to let out much steam. Read the volume of water collected at the end of every minute, and construct a curve with abscissas equal to the times, and ordinates to the volume of water condensed. Repeat the experiment with the other pipes closing the valve of each when its test is complete. A series of curves is thus obtained, and a comparision shows the relative efficiency of the various coverings. These curves first rise rapidly until the pipes are well heated, and then become nearly straight lines, their inclination showing the relative efficiency.

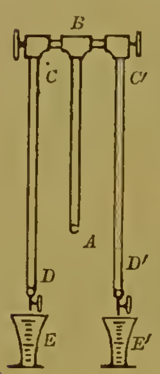


Fig. 99.

From them it will be noticed that while the loss from an uncovered pipe is the greatest, that it takes longer to heat up a covered pipe, so that sometimes when the steam is required very quickly, or for a very short time, the uncovered pipe may be the most effective. To determine the actual condensation per foot of length, a straight line must be drawn nearly coinciding with the curve, and the increase of volume of the water per hour, noted. Dividing this by the length of the pipe in feet, gives the rate of

condensation. From the latent heat, it is easy to reduce to actual heat units. The experiment may be varied, not only by using different coverings, but by varying the pressure of the steam, or the temperature of the surrounding air.

If the pressure of the steam is liable to vary, a valve should be inserted in *A* and a steam gauge connected with *B*. The reading of the gauge is then kept constant by opening or closing the valve.

150. Covering Steam Pipes. II.

Apparatus. A number of pieces of steam pipe of the same diameter and length, closed at one end by caps, and at the other with corks, through which thermometers pass. The pipes are covered, as in the last Experiment, with various substances to be tested, and are all placed side by side on the table but far enough apart not to heat each other.

Experiment. Fill each tube with boiling water, insert the cork and note the temperature every minute, as in Vol. I, Experiment 5. If four thermometers are to be observed, read one at the beginning of each minute, a second, quarter of a minute later, the third at the half minute, and the fourth at the three quarters, so that they shall be read in turn, each one at intervals of precisely one minute. Next, construct curves with ordinates equal to the logarithms of the excess of temperature above the surrounding air, and abscissas to the time. The relative inclination of the various lines gives the comparative rate of cooling.

151. Testing Gauges.

Apparatus. The apparatus described in Vol. I, Experiment 55, and represented in Fig. 44, is well adapted to this experiment. If an open mercury gauge is not available, a *T* may be screwed on to the outlet of a small force pump, the gauge to be tested attached to one branch, and a standard gauge to the other. The best form of gauge for a standard, next to an open mercury column, is a graduated glass tube closed at one end and containing air, the lower part of the tube being filled with mercury. After calibrating the tube the graduation may be reduced to millimetres of mercury by Mariotte's law. Any good gauge may be employed as a standard, if its errors are first determined by comparison with one known to be correct. A still simpler method of comparing two gauges is to connect both with the same steam-pipe, and compare the readings under various pressures.

Experiment. By working the pump, any desired pressure may be applied to the gauges, and they may thus be compared directly. The readings with the mercury gauge may be reduced to pounds to the inch, by the rule that 51.7 mms. of mercury produce a pressure of one pound per inch, hence, the readings in millimetres must be divided by this number to give the pressure in pounds. To obtain the total pressure, the height of the barometer should be added, but it is generally sufficiently exact to add 15 lbs. Care must be taken that the joints are tight, and that the pressures when high, remain constant long enough to read the gauges accurately. Metallic gauges often give erroneous readings when exposed to sudden changes of pressure owing to their imperfect elasticity, and a similar effect is observed with air-gauges, owing to the change in temperature due to sudden condensation or rarefaction. Time should therefore be allowed for the readings to become constant.

Take a series of simultaneous readings, of both gauges, and construct a curve with abscissas equal to the true reading and ordinates to their difference, or the error. This curve may be used directly to correct all readings taken with this gauge, and as the error is likely to alter from time to time it is well occasionally to repeat this experiment.

152. Pressure and Temperature of Steam.

Apparatus. Besides the furnace, boiler, and pressure gauge the only other apparatus needed is a thermometer which can be placed inside the boiler. The best way to insert a thermometer in the boiler, is to bore a hole in the side and cut a thread in it, then screw in a tube from the inside, and close the inner end with a cap. The thermometer is then placed in this tube, and soon attains the interior temperature without being subjected to the pressure of the steam. To ensure good contact, the tube should be filled with mercury or oil. Other methods of determining the temperature may also be employed, as an air thermometer, a thermo-electric pyrometer, or a Siemens' electric resistance thermometer.

Experiment. Start the fire, and as steam is formed, note the corresponding temperatures and pressures. Construct a curve with these quantities as coördinates, and compare it with the

results found by Regnault, and given in the table of the pressure of steam at various temperatures.

153. Indicator Diagrams.

Apparatus. The steam engine described above, and a steam indicator.

Experiment. The steam indicator consists of a small cylinder whose piston is held down by a spring like that of a spring balance, so that the height to which it rises at any instant, is proportional to the pressure of the enclosed gas or steam. A hole is bored in the cylinder head, and the indicator attached by a pipe with a valve, so that when the valve is opened, the height to which the piston rises, denotes the pressure at the instant, in the cylinder. To record this pressure, a pencil, or metallic point, is attached to the piston and moves over a cylinder on which is stretched a piece of paper, so prepared that the passage of the point will make a black mark. The cylinder is connected with the piston of the engine, by a string and lever, so that it shall turn by an amount proportional to the distance traversed by the piston. A spring keeps the string tight and turns the cylinder back when the piston returns. Evidently as the piston moves, the pencil will describe a curve whose abscissas show the position of the piston, and ordinates the pressure throughout the stroke. To draw the curve, or indicator diagram, attach the paper to the cylinder, and open the indicator valve without connecting the string with the piston. The pencil will then simply rise and fall during each stroke, drawing on the paper the axis of Y. Now close the valve and attach the string to the piston. The cylinder will then turn forwards and backwards, and the pencil will describe the axis of X. Now after seeing that the engine is running as uniformly as possible, open the valve, and the pencil will at once describe a diagram, and repeat it again and again as long as the engine continues to work uniformly. After drawing the curve once, renew the paper and repeat; after a few trials a good curve will be obtained. Record the time, pressure of steam in the boiler, and number of revolutions per minute. To reduce the result we must also have the interior diameter of the cylinder and the length of stroke.

The indicator is a most important instrument in studying the steam engine, as almost all the defects or peculiarities of the latter are rendered visible by it. On this account it is necessary to study the form of its diagrams a little more in detail. At the beginning of the stroke the steam enters and the pressure rises almost immediately to that in the boiler, forming a line nearly vertical; it then becomes horizontal till the end of the stroke, when it quickly descends to the line of atmospheric pressure and remains there during the return stroke. If there is a cut-off the pressure begins to descend at the point of cutting off, at first rapidly, and then more slowly, forming, by Mariotte's law, a curve nearly coinciding with a hyperbola. If a condenser is used, the line on the return stroke descends below the atmospheric line approaching the true zero of pressure.

In practice these forms are never perfectly attained, and all the corners of the diagram are rounded, instead of angular. Often, especially at high speeds, the pressures seem to alternately increase and diminish, an effect really due to the vibrations of the spring. The area, however, is sensibly unchanged, and may be found by drawing a line through the centre of the vibrations.

The work done by the steam during any short portion of its stroke equals its length multiplied by the total pressure, or pressure per square inch multiplied by the area of the piston. Hence, it may be measured by the area included between the curve and the axis of X, or line of no pressure. Since the pressure on the back stroke is prejudical, the area between the lower part of the curve, and the axis must be subtracted. Therefore the area enclosed within the complete curve is a measure of the total work done. To obtain this in foot-pounds, the area of the diagram must be divided by the area of a rectangle whose height is one pound, and whose length is one foot, on the scale to which the diagram is drawn. Multiplying this quotient by the area of the piston in inches, gives the total work done by the piston in one stroke. To reduce this to horse-power, multiply by the number of single strokes per minute, and divide by 33,000. To express it mathematically, let r be the radius of the cylinder, n the number of double strokes, which are more easily counted than single strokes, A the area of the diagram in inches, d the distance

on the diagram representing 1 pound, d' that representing a motion of 1 inch of the piston. Then the work of each stroke in foot-pounds, will equal $\frac{A}{12d'd}$, and the horse-power will be,

$$H = \frac{A}{12dd'}\,\frac{2n\pi r^2}{33000} = \frac{2\pi}{12 \times 33000} \times \frac{r^2}{dd'} \times nA.$$

In which, with the same engine and indicator, nA is the only variable.

Any of the methods mentioned in Vol. I, p. 22, may be employed to determine the area of an indicator diagram, the most common being to draw equidistant vertical lines and take the sum of the trapezoids thus formed. This equals the product of their common distance apart, multiplied by the mean of the first and last, plus the sum of all the others. A much more accurate way is to divide the whole length into an even number of equal parts. Then calling $a_1, a_2, a_3 \ldots a_n$ the various ordinates, and b their mutual distance apart, the area by Simpson's rule will be

$$A = \tfrac{1}{3}\, b\, (a_1 + 4a_2 + 2a_3 + 4a_4 + 2a_5 + \ldots . a_n).$$

Many of the defects of an engine are shown by an indicator diagram. Thus if the supply pipe is too small, or the steam wire-drawn, as it is called, the curve will not attain its full height until the piston has moved some distance. To show this, take a diagram when the engine is doing a large amount of work, with the valve in the steam pipe opened to the full, and again, when doing no work, the steam being cut off either by the governor or by partly closing the valve. If the exhaust is too small there is a back pressure during the return stroke. If the exhaust is opened too soon, the pressure falls before the end of the stroke; if too late, an increased back pressure is shown at the end of the return stroke. To show this, take diagrams, changing the position of the eccentric, when its best position is readily deduced. If the engine has a variable cut-off, take a series of readings with it in various positions, and compare in each case the amount of work done with the steam employed. Although diagrams properly taken show many peculiarities in an engine which cannot otherwise be well detected, yet too much reliance must not be placed on them, as it is possible to make an engine give diagrams of almost any desired shape without thereby rendering it very efficient.

154. Friction-Brake.

Apparatus. The steam engine, or any motor, as a gas, hot-air or electro-magnetic engine, a turbine, or water wheel, may be used for this experiment. On the main shaft, the brake represented in Fig. 100 is attached. *AB* is a piece of wood which may be screwed against the shaft by two bolts which pass through it and *CD*. It is upheld at *B* by a spring balance *E*, and is prevented from vibrating by a disk *G* which passes into a vessel of water. A tube *H* allows a small stream of oil to flow over the axle to prevent its becoming heated. To prevent *B* from rising when the engine is started, weights *F* are applied to hold it down, and two stops should be inserted to limit its motion.

Experiment. The friction-brake affords a means of measuring directly the amount of work done by an engine or other motor. Unscrew the bolts holding *CD*, remove the weights *F*, and read the spring-balance. The reading is that due to the weight of the beam *AB*, and must be subtracted from all the following readings. Suppose now we wish to determine the greatest amount of work which can be obtained from the engine when running at a given speed under a given steam pressure. Start the engine, screw *CD* against *AB*, and allow oil to flow from *G* on to the axle, or it will be heated by the friction. If water is used instead of oil the brake is liable to *chatter*, making such a jarring as to endanger the machine. The direction of the motion, if the brake is placed as in Fig. 100, must be that of the hands of a watch, so that *B* will tend to rise and press against its upper stop. The amount of this upward pressure may be regulated by the bolts pressing *CD* against *AB* and will be proportional to the friction around the axle. Continue to tighten the bolts until the full power of the engine is expended in overcoming the friction, which is shown by its beginning to labor and run more slowly, although the governor allows the full supply of steam to pass. Now add weights to *F* until the beam is held balanced between

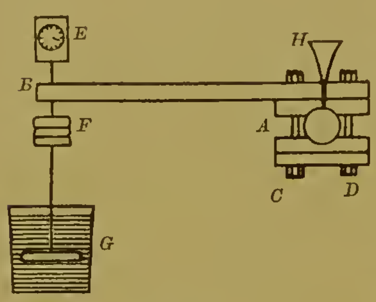


Fig. 100.

the two stops. If no spring balance is used it will be impossible to attain this condition perfectly, as, owing to continual variations in the friction, the beam will sometimes rise and sometimes fall with the same weight; while if we depend wholly on a spring-balance, instead of on the weights, it will begin to vibrate, and the hand will not come to rest. It is best therefore to depend mainly on the weights, adding the spring to enable us to judge better of the mean value of the friction. The reading of the balance is of course always subtracted from the weight to obtain the upward tendency of B. The vibrations of the beam are materially checked by the disk G which can move but slowly, owing to the liquid resistance. A number of readings should now be taken of the balance and weights, and the speed of the shaft observed, as in Experiment 158, loosening the bolts and again tightening them after each observation. To determine the work in foot-pounds let n be the number of revolutions of the shaft per minute, P the change in the force, acting on the end of the beam B, that is the first reading or downward pull of the beam added to the force required to hold it down during the experiment. Call d the perpendicular distance, in feet, from B to the centre of the shaft, and H the required horse-power. Then the work expended in friction, per minute, is evidently the same that would be required to pull B around n times with a force P, or exert a force P through a distance $2\pi n$. This amount of work equals $2\pi nP$ foot-pounds and since 33,000 foot-pounds equal one horse-power, we must have $H = \frac{2\pi nP}{33,000}$, or proportional to n and to P. If the engine can be run at various speeds, measure the amount of work it will do at these speeds, taking care that the full pressure of steam is attained in the cylinder, and that it is not reduced by the governor or by a valve. It must be remembered, however, that a large engine cannot be run above a moderate speed without danger.

155. Transmission Dynamometer.

Apparatus. Any form of transmission dynamometer may be employed, but that represented in Fig. 101 is cheap and convenient when the power to be transmitted is not very great. The arrangement is similar to that devised by Huyghens for winding

astronomical clocks without stopping them. A is a pulley driven by the engine, or other source of power, B a second similar pulley to drive a lathe, planer, or other machine to be tested. An endless belt passes over both, and also over the two pulleys C and D. C is held down by a weight E which measures the force tending to pull it up, and a weight F is attached to D to keep the belt stretched and prevent its slipping on A or B. A better arrangement is to use a chain like that used with large clocks, and for A and B, wheels with projections to fit the chain instead of pulleys. All slip is thus avoided. Another plan is to use bevel wheels instead of A and B, and connect them by an axle also carrying two bevel gears. The latter turn in opposite directions, and tend to turn their axle, end for end, with a force whose moment equals that of the transmitted force. This may then be easily measured by a lever-arm with weights holding the axle at rest. In a third form of dynamometer, the driving and driven pulleys are on the same shaft, one attached to it, the other free to turn; they are connected by some form of spring, and its amount of deflection serves to measure the moment of the transmitted power.

Experiment. When power is furnished to run several machines, it is often desirable to know how much is required for, or consumed by, each. A transmission dynamometer is used for this purpose, and in its simplest form merely shows how much the driving axle is twisted. Other dynamometers do more than this, and record the product of the twisting force by the distance traversed, and thus give the work done directly. To measure the amount of work required by any machine, connect it by a belt with a pulley to the right of B, and the engine with a similar pulley to the left of A. The connection must be such that C shall tend to rise and D to descend. Vary the weight E, until it is just sufficient to balance F, or until its moment equals that of F, plus the moment of the force transmitted by the shaft. Then calling P the difference of E and F, and r the radius of A or B, the moment of torsion will equal $\frac{1}{2} Pr$, and if the shaft makes n turns per minute, the work transmitted will equal πrnP, which represents the amount of work required to run the machine at the given speed. This may be reduced to horse-powers by di-

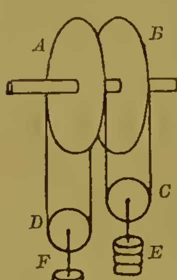


Fig. 101.

viding by 33,000. Now make the machine do work, and F will at once rise, and must be increased to balance the increased moment. The increased work represents that done by the machine. Determine in this way the work done by a lathe when a shaving is being cut, of a planer when at work, and of a circular saw when cutting a board. We see also by these measurements how much of the power is lost by friction.

It is sometimes more convenient to combine the preceding Experiment with this, and let the friction brake absorb the power transmitted by the dynamometer described above. One instrument may then be tested by the other, and the two measures of the power compared. By varying the tightness of the screws various readings of both may be obtained without altering the engine or other motor driving them.

156. Speed of Piston Rod.

Apparatus. A table is placed in line with the piston of the steam engine, and on it is a sheet of paper over which a small carriage may be drawn by a string attached to the piston. On the carriage is a small electro-magnet to whose armature a pencil is fastened, which makes a dot whenever the circuit is closed. A galvanic battery is connected with the magnet and a tuning fork inserted in the circuit so as to make and break the circuit 100 times a second. Instead, a tuning fork and style may be drawn over a long strip of smoked glass or paper precisely as in Vol. I, Experiment 64. A second string is attached to the carriage and passing over a pulley on the edge of the table, carries a heavy weight at the other end. By the motion of the piston, therefore, the carriage is drawn over the table, and is carried back by the weight.

Experiment. See that the pencil is so adjusted as to make dots when the circuit is closed through the magnet, and, after connecting the string with the piston, that the carriage moves smoothly over the table. When the piston is nearly at the end of its stroke, close the circuit, and a series of dots will be formed on the paper near together at the ends, and far apart in the centre. Break the circuit at the end of the stroke, move the paper a short distance sideways, and repeat. Measure the distance of each dot from the end one, and write the results in a column. Write the first differences in a second column, and the mean of each two

consecutive numbers in the third. Construct a curve with abscissas equal to column three, and ordinates to column two enlarged. The result shows the velocity at any point of the stroke. A more accurate method theoretically, is given in the next Experiment, but that given above is sufficiently exact unless the speed of the piston rod is very great so that the number of dots is small. This experiment has great value also in a steam pump or water-pressure engine, to see if the delivery is uniform.

157. Speed of Fly-wheels.

Apparatus. A spur-wheel with 36 teeth is placed on the same shaft as the fly-wheel, and a metal bar allowed to press against it. A chronograph and galvanic battery are the only other instruments needed for this experiment.

Experiment. Connect one pole of the battery with the metal bar and the other with the spur-wheel, or with the engine, interposing the magnet of the chronograph in the circuit. As now the engine turns the fly-wheel, the circuit will be made and broken 36 times in every revolution, or every 10°. If the motion is uniform, the marks on the chronograph should correspond to equal intervals of time, and a curve constructed with these angles as ordinates, and times as abscissas, should be a straight line. If not, the inclination of the curve to the axis of X at any point, gives the velocity at that point. To find this, draw a tangent at the point and measure its inclination, or the change in ordinates, when the abscissas alter by unity. Make this measurement for every 30° and construct a new curve with angles as abscissas, and velocities as ordinates. It is very well to combine this experiment with the last and see if they agree, as in Vol. I, Experiment 28.

158. Speed of Shafting.

Apparatus. A shaft-speeder, and two shafts connected by a belt and revolving uniformly. To test the results the following apparatus will be found extremely convenient. Three vertical gas-pipes are connected together below, and to the centre one a glass tube is attached. They are then filled half full of mercury, and some colored water poured into the glass tube until it is nearly full. If now the tubes are attached to a vertical shaft so that the centre tube shall lie in the axis, and the shaft is caused to revolve,

the mercury will, by centrifugal force, be thrown into the outer tubes, and the water-level will descend. A suitably graduated scale is then placed near the revolving glass tube, when the position of the water-level will mark the rate at which the tube is turning. As the centrifugal force increases as the square of the velocity, the divisions of the scale for high speeds will be much larger than those for low speeds. To render them more nearly uniform, and also to prevent the mercury from being thrown out of the outer tubes, the latter should be bent in towards the centre. To measure the speed of a horizontal shaft the tubes should be mounted so that they are free to turn, and connected with the shaft by a flexible spring like that of a dental lathe, by a pulley and belt, or better still, by bevel-gears.

Experiment. A shaft-speeder consists of a steel rod with a sharp three-sided point, free to turn, and whose revolutions are marked by one or more indices like those of a gas-meter or sirene. It will be noticed that there is a depression in the end of every shaft, called a dimple, by which it is held while made. To measure its speed, the point of the shaft-speeder is inserted in this dimple for one minute, and, being pressed against it, turns, owing to the friction. The number of revolutions, as given by the index, gives the speed of the shaft. As it is a little difficult to bring the point in place exactly at the beginning of a minute, it is more accurate to hold it against the shaft, and note the reading exactly at the beginning and at the end of the minute; or, at the end of the time suddenly remove it, and read the position in which the index has stopped. Greater accuracy is attained by prolonging the time, if the shaft moves uniformly. Measure the speed of the two connected shafts, and also the radii of the pullies over which the belt connecting them passes; or more simply, measure the circumference of each pulley by passing a steel tape around it. If there was no slip of the belt, the ratio of the speeds of the shafts would be the same as that of the radii or circumferences of the pullies. The difference between the observed and computed speed of the driven shaft will therefore measure the slip. These results are easily tested by the liquid speeder. It is only necessary to observe the height of the liquid when the scale gives the speed at once. To make, or test, the graduation, a scale of equal parts should be placed against the glass tube and the shaft run at various speeds. The number of turns per minute is then carefully

measured in each case with the ordinary shaft-speeder, and the corresponding level of the liquid observed. A curve is now constructed with ordinates equal to the scale readings, and abscissas to the speed of the shaft. From this, the correct scale is easily constructed by graphical interpolation.

159. Strength of Materials.

Apparatus. In Fig. 102, AB is a large screw by which any object to be compressed may be forced against the steel plate C. EC is a steel lever with two knife-edges resting against D, which forms the bed-plate of the dynamometer, and to which the nut, through which the screw passes, is fastened. Two other knife edges also rest against C, and take up the pressure from it. To E is attached a stout wire, by which it is hung from the short end of the steelyard FGH. To H is fastened the spring balance I, capable of reading to 30 lbs. by single ounces. Any similar form of lever balance may be employed, but that here shown is simple and inexpensive. If the proportions are such that one pound on I corresponds to ten on EF, and to one hundred on BC, forces up to 3000 lbs. may be measured. J is a small reading microscope with an eye-piece micrometer to measure changes in form of BC. Besides the various objects to be tested which will be described below, a stiff bar K, Fig. 103, with two knife edges on it, and a curved piece of cast iron M are also needed.

Experiment. The dynamometer here described may be applied to a great variety of purposes which will be described below in order. First, to measure the compressibility of any substance, prepare a cylindrical bar of it with flat ends, and placing it between B and C hold it by turning the screw. Mark a point on the bar and read its exact position with the microscope J, or better use two microscopes, and observe the change in distance of two points one near each end of the bar, when compressed. Now remove the spring balance and apply a weight of five pounds to H. It

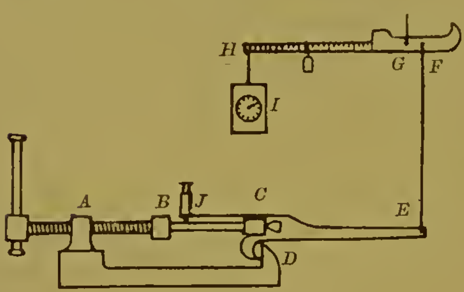


Fig. 102.

will probably at once descend against its stops. Turn the screw until H rises and remains balanced, when the pressure will equal that due to the weight. Observe the distance apart of the points as in Vol. I, Experiment 20. Take in this way a series of readings of the distances corresponding to various pressures. Determine from them the most probable value of the compressibility by the method given in Vol. I, p. 4, assuming that $l = l_0 + ap$, in which l_0 is the length when no pressure is applied, l the length under the pressure p, and a the coefficient of compressibility. The various observed values of l and p must be substituted in this equation and from them the best values of l_0 and a deduced. A simple, but less accurate method, is to construct a curve with ordinates equal to the pressures, and abscissas to the lengths, which should give a nearly straight line; a is then equal to the tangent of the angle this line makes with the axis of X. We must next determine the modulus of compression, C. For moderate pressures, the diminution in length of a bar is proportional to the pressure. Hence if this law held at all pressures, a certain force per square unit would reduce its length to zero, although there is no substance for which this is actually the case, since crushing takes place or the law changes, long before this limit it reached. Calling s the cross-section of the bar, P the load per square inch, l_0 its length under no pressure, and l its length under pressure Ps, we must have $l = l_0 - asP$ in which, if we make $l = 0$, P will equal C, or $C = \frac{l_0}{as}$.

With a large pressure it will be found that the two points in the bar gradually approach each other, and observing their distance at various times we may construct a curve which will represent the permanent set. By using bars of various lengths and cross-sections, we may prove that, for a given pressure, the change in length is with cylindrical or prismatic bars proportional to the length and cross-section, whatever may be their shape. In all the above work it is essential that the section shall be sufficient to prevent the bar from bending under the pressures employed.

To determine the laws of crushing, replace the spring-balance, and observe its index, as the pressure is gradually applied by the screw. When the pressure is considerable, the reading of the bal-

ance will begin to diminish as soon as the screw stops, owing to the permanent set, until finally a point is reached beyond which the index will not move, and the body breaks. This maximum position of the index marks the breaking weight, and is simply proportional to the cross-section of the body. Different substances break very differently, some are brittle, give way suddenly, and the index going down at once to zero, the beam falls against its stop. With a sudden jar, their fracture often takes place at much lower pressures. Other bodies are plastic, yield slowly, and often have no definite breaking points, but will support a much larger weight for a short time than if it has to be sustained very long.

If the length of the body is great compared with its diameter other laws hold. The strength of a column built in at both ends, built in at one end and simply supported so that it is free to turn at the other, and simply supported at both ends, are readily compared by similar rods flat at both ends, rounded at one end, and rounded at both ends. The laws of the length, of the diameter, and the superiority, for a given weight, of hollow columns, are also readily found.

The laws of transverse elasticity may be tested on a much larger scale than in Vol. I, Experiment 35, by the help of K, Fig. 103. A stout bar is laid against C, and two knife-edges slide over it, while a third knife-edge rests against B. The bar to be tested is placed between the three knife-edges and bent by the screw. The pressure is measured by the steelyard as before, and the amount of pressure by the microscope J. Metal bars of considerable size may thus be used and the laws for the length, thickness and width, tested. Castings may also be made of various forms of girders

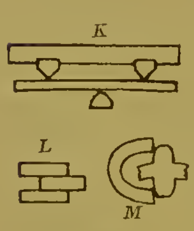


Fig. 103.

at a cost little above that of the patterns, and the metal used over and over again. The shearing strength of various kinds of glue, cement, mortar or other similar substances, is readily found by joining three blocks of wood, metal or stone as in L, inserting them between B and C, and measuring the force required to make them slide over each other. Still another application of this dynamom-

eter is to testing the strength of various forms of teeth for wheels. Castings are made of the form shown in M and the curved piece being placed against B, and the casting against C, the screw is turned until the teeth are broken.

160. Friction of Belts.

Apparatus. A shaft on which are several pulleys of various sizes, either of wood or metal, and which may be turned very slowly by power or by hand. Several belts of various widths and material are also needed, some 10 lb. weights, a spring balance reading to 30 lbs. by single ounces, and a clinometer or level with a graduated circle attached for measuring slopes.

Experiment. Pass one of the belts over a pulley, attach a 10 lb. weight to one end, and the hook of the spring balance to the other. Fasten the other end of the balance to the floor so that it shall hang vertically. If there was no friction of the bearings of the shaft, the reading of the balance should now be 10 lbs. but it may in any actual case be greater or less than this amount by any quantity less than the friction. Now turn the wheel so that the belt shall be carried from the balance. The reading will increase until it equals 10 lbs. plus the friction of repose; the belt will then begin to slip, and as the wheel continues to turn, the reading will remain equal to 10 lbs. plus the friction of motion. See whether the friction is dependent upon the velocity. Next, turning the wheel backwards the reading equals 10 lbs. minus the friction. Care must be taken that the reading of the balance is zero when no weight is applied, or if not, a correction is necessary. Repeat the experiment, adding, a 10 lb. weight to each side so that the strain shall be 20 lbs. on one side, and 10 lbs. plus the reading of the balance on the other. Do the same with heavier weights, also with other pullies and belts. In the case of leather belts try both sides and see which gives the greatest friction. The friction when the belt touches a greater or less portion of the circumference of the pulley is found in a similar manner, by fastening the balance in such a position that the two parts of the belt shall be inclined at the required angle. Adjust this angle by the clinometer, which consists of a spirit-level free to turn in a graduated circle, so that it may be inclined at any angle

to one side of the instrument which is planed smooth. Make this angle 60°, and set the balance in such a position that, when the clinometer is laid on the belt, the bubble of the level will be in the middle. The other part of the belt being vertical, the two sides will be inclined at an angle of 30°, or 150° of the pulley will be covered. Make a series of measurements with angles of 30°, 60°, 90°, 120°, 150° and 180°. For larger weights a stronger balance must be employed, or a cord passing over a pulley attached to the end of the belt and the weights hung to it. This is, however, open to the objection that an error is introduced due to the friction of the pulley. Care should be taken not to turn the shaft too rapidly, or the belt will heat.

To compare the results with theory, let t be the tension of any part of the belt, v the corresponding angular distance from the point where the belt touches the pulley, and f the coefficient of friction. Then the normal pressure dp of the belt on the minute portion dv of the pulley will be the resultant of two equal forces t, inclined at an angle of $180° - dv$, or $dp = tdv$. The increase of tension dt equals dp multiplied by the coefficient of friction, or $dt = fdp = ftdv$, and integrating, $\log t = Mfv$. When the belt covers 180°, $v = \frac{1}{2}\pi$ and calling t' and t'' the tensions at the ends,

$$\log t' - \log t'' = \log \frac{t'}{t''} = \tfrac{1}{2}\pi f M.$$

161. Friction of Pullies.

Apparatus. Several pullies of various sizes and materials placed on a shaft which replaces their pins, and which may be turned in either direction. A set of weights, a spring balance, a flexible cord and a clinometer.

Experiment. This experiment is nearly the same as the last, the cord is used like the belt, and the difference in the tension of its two ends is measured when the axle is turned. The calculation is, however, quite different, as the friction depends only on the resultant of the two tensions. The magnitude of this equals the sum of the tensions if the cords are parallel, otherwise, it is found by the parellelogram of forces. If the pulley is large its weight should be included in finding this resultant. The ratio of the difference in tension to the resultant pressure may be called the coefficient of friction of the pulley, and should be constant.

METEOROLOGY.

The science of Meteorology treats of the heat, pressure, moisture and other properties of the atmosphere with which our globe is surrounded. To determine its laws, observations have been made at various points of the earth at regular intervals for long periods of time. The graphical method is largely used in the discussion of the results. This is most readily done by drawing a curve with abscissas proportional to the times, and ordinates to the temperature or other quantity measured. Sometimes this curve is drawn automatically and the instrument is then called self-registering. The general method in this case is to move the paper uniformly by clock-work, and give the pen or pencil a motion at right angles to it by the thermometer or other instrument to be recorded, taking care that the motion of the pencil shall be proportional to the change in reading. This is easily accomplished with a metallic thermometer or aneroid barometer by allowing them to move the pen or pencil directly. In some cases, however, this cannot be done, and it is always objectionable on account of the friction. It is therefore more common to employ an electrical attachment for which the mercury of the thermometers and barometers is especially convenient in effecting electrical contact. Sometimes, as with magnetic observations, even this method is not available on account of the minuteness of the forces to be observed. In this case a spot of light from a lamp is reflected by a mirror attached to the instrument on to the paper which is rendered sensitive photographically. The slightest motion is thus made visible and recorded permanently without in the least interfering with the action of the apparatus. Self-registering records are much more valuable than those obtained by single observations since they are much more complete, and give all the variations of short duration which occur in all meteorological

phenomena and quite escape a common observer. They effect also a great saving in time both in taking the observations and in constructing the curves. To give the results in numbers they sometimes also print the results (Dudley Observat., Rep. II, p. vii). The most complete self-registering instrument is the Meteorograph of Secchi, exhibited at the Paris Exposition of 1867 (Barnard's Report, p. 571) in which various meteorological phenomena are recorded side by side on the same sheet.

To eliminate the effects of the small variations, the average or mean of a long series of observations is taken. In the same way to determine the changes due to any cause we group together those observations where this cause should have its greatest effect, and in a second group those where the effect is least, and so on for each intermediate value. For instance, suppose we have a series of observations of the temperature of a given place for every hour for ten years. The mean of all these, or their sum divided by their number gives the mean temperature. Now suppose we wish to know if the height of the barometer affected the thermometer. We should group together and take the mean of all these observations of the thermometer taken when the barometer stood between 700 and 710, form a second group of all those between 710 and 720, a third between 730 and 740, and so on, and then see if these means seemed to follow any definite law. This would be done most easily by drawing a curve with abscissas equal to the pressures 705, 715, etc., and ordinates to the observed means. If the variations from a horizontal straight line did not much exceed the accidental errors, we should conclude that there was no relation, or at least that the effect was too small to be shown without a still greater number of observations. Generally the cause is periodic, as the motion of the sun or moon. Thus to find the effect of the rotation of the earth we group all the observations at the same hour of the day and compare their mean with those taken at other hours. The effect of the motion of the earth around the sun is similarly shown by comparing the mean temperature for each month.

The true mean for any given time is of course obtained more accurately from the curve of a self-registering instrument. In this case the area of the space included between it and the horizontal

axis must be determined and divided by its length, that is, by the time; the result is the required mean.

Another most important application of the Graphical Method is to represent the conditions at different places. The method of contours, Vol. I, pp. 14 and 34, is here used, all the points where the quantity observed is the same being connected with a curved line like a contour line. This plan is largely used by the Signal Service Office in predicting the weather and marking the progress of storms. These lines have different names according to the phenomena they represent.

Isothermal lines are those of equal temperature.

Isochimenal lines are those of equal winter temperature.

Isotheral lines are those of equal summer temperature.

Isobaric lines are those of equal barometric pressure.

Isogonal lines are those of equal magnetic declination.

Isoclinal lines are those of equal magnetic dip or inclination.

Isodynamic lines are those of equal magnetic intensity.

162. Temperature of the Air.

Apparatus. A good thermometer, the various maximum and minimum thermometers described below, and Joule's arrangement for determining the temperature of the air. This consists of two thin copper tubes, one inside of the other, so connected that the intermediate space may be filled with water, whose temperature is measured by a good thermometer. In the interior a spiral wire with a mirror attached, is hung by a filament of silk to show if the air currents make the wire twist. The tube may be closed by a cap placed over the lower end. The thermometers should be protected from radiation and be hung at least ten feet from the ground, and a foot from the wall. The arrangement employed at the Greenwich observatory consists of a stand which somewhat resembles a high writing desk, and consists of an inclined board on a support, to the upper edge of which the thermometers are hung. To prevent the board from becoming heated by the sun's rays a second board is placed parallel to it, an intervening space being left to allow the air to circulate and an inclined roof is attached over the thermometers to protect them from rain. The whole is free to turn and should always be placed with the first board towards the sun.

Experiment. The most prominent meteorological phenomenon, and that most commonly measured, is the temperature of the air.

This may be determined by a thermometer whose correctness is tested and its errors determined as described in Experiment 122. Asuming however, that the thermometer is exact, it is not an easy matter to determine the true temperature of the air. Of course the thermometer must be protected from the sun, and observed in the shade, and care should be taken that it is not exposed to other radiations, as that of a brick or white-washed wall on which the sun is shining, or to light reflected from the water or ground. Exposure to the sky when clear, is nearly as bad, though the effect is the opposite, since the thermometer then radiates its heat into space and its temperature, especially after sunset, is often lowered several degrees. The effect is diminished by the black tin stand which partially covers the bulb, but if this is unduly heated or cooled by radiation the bulb is affected also. Holding the thermometer in the hand or breathing on it also soon alter its temperature.

To determine the temperature of the air by Joule's method, fill the space between the copper tubes with water, and close their lower end by the cap. The air currents are thus cut off and the spiral wire inside will now come to rest so that there shall be no torsion of the suspending fibre. Mark the position of the mirror either by a scale and spot of light, or simply by the eye. Now move the cap, and if the water is warmer then the surrounding air, the tubes will act like a chimney and the ascending air current will twist the spiral wire, and with it the mirror. If the water is colder than the air the mirror will turn in the opposite direction. Observe the effect with cold and warm water, then vary the temperature of the latter until the mirror remains in the same position, whether the cap is on, or off, taking care to stir the water briskly so that its temperature shall be uniform. The reading of the immersed thermometer will then give the true temperature of the air.

We often wish to know, not only the actual temperature at any given time, but the highest and lowest temperature attained during the day or other interval. Maximum and minimum thermometers are employed for this purpose. Rutherford's maximum thermometer consists of a common mercury thermometer placed horizontally, with a small index of steel or graphite in the tube

above the mercury. If the temperature increases, the mercury pushes the index in front of it, but as it cools leaves it behind, since there is little adhesion between the mercury and index. The latter, therefore, remains at the point where the temperature was highest. To bring the index back to the mercury or to reset the instrument it is only necessary to incline the thermometer and tap it, or if the index is of iron, draw it back with the magnet. Phillips' maximum thermometer differs from the above in replacing the index by a small bubble of air which separates a part of the mercury column from the remainder. When the mercury expands, it pushes the column in front of it, but when it contracts the elasticity of the air prevents the motion of the detached portion. Rutherford's minimum thermometer is filled with alcohol, and carries an index of glass which remains in the liquid, allowing the latter to expand past it when the temperature rises, but by capillarity being drawn back when the surface touches it, owing to the contraction of the liquid. It is set by inclining the tube and tapping it until the index slides down to the surface. Commonly the maximum and minimum thermometers are placed on a board side by side, their bulbs turned in opposite directions so that both may be set by inclining the board the same way.

To avoid the trouble of a index, which sometimes sticks to the mercury or catches and cannot be moved along the tube, Negretti and Zambra make a maximum thermometer with the tube bent at right angles near the bulb, and partially contracted at this point. The tube is then inclined downwards and when the mercury expands it forces itself past the contracted part, while when the temperature falls, separation takes place there. The reading is, therefore, that of the highest temperature attained since the instrument has been set. The latter operation is performed as before by inclining the thermometer.

Sixe's maximum thermometer consists of a **U**-tube closed at one end and terminating at the other in a large bulb filled with alcohol. The **U** is half filled with mercury over which are two steel indices with little hair springs which hold them in place when left to themselves. As the temperature rises, the alcohol expands and pushes the mercury down on one side and up on the other. The first index, therefore, being held by the spring, is

left hanging in the tube, the alcohol passing it freely, while the second index is pushed up by the mercury column until the maximum is reached. When the temperature is lower than it has been since the instrument was set, the opposite effect is produced, the first index being pushed up by the mercury, and the second left hanging by its spring. The indices are drawn back to the mercury by a magnet, and the position of their lower ends denotes the maximum and minimum temperatures attained.

One of the best forms of maximum thermometer is Walferdin's, in which the upper end of the tube terminates in a fine point enclosed in a small glass chamber. If now the temperature rises, part of the mercury overflows into the chamber and remains there when the temperature falls. The maximum temperature is determined by immersing the thermometer in a vessel containing water, and heating the latter until the mercury again reaches the top of the tube, when the temperature of the water as shown by a common thermometer equals the required maximum. Or, the tube may be graduated from the point, and the maximum temperature attained equals the reading of the thermometer plus its temperature, which is readily found by placing it with another thermometer in a vessel of water. To set the instrument, warm it until the tube is full and invert, when as it cools, the mercury is drawn back into the bulb.

A minimum thermometer is made by slightly altering this and inverting it, filling the bulb, which is now uppermost, partially with alcohol and the lower part with mercury and alcohol. The stem is filled partly with mercury and partly with alcohol by warming and inverting it. As the temperature falls, the mercury is drawn out of the tube, but as it rises is replaced by alcohol.

To compare the climate of different places, and for other purposes, it is often desirable to compare their average or mean temperature. If observations are made at short and equal intervals, as every hour, it is seen that the temperature attains a maximum at about 2 P. M., and a minimum shortly before sunrise. The mean temperature of the day may then be found by taking the sum of all the observations and dividing by their number. Such observations, however, are exceedingly laborious, unless made with a self-registering instrument, and other less accurate methods are

therefore generally preferred. The mean of the maximum and minimum temperatures gives approximately the mean temperature, but the result is generally a little too high. Evidently twice during the day the temperature must coincide with the mean, but the hour will vary with different localities. It is generally about 8 or 9 in the morning and evening. The mean of two observations at intervals of twelve hours gives nearly the mean, the best hours being 10 A. M. and 10 P. M. Still better results are attained by three daily observations, at 7 A. M., 2 P. M., and 9 P. M., and adding the sum of the first two to twice the last, and dividing by four.

To determine the mean temperature during any given time an ingenious device has been proposed by Jurgensen. A watch is made in which the balance wheel instead of being compensated, has the two metals reversed, so that a slight increase of temperature makes it run very slowly, and a decrease makes it gain rapidly. It is now kept first at a high, and then at a low temperature, and its rate being accurately determined in each case, gives nearly the temperature necessary to produce any required rate. Now deduce the mean rate during the given time, and from it we obtain the mean temperature with great exactness, since it allows for every change in temperature of the balance wheel even if lasting for but a few seconds.

Another method of determining the mean annual temperature is from the temperature of deep wells or springs or from the temperature of the ground at considerable depths.

Besides measuring the temperature of the air in the shade, many observations have been made to determine the relation of the temperature to the height, its variations at different depths at sea and in the ground. It is found that at great depths the temperature rises in the earth about 1° C. for every fifty to one hundred feet, but at small depths the temperature is affected by the diurnal and annual variation at the surface, being, however, behind them in time, so that at a depth of about twenty-five feet the changes are six months behind hand, or the temperature is greatest in winter, and least in summer.

163. Solar Radiation.

Apparatus. A solar radiation thermometer, a Pouillet's pyrheliometer, and a lens pyrheliometer, or a large burning-glass and a thermometer with blackened bulb.

Experiment. The solar radiation thermometer consists of a mercury maximum thermometer with a blackened bulb, contained in a larger bulb from which the air has been completely exhausted. To use it, expose it to the sun's rays, and record the maximum temperature attained; observe also the temperature of the air in the shade. The most common method of determining the absolute amount of heat received from the sun is Pouillet's pyrheliometer. This consists of a flat circular tin box, blackened on one side and filled with water in which is immersed the bulb of a thermometer. The instrument is placed on a universal joint so that it can be turned in any direction. To make a measurement, the water being nearly at the temperature of the air is placed in the shade for four minutes and exposed to the radiation of the sky; during the next minute remove it into the sunlight and adjust it so that its face shall be perpendicular to the rays of the latter, but so covered that it shall be protected from its heat. Call the change of temperature during these five minutes, t. To adjust the position of the instrument a circular disk is placed behind the box and parallel to it, so that it is set in position by merely turning the instrument until the shadow of the box shall exactly cover the tin disk. Now expose the blackened surface of the box to the sun for five minutes, call the change of temperature s, and again place it for five minutes in the shade, and call the change of temperature t'. Then the true increase may be taken equal to $s - \frac{1}{2}(t + t')$. If w is the weight of water, w' the weight of the tin box containing it, and s' its specific heat, $w + w's'$ will be its water-equivalent, and if r is the radius of the circle exposed to the sun the number of units of heat received from it per minute will be nearly equal to, $\frac{(w + w's')\,[s - \frac{1}{2}(t + t')]}{5\,\pi\,r^2}$. This forms a standard by which any solar radiation thermometer may be graduated. The lens pyrheliometer consists of a calorimeter in which the water is heated by a large burning-glass. The change in temperature is here much greater, but a correction must be made for the heat lost by the lens. By simply placing a thermometer with a blackened bulb a short distance within the focus of a large burning-glass, the comparative heat of the sun, at different times, is readily observed. The variations with its altitude, and during the progress of an eclipse, are thus well studied.

164. Atmospheric Pressure.

Apparatus. Examples of the various barometers described below, and a thermometer for determining the temperature of the air. While most meteorological observations must be made in the open air, that of atmospheric pressure is made equally well indoors, as the pressure is easily transmitted through the cracks in the doors and windows.

Experiment. A standard barometer consists of a glass tube at least a centimetre in diameter, filled with mercury from which the air has been expelled by boiling, and the tube is then inverted over a cistern containing mercury. The height is read as described in Vol. I, Experiment 12, or the scale may be attached directly to the steel point. The latter is then screwed down until it just touches its reflection, and the height of the column read by a vernier or cathetometer telescope. To determine from this reading the true height of the barometer, corrections must be applied for capillarity, for temperature of the mercury, and for latitude, as described in Vol. I, Experiment 58. The formula there given may also be employed to correct the elevation above the sea-level, since having E and p we may determine p'. Generally, however, it will be sufficiently exact, if the height is not very great, to subtract a certain constant quantity from all the readings, and thus correct for elevation and capillarity.

The principal objections to this instrument are, that it is not portable, and that an error is likely to occur in making the point coincide with the surface of the mercury. One method of remedying this difficulty is to attach a plunger to the vernier, so that raising the latter depresses the former, by an equal amount, into the mercury of the cistern. The surface of the latter will always remain at the same height when the vernier is set, if the plunger has the same diameter as the tube.

Another form of barometer, known as the Kew standard barometer, has a cistern with a cross-section precisely 25 times that of the tube. The scale on the latter is made $\frac{24}{25}$ of its true size, hence the rise of the mercury in the cistern is corrected at once by the scale. Each inch on the scale is divided into twenty parts or to .05″ and the vernier divides these into 25 or reads to .002″.

To set the instrument, raise or lower the vernier by the milled head on one side, until the top of the mercury seems just to touch the lower edge of the vernier, when the eye is brought on a level with it. The reading of the latter then gives the height.

Gay Lussac's siphon barometer consists of a bent glass tube of which the larger end is closed and forms the barometer tube while the shorter end is open and forms the cistern. Evidently as the mercury falls in the long tube it will rise a nearly equal amount in the short one, hence, the range of either end is only about one half that of the common barometer. The scale is divided both ways from the centre, and there is a vernier to read each mercury surface; the height equals the sum of the two readings. This barometer may be carried by inclining it until the mercury reaches the top and then inverting it. The tube being then full of mercury there is little danger of injury, while if carried right side up the mercury rises and falls and is very liable to break the tube.

One of the most common forms of barometer is Fortin's, in which the cistern is closed below by a leather bottom, which may be raised by a screw. The sides are of glass and an ivory point dips down so that it may be made to touch the mercury by turning the screw. To take a reading, turn the screw until the point touches its reflection, and read the verniers. It may be transported by turning the screw until the mercury is pushed up to the top of the tube and then inverting the instrument. This makes an excellent mountain barometer. When used for very great heights only, the tube is sometimes made shorter, thus rendering it more portable.

The aneroid barometer is constructed on a wholly different plan from the above. A circular metallic box is closed above with a cover of corrugated metal and exhausted of air. As the pressure of the outer air varies, the cover rises or falls and its motion is magnified by an index moving over a scale. The latter is divided arbitrarily and the index is turned by a screw in the back of the instrument until it agrees with the reading of a standard barometer. Both are then subjected to a different pressure and the length of the short arm of one of the levers altered until the reading is again the same. In this way it may be made to agree very nearly with the standard, as long as the temperature is unchanged. If

now the temperature rises, the elasticity of the metal diminishes, and the cover sinks in, as if the pressure had increased. To remedy this source of error a little air is admitted into the box which, when heated, expands, and increasing the interior pressure tends to counteract the effect of the diminished elasticity of the metal. Evidently by varying the amount of air, which is easily measured by the pressure it produces, we can compensate almost exactly for the changes in temperature. An aneroid is read directly from the position of its index, and gives the pressure approximately without correction. The best instruments are, however, liable to an error of some hundredths of an inch. They will generally give different results when hung up and when laid down, and also alter if tapped with the finger. In the latter case the reading may be greater or less than the true reading by an amount not exceeding the friction of motion, while if not tapped, it may vary as much as the friction of repose.

165. Wind.

Apparatus. A weathercock to show the direction of the wind, and various anemometers to measure its velocity. They should be raised above surrounding objects, otherwise eddies will be formed, and errors thus introduced.

Experiment. The direction of the wind is most easily shown by an ordinary weathercock. The centre of gravity of the vane should lie in its axis and the surface exposed to the wind be as large at one end and as small on the other as possible. The friction also must be reduced to a minimum. The direction of the wind in the upper part of the atmosphere may sometimes be measured by the motion of the clouds. For this purpose lay a sheet of looking-glass horizontally, and observe the reflection of a cloud in it. Then lay a ruler on the glass and holding the head perfectly still, turn the ruler until it coincides with the direction in which the cloud appears to move. Its position may then be measured by a compass, and gives, after correcting for the magnetic variation, the direction of the wind.

In determining the velocity of the wind, much will depend on the height of the anemometer above the ground or surrounding objects. At the height of eight feet, the velocity is often double

that within a foot of the ground. Most of the following anemometers must be kept turned towards the wind, and this is most easily effected by attaching them to the weathercock. One of the simplest anemometers is that of Lind, which consists of a simple U tube with one end bent down horizontally and turned towards the wind. When half filled with water the liquid will be pressed down on the side towards the wind, and the difference in level x gives the velocity v of the wind by the formula $v = 3.26\sqrt{x}$; x is here given in inches, and v in miles per hour. To reduce the oscillations, the tube may be contracted at the lower part, and for observations at sea, a valve is sometimes connected with the tube so that when the liquid has attained its proper position, the valve may be closed so that it will not return, and the reading then taken at leisure. The maximum pressure is readily obtained by making a small hole in the leeward arm of the tube at the water line. As the pressure increases, the water is forced out and the level of the remaining liquid shows the maximum pressure attained.

Bouguer's anemometer consists of a single plate held at right angles to the direction of the wind, and the pressure measured directly by a spring. The velocity is then given by the formula $v = 1.3\sqrt{p}$ in which v is the velocity in miles per hour, and p the pressure in pounds per square foot. A modification of this instrument is that of Taupenot, in which a board is allowed to swing out at an angle, whose magnitude measures the pressure. Both these instruments are especially open to error from the varying strength of the wind, which is liable to set them vibrating.

One of the most common forms of anemometer is that of Robinson, which consists of four sheet-metal hemispheres connected together so that they can turn around a vertical axis. As the pressure is always greater on the concave than on the convex side, they will continue to revolve in the same direction, whatever is the direction of the wind. The velocity of each hemisphere will be one third of that of the wind, so that if placed at a distance of 6.72″ from the axis, 500 revolutions will equal one mile. A set of wheels and indices serve to measure the total number of turns. The total distance traversed by the wind is given by subtracting the reading at the beginning from that at the end of the time, and the average velocity by dividing this distance by the time. An-

other excellent anemometer is that described in Vol. I, Experiment 60. All anemometers should be tested before relying too implicitly on their readings, by carrying them on a calm day either on a railroad car or in a wagon at various known velocities, or attaching them to a long arm free to revolve around a vertical axis. A curve may then be constructed which will give the reading in terms of the velocity.

The direction and velocity of the wind is so constantly changing that self-registering instruments are needed to give really satisfactory results. A great variety of methods have been employed and will be found described in detail in the proper works, but as they are of special, rather than general interest, they need not be enumerated here.

166. Moisture.

Apparatus. A hair-hygrometer, wet and dry bulb thermometers, a hygrodeik, Daniell's hygrometer, Regnault's hygrometer, an aspirator, some drying tubes, a balance and weights.

Experiment. The amount of moisture in the air may be stated in three different ways. First, the absolute amount may be given, as so many grammes per cubic metre, or grains per cubic foot; secondly, by giving the pressure of the vapor in millimetres or inches; and thirdly, the amount may be compared with that required to produce saturation. Thus if the air contains half as much moisture as is needed to saturate it, or produce condensation, the moisture is said to be 50 per cent. Since cold air will hold less moisture than warm, if the latter is cooled, a temperature, called the dew-point, is soon reached at which the moisture is deposited as dew.

The simplest instrument for measuring moisture is Saussure's hair hygrometer. This consists of a hair fastened at one end and carrying an index at the other. When exposed to moisture it expands, and the index shows the change. Unfortunately, the motion of the index is not proportional to the amount of moisture, and what is worse, is not the same for any two hairs. Accordingly, a table must be determined for each instrument, or whenever the hair is changed. Even then, accurate results cannot be

obtained, since, if exposed to a perfectly dry atmosphere, the zero point will slowly change. To obtain the best results the hair must be steeped in ether or boiled in carbonate of soda, to remove all traces of grease.

The most common method of measuring moisture is by means of the wet and dry bulb thermometer. This method depends on the principle that the dryer the atmosphere, the more rapid the evaporation of water, and hence the colder the water becomes, from the absorption of the heat required to vaporize it. The arrangement employed consists of two similar thermometers placed side by side, one being covered with a piece of cloth kept wet from a vessel below containing water. The wet bulb thermometer will now always read lower than the dry bulb, and by the two readings the amount of moisture may be determined from a table. To measure the moisture therefore, it is only necessary to see that one bulb is wet, to read the two thermometers, and determine the moisture from the table. Care should be taken that the wet bulb is not exposed to a current of air, as this accelerates the evaporation and diminishes the apparent amount of moisture.

An ingenious modification of the wet and dry bulb thermometers is the hygrodeik. In this instrument the two thermometers are placed side by side on a stand, and carry two indices of which one is brought to the level of the mercury in the dry bulb by raising or lowering a milled head, and the other is brought to coincide with the wet bulb by turning the milled head. A pointer is attached to the latter which may thus be directed to any part of the space between the two thermometers. In this space a card is placed on which are drawn three sets of curves of such a form as to show the amount of moisture corresponding to any readings of the two thermometers. One set of curves gives the absolute amount of moisture and its pressure, the second the relative humidity, and the third shows the dew-point. To make a reading, raise or lower the milled head until the index is opposite the mercury of the dry bulb thermometer, then turn it until the second index coincides with the mercury of the wet bulb, and the position of the pointer shows by inspection the amount of moisture.

All the above instruments give results which have to be re-

duced by the aid of previous experiments, and cannot be relied upon when great accuracy is required. These objections are avoided in hygrometers depending on the determination of the dew-point. Daniell's hygrometer consists of a glass tube bent at right angles, with a bulb at each end, one of blackened glass, the other covered with muslin. A thermometer is enclosed in this tube with its bulb in the black glass, which is half filled with ether and the air expelled by boiling it before sealing. If now a little ether is poured upon the muslin, by its rapid evaporation and consequent absorption of heat from the glass, the latter is cooled. Condensation, therefore, of the ether vapor inside of it takes place, and evaporation of the ether in the black glass, which in turn is cooled as in a cryophorus. The thermometer, therefore, begins to fall, and descends until it has so far cooled the surrounding air, that the dew-point is reached and condensation takes place on the surface of the glass. The temperature is then read by the thermometer but will be somewhat below the dew-point, since the dew is probably not noticed at first. In a few seconds the glass will become warmed and the condensed dew will evaporate; read the temperature which may be a little above the dew-point, add more ether, and repeat until the dew-point is determined exactly. Care must be taken not to breathe on the blackened bulb or to allow the moisture of the body to be deposited on it. In warm climates, alcohol may be used instead of ether.

Regnault has modified the above instrument so as to render it much more accurate. Two thin polished silver tubes, like test tubes, are closed by corks through which pass the stems of two similar thermometers. One of the tubes is partly filled with ether, and through its cork pass two glass tubes, one opening in the upper, the other in the lower part of the silver tube. The first glass tube is connected with an aspirator by which air may be drawn through the ether. When this is done the latter evaporates rapidly, and its temperature falls until dew is deposited on the silver. The air-current is then stopped, the ether grows warmer and the dew evaporates; the true dew-point is thus determined with great accuracy. When an aspirator cannot be conveniently used, a rubber tube is connected with the glass tube passing to the bottom of the silver tube, and the ether cooled by blowing air through it.

The ether vapor escaping from the other tube should in this case be carried off and down by a second rubber tube so as not to interfere with the deposition of the dew. The advantages of this instrument over Daniell's hygrometer consist in the ease with which the dew can be detected on the surface of the silver, the more so from the presence of the second silver vessel, which is always dry; again, the passage of the air through the ether stirs it up thoroughly, and renders its temperature very nearly uniform throughout.

The most accurate method of determining the moisture in the air and that with which all others are compared to see if they are correct, is the chemical method. In this, a known volume of the air to be measured is drawn by an aspirator through three tubes filled with some drying substance, as pumice stone moistened with sulphuric acid. The first tube collects the moisture, or if any escapes this, it is stopped by the second, and the third prevents moisture from passing back from the aspirator. The amount of moisture is found from the increase of weight of tubes one and two.

167. Rain and Dew.

Apparatus. A rain gauge, a vessel for determining the evaporation, some cotton wool, thermometers, an aethrioscope and an actinometer.

Experiment. A rain gauge may be made of a simple funnel inserted in a graduated vessel so that the rain falling on it may be collected and measured. It should be placed within a few inches of the ground, as the quantity of rain received diminishes rapidly with the height, and it should be placed as far as possible from all buildings and other objects liable to produce eddies. By a simple proportion, the water received is reduced to the depth it would have if distributed uniformly over the exposed surface. To compare the amount of rain received, with the moisture passing into the air by evaporation, a cylindrical vessel containing water is exposed directly to the air. The lowering in level from day to day measures the evaporation, and may be observed directly. To prevent birds and other animals from drinking the water, it is customary to surround the vessel with sharp pointed wires. This

instrument is called an atmidometer. To measure the evaporation with precision, the apparatus described in Vol. I, Experiment 13, may be used.

Many objects exposed to the sky on clear nights, especially in the summer and autumn, cool by radiation until their temperature is below the dew-point of the air. Moisture is then deposited on them, and is called dew. The amount may be measured by exposing a plate of glass or metal painted black to the sky and collecting the water deposited. Or, pieces of cotton wool may be employed, and the increase in weight observed. Such an arrangement is called a drosometer. To measure the amount of nocturnal radiation a thermometer with blackened bulb is exposed to the air and protected from radiation from the earth by placing under it a box filled with eider-down. This instrument is called an actinometer. The radiation to the sky may also be measured by the aethrioscope, which consists of a vertical glass tube terminating in bulbs, of which the lower takes the temperature of the air, and the upper is blackened and exposed to the sky. A concave mirror cuts off radiation from the earth and an index of water in the tube shows the relative temperatures of the two bulbs. When the upper bulb is covered, both take the same temperature, and the drop of water comes to the zero point, but on exposing the upper bulb to the sky, the drop at once rises.

168. Tides.

Apparatus. A tide-gauge, which may consist of a simple float attached to a cord passing over a wheel whose position marks the level of the water. Or a flexible rubber bag sunk below low-water mark, and connected by a tube with a mercury column or steam gauge may be employed.

Experiment. Although not strictly a meteorological phenomenon the rise and fall of the tide is often associated with Meteorology, and this is especially the case, from their close connection with the rain fall and evaporation, with the variations of level of lakes and rivers, which are observed in precisely the same way. To observe these changes of level, it is only necessary that the water should communicate with an adjacent well or pit, and that sudden variations of level due to waves or other disturbing causes should

be cut off by diaphragms or by contracting the connecting pipe. The simplest way to measure the level of the water is by a vertical graduated rod immersed in the water, from which the level is read directly. Another method is to use a float with a cord passing over a pulley and stretched by a weight at the other end. The position of the wheel, which may be transmitted to indices, marks the height of water.

An ingenious form of tide-gauge has been used by the Coast Survey, for measuring the rise and fall of the tide when observations could not be made on shore but only by vessels at anchor. It consists of a flexible air-tight bag which is filled with air and thrown overboard, being weighted so that it will sink, and connected with the surface by a long flexible tube. To the upper end of the latter is attached any form of pressure-gauge, and from its reading the depth of water is at once deduced. As, therefore, the vessel rises and falls with the tide, the index of the gauge moves to correspond. The same instrument is well adapted to taking soundings, if the water is not too deep. The bag is thrown overboard and towed by the boat, when the index always denotes the vertical depth of water above the bag.

169. Magnetic Declination.

Apparatus. In this experiment, and in the three that follow it, all the observations should, if possible, be made in a small detached building or magnetic observatory, constructed entirely without iron, and the instruments should be mounted on stone piers disconnected from the rest of the building, to protect them from jars. When this cannot be done, a room should be assigned to them, preferably in the cellar, to secure a uniform temperature and a steady foundation. All iron must be removed to a distance, especially when determining the absolute magnitude of the elements, and the observer should take care that he has no iron about his person, as a pocket knife, keys, or steel-mounted eye-glasses. The instruments should be protected by cases with plate glass windows through which they may be observed.

To measure the magnetic declination, a common surveyor's transit is needed, which may be placed near a northern window so that the pole star shall be visible. A vertical mirror is attached to the wall opposite it in such a position that the observer on looking through the telescope will see its reflection in the mirror. Instead of the mirror a collimator may be used, or telescope without an

eyepiece having cross-hairs illuminated by a light placed behind them. A still simpler substitute is a very distant object. The direction of the meridian must be determined once for all, as explained in Experiment 182, and the reading of the horizontal circle of the transit observed. The telescope is then turned towards the mirror until, on looking through it, its cross-hairs bisect the reflection of its end. This may be determined more precisely by hanging a plumb line in front of its centre, or if a collimator is used, bringing the two sets of cross-hairs to coincide. The horizontal angle will now give the direction of the line normal to the mirror. North or south of the transit a bar magnet is hung by one or more filaments of silk, the upper end lying in the thread of a horizontal screw turning in a fixed nut, so that the magnet may be raised or lowered in a perfectly vertical direction. To remove the twist, the screw and nut may be turned around a vertical axis over the silk fibres. To one end of the magnet a lens is attached and, at a distance equal to the principal focal distance, a set of cross-hairs. The magnet is hung from the silk by a stirrup so that it may be turned over at will. A brass bar of the same weight as the magnet should also be provided.

To measure changes in the magnetic declination, a mirror is attached to a small magnet suspended by a long filament of silk and its motion observed by a telescope and scale.

Experiment. In studying the magnetic condition of the earth we find that its total effect is equivalent to two equal and opposite forces acting on the two poles of the magnet. To determine the direction of this force we must measure its declination, or variation to the east or west of the true meridian, and its dip or inclination to the vertical. Then, finding the magnitude of the force or the magnetic intensity, the magnetic condition is fully determined. The three quantities, the declination or variation, the inclination or dip, and the intensity, are called the magnetic elements. On measuring them, it is found that they vary not only in different parts of the earth, but also with the time, undergoing variations with the hour of the day, the season of the year and from year to year. These variations are called diurnal, annual and secular. Besides these there is a fourth, irregular variation which cannot be predicted, keeping a suspended magnet in constant motion, the changes sometimes being very great. The latter are called magnetic storms. Two classes of instruments are therefore required, the first to determine the absolute magnitude of the three elements, and the second to study their changes.

Set up the transit, so that it shall lie in the same meridian as the suspended bar magnet and in the line perpendicular to the mirror. Turn the telescope towards the latter until the cross-hairs coincide with the reflection of the centre of the end of the telescope, or of a plumb line hung in front of it, and read the horizontal angle. To eliminate torsion replace the magnet by the brass bar and turn the suspending fibre by its support at the upper end until the stirrup points directly towards the transit. Then replace the magnet and placing a light behind its cross-hairs, turn the telescope towards it and bring the two sets of cross-hairs to coincide. Turn the magnet over in its stirrup and repeat. The mean of these two readings gives the true direction of the magnetic meridian, since it eliminates any error due to deviation of the magnetic axis from the line connecting the cross-hairs and centre of the lens attached to the magnet. If now we call a the angle between the true meridian and the line perpendicular to the mirror, and b the angle between the mirror and the mean position of the magnet, $a - b$ will equal the magnetic declination.

To determine the variations of the declination, it is only necessary to observe, with the telescope, the reading of the scale reflected from the mirror. To reduce these readings to absolute angular readings, we must know the distance of the nearest point of the scale from the magnet-mirror and the scale reading of this point. For this purpose hang a plumb line over the centre of the telescope, and turn the mirror until its reflection where crossing the scale is visible in the telescope. The scale reading s', gives the required point. Its distance d, from the mirror is then measured directly by a millimetre scale. Call s the scale reading corresponding to any angle a, and a' the declination when the scale reading is s'; then $\frac{s' - s}{d} = \text{tang } 2\ (a' - a)$. To determine a', measure the absolute declination by the instrument described above, and read s at the same time. Then substituting the value of a deduce a'. As the declination varies but a small amount it will generally be sufficiently accurate to assume that the tangent is proportional to the arc, in which case each scale division will equal $\frac{D}{1719}$ minutes, or if $D = 1719$ mms., 1 division will equal a minute. If the scale is observed continuously, it will be seen to be con-

stantly in motion, and this is the case to a surprising extent during displays of the *aurora borealis.*

170. Magnetic Dip.

Apparatus. A dipping needle, of which the best form is that proposed by Joule, and a magnet to reverse the polarity of the needle.

Experiment. If a piece of steel is placed north and south, balanced on a knife-edge and then rendered magnetic, the north end will seem to have become heavier than the south, owing to the inclined direction of the magnetic force of the earth. In the southern hemisphere, however, the other end descends or dips. To measure the angle of inclination, a dipping needle is employed. This consists of a magnetic needle resting on a circular axis so that it can move very freely in a vertical plane. The ends are pointed, and a vertical graduated circle is placed near them to show the angle of dip. Sometimes a plate of looking-glass is placed behind the needle, and the graduation etched on it. The parallax is eliminated by bringing the needle to coincide with its reflection. The friction should be reduced to a minimum by using a very small axis of hardened steel, resting either on two knife-edges, or on friction rollers. The whole is mounted on a vertical axis free to turn, the angle of rotation being measured by a horizontal graduated circle. It should also carry a level, and be mounted on levelling screws.

In Joule's dipping needle, the axis, instead of resting on steel supports hangs on two loops formed by hanging filaments of silk from the ends of a delicate balance. The friction is thus reduced to a minimum and alters the position of the needle by less than a minute of arc. Maxwell proposes to read the position of the needle, by placing two prisms acting like mirrors in front of the telescope, so that they shall reflect the two ends of the needle into the field at the same time, and to take the reading by measuring the angle through which they must be turned in order that the two ends may coincide.

To measure the dip, level the instrument in the usual way, by bringing the level parallel to two of the levelling screws, and turning one of them until the bubble is in the middle; then turn the

level 90°, or until it is perpendicular to these screws, and again bring the bubble to the centre by the third screw. Repeat until the bubble remains in the centre, however the vertical circle is turned. If, now, the needle is set vibrating, it should continue to move for a long time, and finally always come to rest at the same point. Place the circle in the magnetic meridian, read the position of the two ends of the needle, and their mean will equal the dip. The circle may generally be set in the proper position by a common compass, but the following method is more common. If the circle is turned completely around, it will be noticed that in two positions, when at right angles to the magnetic meridian, the needle is vertical, and when in the meridian its reading is least. The proper position may therefore be found by turning the circle until the needle is vertical, then turning it exactly 90°. When the needle is in the meridian it is acted by the two components $H = I \cos i$ acting horizontally, and $V = I \sin i$ acting vertically, calling I the total force, H and V its two components, and i the angle of inclination or dip. If the needle is turned into a plane inclined by the angle v to the magnetic meridian, the vertical component is unchanged, while the horizontal component is reduced to $H' = H \cos v = I \cos i \cos v$. Hence, if i' is the angle of inclination of the needle in its new position, we shall have,

$$\text{tang}\ i' = \frac{I \sin i}{I \cos i \cos v} = \frac{\text{tang}\ i}{\cos v}.$$

Evidently the minimum value of i' is i, when $v = 0$, and $i = 90°$ when $v = 90°$, as stated above. If two readings i' and i'', are taken when the circle is turned 90°, we have $\cot i' = \sin v \cot i$ and $\cot i'' = \cos v \text{ tang } i$, or $\cot^2 i' + \cot^2 i'' = \cot^2 i$, which furnishes a third method of determining i.

In the above measurement of the dip we have made two assumptions, neither of which is likely to be correct. First, that the centre of gravity coincides with the centre of the axis supporting the needle, and secondly, that the line connecting the north and south poles of the needle, is parallel to that connecting the two ends, which serve as pointers to read the graduated circle. The first source of error is eliminated by turning the vertical circle 180°, when the needle turns over, so that the other side is uppermost. If, then, in the first case the centre of gravity is below the

axis and tends to diminish the inclination, in the second it is above it, and increases the inclination by nearly the same amount. Hence, the mean of the two gives very nearly the correct reading. To eliminate differences in the pivots the needle should also be turned over, and the readings repeated in that position. To eliminate the incorrect position of the magnetic axis, the magnetism must be reversed. This is done by stroking the needle several times, from the centre outwards, first on one end and then on the other, with a permanent magnet. As the polarity is to be reversed, the north end of the needle must be stroked with the north pole of the magnet, and the south end with the south pole. After reversal, the dip is again observed, the other end of the needle now pointing downwards.

The dip may also be found from the time of vibration of a dipping needle, when placed first in the meridian, and then, in plane at right angles to this. The force acting on the needle will be, in the first case, the total magnetic force I, and secondly its vertical component only, or $I \sin i$. But if n and n' are the number of vibrations the needle makes in a given time in the two cases, the forces will be proportional to n^2 and n'^2, or $I : I \sin i = n^2 : n'^2$, hence, $\sin i = \frac{n'^2}{n^2}$, or the sine of the dip equals the square of the ratio of the number of vibrations.

171A. Horizontal Component.

Apparatus. A mirror is attached to a rectangular steel magnet about a decimetre long, which may be suspended by a bundle of filaments of silk. A telescope and scale serve to mark the motions, as in Experiment 169. To determine the moment of inertia, two cylindrical brass weights may be attached to the magnet at known distances from its end; a good compass, a wooden or brass bar a metre in length and divided into decimetres, and a bifilar magnetometer are also needed.

Experiment. If a dipping needle is moved from its position of equilibrium, it will vibrate under the influence of the magnetic attraction of the earth, like a pendulum, and the square of the time will be inversely proportional to the magnitude of this force. Owing to friction, however, an accurate measurement cannot be obtained in this way, and accordingly its horizontal component and

direction are determined instead. The total force equals its horizontal component divided by the cosine of the dip. The horizontal component is measured as follows. A magnet is suspended by a bundle of filaments of silk, and its time of vibration determined. This gives the product of H, the horizontal component of the earth's magnetism, by M, the magnetic moment of the magnet, if we know I the moment of inertia of the latter. The ratio of M to H is next determined by seeing how far the magnet will deviate a compass needle from the meridian. Having thus determined MH and $\frac{M}{H}$ we readily deduce H. Let us now see more exactly how these two experiments are made, and then how the value of H is computed from them.

To determine its time of vibration, the magnet is placed in its stirrup, a black thread is hung over the scale near the point marked by the cross-hairs of the telescope, and the magnet set in vibration, by holding another magnet near it for a few seconds. Care must be taken not to touch it, or it will be set swinging like a pendulum. The time of transit of the thread past the cross-hairs is next taken, as in Vol. I, Experiment 15. Six transits are thus observed to the nearest tenths of a second, an interval of several minutes is then allowed to elapse, and a second series of transits taken. From these the true time of transit may be determined with great accuracy. For this purpose, take the mean of the first and second, the third and fourth, and the fifth and sixth observations of each series. This will give the turning points of the magnet. Then take first differences, which will equal approximately the time of vibration. Take the mean of the four times thus obtained and call it t. Then take the mean of the six original observations of the first and of the second series, and call their difference T. Dividing T by t gives the number of intermediate vibrations, which should be a whole number. Owing to slight errors in t, it will not come out exact, but dividing T by the nearest whole number gives the time with great accuracy. Care must be taken not to make T so great as to render the number of vibrations doubtful, or other intermediate observations become necessary. Often sufficient accuracy is attained by observing two transits of the thread past the cross-hairs in the same direction with an

interval of several minutes between them, and counting the intermediate passages. The time is then found by a single division.

To determine the ratio of M to H, two observations are taken with the magnet placed at different distances from the compass needle. The compass is placed in the centre of the divided bar, which is turned at right angles to the magnetic meridian. The magnet is then placed with its centre on the 1, 2, 8 and 9 dms. points, its north pole being turned first toward the compass, and then in the other direction. All the deflections of both ends of the compass needle are measured, and the mean of those produced when the magnet is distant 3 dm. taken, also when 4 dm. distant. It is only absolutely necessary to take two readings at different distances, but by the repetition recommended above, errors of eccentricity and want of symmetry of the magnet or compass needle are eliminated. The deflection should be as great as possible, but the least distance of the magnet should be at least three times its length, and ten times that of the compass needle. The greatest accuracy is attained when the greater distance is to the smaller, as four is to three. Instead of placing the magnet east and west, it may be placed north and south of the compass, and the observations made as before, only the result, if the same formula is used, will be twice as great as in the first case, that is will equal $2M$ divided by H. Of course the bar must now be turned east and west, as, if placed north and south no deflection will be produced.

The horizontal component H of the earth's magnetism is next computed as follows. Let M be the magnetic moment of the magnet, or the intensity of magnetism of the poles multiplied by their distance apart. For any pendulum, $t = \pi \sqrt{\frac{l}{a}}$ in which t is the time of vibration, l the radius of gyration or length of an equivalent simple pendulum, and a the acceleration of the force causing it to vibrate. From this we deduce, $a = \frac{\pi^2 l}{t^2}$ or multiplying each side by the mass m, and by l, we obtain $mla = \frac{m\pi^2 l^2}{t^2}$ But $mla = MH$, since it equals the force tending to bring the

magnet into the meridian from a position at right angles to it, or replacing ml^2 by the moment of inertia I, we obtain $MH = \frac{\pi^2 I'}{t^2}$. Commonly we have given the weight w, length l, and breadth b, of the magnet, and in this case,

$$I = \frac{w\ (b^2 + l^2)}{12\ g} \text{ or } MH = \frac{\pi^2 w}{12 g} \cdot \frac{b^2 + l^2}{t^2}.$$

If the magnet, instead of being rectangular, is of such a shape that its moment of inertia cannot be computed, it may be determined experimentally by hanging two cylindrical weights at equal distances from the centre and observing the new time of vibration. Calling t' this time of vibration, $2w'$ the weight of the cylinders, and l their distance from the centre, we have the proportion,

$$t^2 : t'^2 = I : I + \frac{2w'}{g} \text{ hence, } I = \frac{2w'}{g} \cdot \frac{t^2}{t'^2 - t^2}.$$

Having thus computed MH, we must next determine $\frac{M}{H}$ from the second observation. Calling r and r' the two distances at which the magnet is placed, or 3 and 4 decimetres in the above example, and v, v' the corresponding mean deflections of the compass needle, it may be proved that $\frac{M}{H} = \frac{r^5 \text{ tang } v - r'^5 \text{ tang } v'}{2(r^2 - r'^2)}$. As stated above, if the deflecting magnet is placed north and south of the compass, $\frac{M}{H} = \frac{r^5 \text{ tang } v'' - r'^5 \text{ tang } v'''}{r^2 - r'^2}$, v'' and v''' being the corresponding deflections. Having thus determined MH and $\frac{M}{H}$ we readily deduce $H = \sqrt{MH \div \frac{M}{H}}$. Great care must be taken to reduce all measures to the same units, or to centimetres, grammes and seconds.

To measure the changes in the horizontal component of the earth's magnetism, a bifilar magnetometer is commonly employed. This instrument differs from that used to measure variations in the declination, mainly in having the magnet suspended by two bundles of filaments of silk instead of by one. Their distance below is regulated by two screws, and above by two pulleys between which they pass. The two threads are connected together above and pass over a pulley, so that the tension of both may be the same. The upper suspending pulleys are turned until the magnet is nearly perpendicular to the magnetic meridian, and the length of the threads, and their distance apart should be such, that they

will thus be twisted through an angle of about 45°. Evidently when the threads are turned so that they hang obliquely, as their length remains unchanged, the magnet is slightly raised, so that the directive force of the earth's magnetism is balanced against the weight of the magnet. The observations are made by the telescope and scale in the usual manner, the scale readings being very nearly proportional to the changes in the horizontal intensity. To reduce them to absolute measure, a magnet of known magnetic moment M is placed north or south of the suspended magnet, at the same height, and at a distance d. The change in the magnetic field will then equal $\frac{4M}{d^3}$, and, as this corresponds to a change of reading of n divisions, the value of one division is readily obtained. The zero of the scale is at once found by measuring the component by the method given above, and by the bifilar magnetometer simultaneously. This comparison should be made frequently as the readings are liable to vary, owing to changes in the magnetic moment of the suspended magnet, and to alterations in the length and distance apart of the two suspending threads, due to changes of temperature. To measure very minute changes in the horizontal intensity, the magnet is sometimes turned 180° into the magnetic meridian with its north end to the south. The delicacy may then be increased indefinitely by varying the distance of the suspending filaments. If they are brought too near together, however, the magnet will be in unstable equilibrium.

171B. Vertical Component.

Apparatus. The only instrument required, is a magnetometer balance with a telescope and vertical scale for measuring deviations.

Experiment. The absolute value of the vertical component of the earth's magnetism is not readily measured directly, but is more commonly deduced from the other magnetic elements. Its variations are, however, easily observed by the magnetometer balance, which consists of a magnet placed in the magnetic meridian, and balanced on knife-edges like the beam of a chemical balance. A mirror is attached, from which the deviations may be observed in the usual way, by a telescope and vertical scale.

The magnitude of the divisions of the scale may be reduced to absolute measure by placing a vertical magnet of known moment M at a distance d above or below the balance, and noting the change in scale reading n. The corresponding change in the magnetic field will be $\frac{4M}{d^3}$ and dividing this by n will give the value of one scale division as in the case of the bifilar magnetometer. The zero of the scale is then determined by comparison with the absolute vertical component, which is deduced by multiplying the horizontal component by the tangent of the dip. The absolute magnitude of the divisions may also be determined by measuring the force required to balance the beam when turned horizontally 90° or 270°, and comparing it with the deflection produced by one milligramme placed at the same distance from the knife-edges. The small magnitudes of the forces to be measured, renders this method unsatisfactory.

The horizontal and vertical components, H and V, the total intensity I, and the inclination i are connected together by the two equations, $I^2 = H^2 + V^2$, and tang $i = \frac{V}{H}$. Hence, if either two are known the other two may be deduced. It will be seen from the above that it is difficult to measure directly the total intensity I, or its variations, the vertical component V, or the variations of the dip i.

172. Electricity of the Air.

Apparatus. Two instruments are required in this experiment of which one assumes the same electrical potential as the air, and the second measures this potential. For the first of these a water-dropping collector is commonly used, or an insulated vessel of water, with a small tube leading from it through which the liquid escapes in a fine stream breaking into drops. A burning match made of a roll of blotting paper dipped in nitrate of lead may be used for the same purpose, or a metallic vessel containing ether, whose vapor escapes through a small aperture in the top. To measure the potential, any electrometer of sufficient range and delicacy may be used. Generally Thomson's quadrant electrometer is to be preferred; or his portable electrometer, if observations are to be made at various places. Peltier's electrometer is also convenient, if less accuracy is required.

Experiment. If the electrical potential of the earth is compared with that of the air, it will be found that the latter is commonly in excess in pleasant weather, or the earth is negative. In stormy weather, especially during thunder-storms, the potential of the air varies very irregularly, being sometimes positive and sometimes negative; even in calm, clear weather the variation per minute often amounts to five or ten per cent. To measure the potential of the air, fill the water-dropping collector, and connect it with one terminal of the electrometer, the other terminal being connected with the earth. If the surface of the stream of water has a potential greater than that of the surrounding air, the excess of electrietity is rapidly carried off by the falling drops. The potential measured, therefore, is that of the air at the point where the stream divides into drops. The heated air from the match and the ether vapor, act in a similar manner.

The method of using the quadrant electrometer is given in Experiment 111. In the portable electrometer, the electrified needle is attached to the centre of a stretched platinum wire, and the angle through which the latter must be twisted to bring the needle into a given position, is noted. The difference of potential is proportional to the square root of the angle of torsion. Peltier's electrometer consists of an insulated compass needle resting against a wire parallel to it. When both are electrified, repulsion takes place and the needle swings off at an angle which may be measured by a graduated circle placed below.

Measure the potential at various heights above the surface of the earth and it will be found that the changes are nearly proportional to the height. Take also a series of readings every minute, and construct a curve with times as abscissas, and potentials as ordinates. It is curious to notice the electrometer, during the progress of a distant thunder-storm, as after each flash of lightning the electrometer will mark a sudden change of potential.

PRACTICAL ASTRONOMY.

173. Sextant.

Apparatus. The only instrument needed for this experiment is a sextant; and, although the adjustments are best made by means of a star, the sun or any well defined distant terrestrial object may be employed. A star catalogue is needed for the latter part of the experiment.

Experiment. A sextant consists of a sixth of a graduated circle with each division exactly one half of its usual size. A small telescope is attached to one side, and opposite is placed a mirror, called the horizon-glass, of which one half only is silvered. The angles are measured by a vernier reading to 10″, attached to an arm free to revolve around the centre of the circle. Upon this arm, and in the same plane as the axis, is attached a second mirror called the index-glass. The arm may be set exactly in any required position by a clamp and tangent screw. A magnifying glass is attached, to read the vernier, and a handle is placed behind the instrument by which it may be held. A set of four colored glasses may be inserted between the two mirrors to moderate the light, when directed towards the sun. A second set of three glasses may be interposed behind the horizon-glass to moderate the light of the direct image. When the sun is partially obscured, one or more of these glasses may be used. The telescope has two horizontal and two vertical cross-hairs in its focus, forming a square in the centre of the field of view, and is directed towards the horizon-glass, which is so placed that it will reflect light received from the index-glass into the telescope. On looking through the latter, therefore, two objects may be seen simultaneously, one through the unsilvered portion of the horizon-glass, the other by reflection

from both mirrors. From the law of reflection it follows that when the two images coincide, the angle will be double that between the two mirrors. That this condition may hold, it is essential that both mirrors should be perpendicular to the plane of the graduated circle, and the telescope parallel to it. Before using a sextant, therefore, it should be subjected to the following tests, and the error, if any, corrected.

1st. Index-glass perpendicular to circle. Turn the sextant around so that the graduation is away from the observer, and holding the index-glass near the eye, observe the reflection of the graduation in it. If the image coincides in direction, and appears to form a continuation of the circle itself, the mirror is in its proper position. There is no provision for adjusting this mirror as it is not often necessary. It may be adjusted by unscrewing the index-glass, and inserting paper, or tin-foil, under one edge of its support, or by filing down the pins against which the mirror rests.

2d. Horizon-glass perpendicular to circle. Bring the vernier near the zero and turn the telescope towards a star or other well-defined distant object. If the images can be brought to coincide by moving the index, no correction is necessary. Otherwise, turn a screw above or below the horizon-glass until this condition is fulfilled.

3d. Telescope parallel to plane of circle. Bring two of the wires in the telescope parallel to the circle, and set the index so that the two images of the star shall coincide with the wire nearest the circle. Turn the instrument until they fall on the other wire, and if they still coincide, the adjustment is exact. Otherwise, move the two screws which fasten the collar holding the telescope to the frame of the instrument.

4th. Index Error. Make the two images coincide exactly by the tangent screw, when the reading of the arc will give the index error. The graduation is extended beyond the zero, forming what is called the arc of excess, and if the reading falls on this, it must always be added, otherwise, subtracted from the observed reading. The index error may be found in the day time by viewing the two images of the sun, first interposing both sets of colored glasses. Bring the two images so that they shall just touch, first with one

uppermost, and then the other. One half of the difference of the two readings equals the index error. This is the most important error of all, and should always be observed before using the instrument.

Now measure the distance between two bright stars at least 45° above the horizon. For this purpose hold the sextant by its handle in the right hand with its plane parallel to the rays coming from both stars, and the telescope turned towards one of them. Then turn the movable arm until the second star is seen at the same time, clamp it and bring the two images together with the tangent screw. The reading of the vernier, when corrected for index error, will equal the required distance. Take from the star catalogue the right ascensions and declinations of the stars. Then in the spherical triangle they form with the pole, we have given two sides and the included angle; for 90° minus the declination of each gives a side, and the difference in right ascension gives the angle between them. From these, compute the third side or distance apart of the two stars and see how nearly it coincides with observations. Calling S, S' and P the two stars and the pole, and D the point where a perpendicular from S will meet $S'P$, we have $\tan PD = \cos SPS' \text{ tang } S'P$ and $\cos SS' = \cos S'D \cos SP \sec PD$. If stars near the horizon are observed, an incorrect result is obtained owing to the refraction of the air, but above 45° this error will be small.

174. Latitude.

Apparatus. A sextant and artificial horizon which consists of a vessel containing mercury, protected from currents of air by a roof formed by two pieces of plate glass. To prevent the mercury from becoming tarnished, a small piece of tin-foil may be added to it, which, being dissolved, forms a film covering its surface. If this film is removed, the liquid beneath will be bright and clear. Glycerine is also recommended for the same purpose, and to diminish the motion caused by slight jars. Another form of artificial horizon consists of a piece of black glass ground perfectly plane and resting on three levelling screws. A very delicate spirit-level resting on three points, one of which may be raised or lowered, serves to render the plate horizontal. It is desirable, though not essential, to have a sea-horizon to the south, and a chronometer giving Greenwich mean time.

Experiment. The most common method of determining the latitude is by measuring the altitude of the sun or of a star when on the meridian. If the observation is made at sea, the telescope of the sextant is directed towards the horizon beneath the object, and the image of the latter brought to coincide with it by moving the index. The sextant is then turned from side to side, when the object will appear to describe a line convex downwards. Turn the tangent screw until at its lowest point the image will just touch the horizon, and take the reading. If the observation is made on land, the artificial horizon is more commonly employed. In this case the telescope is turned down until the reflection of the object in the mercury is seen, the index is then moved until the second image is brought into the field, the instrument clamped and the images brought to coincide by the tangent screw. The angle as given by the vernier is that between the object and its reflection, or twice the altitude. If practicable, and always where the greatest accuracy is required, two observations should be made, turning the artificial horizon around 180°, so as to eliminate want of parallelism of the plates of glass.

If the glass horizon is used, it must be levelled as follows. Place the level parallel to two of the screws, and raise or lower one of them until the bubble is in the centre. Turn it end for end, and if the bubble goes toward one end of the tube, bring it half way back by the adjustable point on which one end of it rests, and level the glass plate again. Now turn the level 90°, and turn the third screw until the bubble is in the middle. It should remain in this position however the level is turned.

The observed altitude by no means equals the true altitude, but should be corrected as follows. The order, though not essential at sea, or when great accuracy is not required, should be strictly that given below.

Index-Error. Add or subtract the index-error according to its sign.

Dip. Owing to the sphericity of the earth, the sea-horizon appears below the true horizon or great-circle with the zenith as a centre. The magnitude of the dip in seconds is $D = 59'' \cdot \sqrt{h}$, in which h is the height of the point of observation in feet, or $\log D = 1.77115 + \frac{1}{2} \log h$. This correction must always be sub-

tracted from the observed altitude. If an artificial horizon is used, this error is reduced to zero.

Refraction. Owing to the refraction of the light passing through the air, objects always appear above their true position. This correction is a large and uncertain one, unless the altitude is considerable, and, on account of it, observations of the heavenly bodies should never be taken near the horizon. The magnitude of the refraction is approximately given by the equation $R = 57'' \cdot$ tang $(Z - 3R)$, in which R is the required refraction, and Z the apparent zenith distance or 90° minus the altitude. To determine R, make it equal to zero in the second side of the equation and thus determine R approximately, then substituting this value gives the more accurate value, $R = 57'' \cdot \tan\,(Z - 3 \times 57'' \cdot \text{tang}\, Z)$. This correction, like that of dip, must always be subtracted from the observed altitude.

Parallax. An error is due to the apparent change in position of the body, since the observer is not at the centre of the earth. The amount of the error equals the angular interval, as seen from the object, between the observer and the centre of the earth. It is called the parallax P, and equals $\frac{R}{D} \cos A$, in which A equals the altitude, R the radius of the earth, and D the distance of the object. The quantity $\frac{R}{D}$ is called the horizontal parallax, and is usually given in the Nautical Almanac. Except in the case of the moon, this correction is small, and with the sun never exceeds 8″. It is always to be added to the observed altitude, and in the case of the fixed stars is always zero.

Semi-diameter. When the lower edge of the sun or moon is observed, the true altitude is determined by adding the semi-diameter, which is given in the Nautical Almanac. This correction is small with the planets, and imperceptible with the fixed stars. In the case of the moon, the semi-diameter must be increased, owing to the observer being nearer than the centre of the earth, the amount of the correction, or the augmentation, equalling 15.″65 sin A.

When the object is on the meridian, the latitude is given by the formula $L = A + D - 90°$, in which L is the latitude, A the altitude, and D the declination of the object, south declinations

being always regarded as negative. In the case of the sun, D is found by interpolation from the Nautical Almanac, which gives the declination every day at Greenwich apparent noon. To this must be added the Greenwich time of the observation multiplied by the hourly change of declination, or subtracted if the declination is diminishing. The Greenwich time is either taken directly from a chronometer or it will equal the longitude west of Greenwich added to the equation of the time. The time need not be found with great accuracy, since an error of a minute will at the most only cause an error of about $1''$ in the latitude. The observation should be made within a minute or two of apparent noon, that is, twelve o'clock plus the equation of time. At sea, however, it is customary to begin to measure the altitude some minutes before noon, and follow the sun with the tangent screw until it begins to descend or dip. The greatest altitude is that employed. If the sun passes near the zenith, its altitude will alter rapidly from east to west. In this case, its distance from the north or south point of the horizon should be measured.

The observation of a star is more difficult on account of its feeble light, but greater accuracy is attainable, and the calculation is much simpler.

If the time is known with accuracy, either by a chronometer or as described in the next Experiment, the latitude may be determined approximately by a single observation of any known heavenly body. This involves our first solution of a spherical triangle which is so frequently employed in astronomy that it is known as the astronomical triangle or as the ZPS triangle, Fig. 104, since it is formed by the zenith Z, the pole P, and the star or other object S. In this triangle ZP equals 90° minus the latitude of the place, PS the north polar distance, or 90° minus the declination, and ZS the zenith distance, or 90° minus the altitude. The angle PZS or angular distance from the meridian is known as the azimuth, or, with terrestrial objects, as the bearing. The angle ZSP is rarely used, it is sometimes called the parallactic or position angle. The third angle ZPS is called the hour angle, and

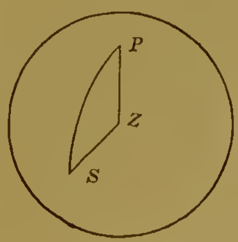


Fig. 104.

equals the time which has elapsed since the star has culminated or crossed the meridian. For objects west of the meridian, this angle will be positive, for those east, negative. When the star culminates, the sidereal time will equal its right ascension, hence the sidereal time minus the right ascension will equal the hour-angle at any instant.

If the mean time t, and the longitude L are known, the sidereal time must be computed from them. A clock giving mean solar time will gain on a clock giving sidereal time 9.8565 sidereal seconds per hour, or 236.5553 per day. The rate of gain may be expressed by the fraction .0027379 of the whole solar interval. Hence any mean solar interval T is reduced to sidereal time, by adding .0027379 T or sometimes more conveniently by reducing it to hours h, and adding 9.8565 seconds. Sidereal time is in like manner reduced to solar, by subtracting .0027304 T or 9.8296 seconds per hour. If s is the sidereal time of mean noon at Greenwich as given in the Almanac, the required sidereal time,

$$t' = t + E + .0027379\ (t + L),$$

and the hour angle is found as before by subtracting t' from the right ascension.

If now any three of the parts of the ZPS triangle are given, the other three may be computed. Thus, in the present case, we determine from the Almanac PS and ZPS, and the altitude as measured by the sextant subtracted from 90°, gives ZS. From these, PZ may be computed by letting fall a perpendicular SD from S upon the meridian PZ, when tang $PD =$ cos SPZ tang SP, and cos $DZ =$ cos PD cos SZ sec SP; again, $PZ = PD + ZD$ and the latitude equals 90° — PZ. Greater accuracy is attained and the calculation simplified by using the pole star, in which case PS is only 1° 25′.

In all ordinary cases, the star should be observed near the meridian, and the calculation may then be greatly simplified. Call a the change in altitude during the first minute after culmination, then $a = \frac{1''.9635 \cos L \cos d}{\sin\ (L - d)}$, in which L is the latitude and d the declination. Then for a small hour-angle t, the change in altitude will be proportional to t^2, or $A = A' + at^2$ in which A is the true, and A' the observed altitude. The common method of finding the true altitude of a star at culmination, is to observe its

altitude at short intervals, before and after, and reduce them by the formula,

$$A = \frac{A' + A'' + A''' + \&c.}{n} + \frac{t'^2 + t''^2 + t'''^2 + \&c.}{n} \cdot a$$

in which n is the total number of observations. The value of A is then corrected and used as an ordinary meridian altitude. It will be noticed that a becomes very large when L nearly equals d, or the star is near the zenith. Stars, therefore, should be selected which do not culminate too near the zenith, since their altitude varies too rapidly.

175. Time.

Apparatus. A sextant, an artificial horizon, and a clock or chronometer.

Experiment. In the ZPS triangle, if we know the latitude of the place of observation, the declination and right ascension of the sun or a star S, and measure the altitude, or $90° - ZS$, the triangle is readily solved. Having the three sides, we may determine the hour-angle ZPS and hence the time, by the formula,

$$\cos ZPS = \frac{\cos ZS - \cos PS \cos PZ}{\sin PS \sin PZ},$$ or more conveniently, calling $M = \frac{1}{2}(PZ + PS + SZ)$, by the formula,

$$\cos \tfrac{1}{2} ZPS = \sqrt{\left(\frac{\sin M \sin (M - ZS)}{\sin PS \sin PZ}\right)}.$$

If the star is near the horizon the error from refraction is large and variable; if near the meridian, the change in altitude is too slow, and a slight error in altitude will produce a large error in the time. Hence the observations should generally be made two or three hours before or after the star culminates. As a single observation is always uncertain, it is best to take a series of readings by seting the index to some even division of the graduation, and observe by a watch or clock the time at which the two images of the star coincide, then move the index exactly 10′ or 20′ and observe the time again. Having thus obtained a number of observations, take the mean of the angles and the mean of the times, and treat them like a single observation.

If the object observed is a star, the sidereal time is very simply found from the hour-angle H. Calling R the right ascension of

the star, the sidereal time $T = R + H$, taking care to make H negative if the object is east of the meridian. This equation is readily proved by recollecting that H hours before the observation, the star was on the meridian, when the sidereal time by definition equalled its right ascension. The local time $T' = T - S$, in which S is the right ascension of the sun at the time of observation, obtained from the Nautical Almanac by interpolation, as in the last Experiment. In the case of the sun, the mean time may be determined directly by the equation, $T' = H + E - .0027304\ H$ in which E is the equation of time.

A much more accurate method than that of single altitudes, given above, is the method of equal altitudes, in which the star is observed before and after passing the meridian. Clamp the index of the sextant and take a series of readings at intervals of 10′ or 20′, when the star is east of the meridian, then, without unclamping the index, wait until the star has culminated, and attained nearly the same altitude west of the meridian. Now take a second series of altitudes, of course in inverse order, as the star descends. The mean of the times, when the star has the same altitude east and west of the meridian gives the time of culmination. The advantage of this method is that it eliminates index error, error in graduation, eccentricity, refraction, dip and parallax, since these quantities are the same in both cases. The calculation, also, is extremely simple and requires no logarithmic tables.

When the sun or a planet is observed, a correction must be applied, since there is generally an appreciable change in declination between the morning and afternoon observation. Calling L the latitude of the place, D the declination of the sun, d its change in declination between the time of culmination and that of the last observation, H the hour-angle, or half the interval between the two observations, and h the correction to be applied. Then it may be proved that $h = \frac{1}{15}\ d$ (tang L cosec H — tang D cot H) which is to be added to the computed time of culmination if the object is moving northward, and subtracted if it is moving southward.

176. Longitude.

Apparatus. The sextant, artificial horizon, and a chronometer regulated to Greenwich mean time.

Experiment. Various methods are employed for finding the longitude, which will be described in detail in Experiment 185. At sea it is most commonly found by determining the local time as described in the last Experiment, and comparing it with a chronometer carefully regulated to Greenwich time. The error and rate of the chronometer must be determined as frequently as possible by comparison with other chronometers, or by determining the local time at points whose longitude is known. The longitude then equals the difference between the local time and Greenwich time.

When the Greenwich time is not known, the longitude may be determined from the position of the moon. The most common method is that known as "Lunar Distances." In the Nautical Almanac, the distance of the moon from several stars is given every day, at Greenwich noon. The motion of the moon is, however, so great, over half a degree an hour, that this distance is constantly altering rapidly. If, then, the distance is observed at any other point, the Greenwich time at that instant may be computed, and comparing it with the local time, gives the longitude. Several corrections, however, have to be applied on account of the small distance of the moon, and hence, this method in practice is both laborious and inexact.

The observations consist in the determination of the local time, a series of readings of the distances of the moon and star, and their altitude found by interpolation from observations before and after. The mean of the distances and of the times is to be used as a single observation and the altitude at this instant determined. The approximate latitude and longitude must also be known. If the latter cannot be otherwise obtained, the method of successive approximations may be used (Vol. I, p. 10). Find from the Nautical Almanac the semi-diameter and parallax of the moon and of the other body, if it is not a star. Add to the moon's semi-diameter its augmentation, or 15.″65 sin A, in which A is its altitude. If the altitude is small, the contraction due to refraction must be subtracted from the semi-diameters of the sun and moon. The observed distance must be corrected for index error and for semi-diameter of the moon, and of the sun also, if the latter is observed. Correct the observed altitude of each body for index

error, dip and semi-diameter, to find the apparent altitude. Find also the true altitude by subtracting the parallax and adding the refraction. Call M and S, Fig. 105, the apparent positions of the centre of the moon and star, and M' and S' their true positions. Then, in the triangle MSZ, we have given the three sides, hence we can compute the angle at Z. But in the triangle $M'S'Z$ we have given $M'Z$ and $S'Z$, equal to the complements of the true altitudes, and the angle $M'ZS' = MZS$, since both parallax and refraction act only in vertical circles. Accordingly we can solve the triangle $M'ZS'$ and deduce $M'S'$, the true distance of the moon and star, as seen from the centre of the earth. This is most conveniently done by the following formulas. Call $A = 90° - MZ$, and $A' = 90° - M'Z$, the apparent and real altitudes of the moon, $B = 90° - SZ$ and $B' = 90° - S'Z$, the apparent and real altitudes of the star, and $D = MS$, $D' = M'S'$ the apparent and real angular distance apart of the star and moon. Make $N = \frac{1}{2}\ (A + B + D)$ and $\sin^2 \frac{1}{2}\, v = \dfrac{\cos A' \cos B'}{\cos A \cos B} \cos N \cos (N - D)$. Putting $N' = \frac{1}{2}(A' + B' + v)$, we have, $\sin \frac{1}{2} D' = \sqrt{\cos N \cos (N' - v)}$. Several other solutions may be used, but these have the advantage that, having only cosines in their second members, they are readily remembered.

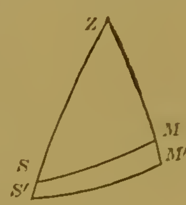


Fig. 105.

177. Meridian.

Apparatus. The sextant, an artificial horizon, chronometer, and a distant terrestrial object. At night a distant lighthouse or other light answers well.

Experiment. The true bearing of any terrestrial object may be determined by the sextant, if we know the latitude and longitude of the place of observation and the local time. Let Z be the zenith, P the pole, O the object, and S the sun or any star whose right ascension and declination are known.

Measure the distance SO by the sextant, or better, set the index at any even division and notice the time at which the two images touch. Increase or diminish the angle 10′ or 20′ and read again.

Take a series of readings in this way and compute the mean of the distances, and the mean of the times.

The altitude of S must also be determined either from simultaneous measurement by another observer, or from measurement of the altitude before and after, from which the true altitude may be found by interpolation. The altitnde may also be found from the ZPS triangle in which we have given ZP, PS and ZPS. Call D the point where a perpendicular from S meets PZ, then tang $PD =$ cos ZPS tang PS, cos $DZ =$ cos PD cos SZ sec PS, and $PZ = PD \pm ZD$. This must be corrected for refraction so as to give the apparent altitude. Again, compute the azimuth PZS by the proportion, sin ZS : sin PS = sin ZPS : sin PZS. Measure the altitude of O or $90° — OZ$, and in the triangle OZS we have given the three sides OZ, OS, and ZS; from these compute OZS as in Experiment 175. Adding OZS to PZS gives the required azimuth PZO.

The azimuth of any other terrestrial object, O', is found directly from that of O, by measuring OO' and the altitudes of O and O', we then have the three sides of the triangle OZO' and can, hence, readily compute the angle at Z, as above. In selecting S and O, we must take care that the angle SOZ is not too nearly 0° or 180° as, otherwise, a slight error in SO may make a large error in the azimuth.

To determine the direction of the meridian subtract PZO from 180°, and set the sextant at this angle. Then move a meridian mark until its image O'' is brought to coincide with that of O. If O and O'' are not in the horizon, the angle between them must be computed from the spherical triangle $OO''Z$.

178. Time by Transit.

Apparatus. A portable transit instrument, a chronometer, and a vessel of mercury. If the transit is mounted permanently, two collimators are convenient, though not essential; a surveyor's transit may be used in this and the three following Experiments, if great accuracy is not required.

Experiment. The most important instrument in an Astronomical Observatory, as far as measurements of precision are concerned, is the transit. This consists of a telescope mounted so that it is

free to revolve in the plane of the meridian. Its axis, which consists of two cones to ensure stiffness, terminates in carefully turned cylindrical steel pivots which rest in metallic *Vs*, resting on substantial stone or iron piers. To adjust the position of the axis, one *V* may be raised or lowered, and the other moved horizontally by screws. In large instruments most of the weight is taken off the pivots by levers and counterpoises. The observation consists in noting the time at which various celestial objects transit, or cross the spider-lines placed in the focus. Generally five or seven vertical equidistant wires are used, and one horizontal wire. A thread, movable by a screw forming a spider-line micrometer is also inserted in the focus, and may be parallel to either the vertical or horizontal wires, according to the use to which it is to be applied. To level the axis, a delicate spirit-level terminating in *Vs* may be laid across from one pivot to the other. To point the telescope at any desired altitude, a small graduated circle is attached either to the axis of the instrument or to the eye end of the telescope. The angle in the latter case is read by an index and vernier with a level attached. The vernier is set to the required angle, and the telescope then inclined until the bubble of the level is brought to the middle of the tube.

At night the cross-hairs will not be visible on account of the darkness of the sky; some method of illumination must therefore be employed, of which the simplest is to place a lamp nearly in line with the telescope, but a little to one side so that its light shall not fall directly into the field. The latter is thus illuminated so that the wires appear dark on a light back ground. A better and more common method of illumination is to place an inclined plate of metal in front of the telescope so as to reflect the light down the tube, and to perforate it so that it shall not cut off the light passing directly into the telescope. In larger instruments, a hole is cut in the tube of the telescope, or the axis is perforated and the light thus admitted, the metallic plate being placed inside. For faint objects a glass plate is inserted in one side of the eyepiece, and the light allowed to shine directly on the wires, which thus appear bright on a dark ground.

Place the instrument approximately in the meridian where there is a clear view to the north and south, focus the eyepiece on the

cross-hairs and then turning the telescope towards a star, move the eyepiece and cross-hairs together until a distinct image is formed. This must be done with care, until, when the horizontal cross-hair is brought over a star, the latter will remain bisected when the eye is moved up and down.

Next, level the axis by placing the spirit-level astride from one V to the other, and turn the screws, altering the height of the V, until the bubble is in the centre, that is, until the reading of both ends of the bubble is the same. Then reverse the level end for end, and, if the bubble remains in the centre, the adjustment is correct, if not, alter the screws of the level until the bubble is brought half way back, and the screw of the transit V until the bubble returns to the centre. Reverse again, and repeat until the adjustment is exact. The level does not now necessarily lie in the same vertical plane as the axis of the transit, and this should next be tested by swinging the level backward and forward so that its *Vs* will slide over the pivots. If the bubble moves, the level must be adjusted by the screws by which one end is moved laterally. It is important to know the angular magnitude of the divisions of the level. For this purpose it is laid on a long rod, one end of which may be raised or lowered by a micrometer-screw, and the other rests on two points at right angles to its length. Bring the bubble first to one end and then to the other of the graduation of the tube, and read the position of the screw in each case. Take the mean of the two ends of the bubble, and call the change in position, in divisions, n. Call a the change in reading of the micrometer-screw, and l its perpendicular distance from the line connecting the two points. Care must be taken to measure both a and l in the same unit, as the centimetre or inch. If s is the number of seconds corresponding to each division of the level, since the length of the radius in seconds is 206265, we must have the proportion, $l : a = 206265 : ns$. A simple method of measuring the divisions of a level is to lay it on a straight-edge set on edge, and raise either end by inserting under it a wire whose diameter is then measured by a sheet-metal gauge.

If the level is not very sensitive, its form is readily investigated by attaching it to a telescope with cross-hairs, and directing the latter towards a distant vertical scale of equal parts. The tele-

scope is then inclined so that the bubble shall rest in different parts of the tube, and a curve constructed with the scale-readings of the cross-hairs as ordinates and the positions of the centre of the bubble as abscissas. If the scale is of millimetres and is distant 20.6265 metres from the object-glass of the telescope, each division will equal 10″. For other distances the readings may be reduced to seconds by a simple proportion. Instead of the scale, the telescope may be directed towards any distant object, and its angular position as the bubble is moved along the tube, measured by a spider-line micrometer.

The cross-hairs should be exactly vertical, and this is effected by turning the ring carrying them until when a distant object is covered by one, it will remain covered, as the telescope is raised or lowered. The horizontal wire may also be tested when the instrument is completely adjusted, by seeing if a star near the equator, when bisected by the horizontal wire, neither appears above or below it, as, by the diurnal motion, it moves slowly across the field.

The next adjustment is to bring the central cross-hair into the plane perpendicular to the axis, otherwise, it will describe a small circle parallel to the meridian. This is called the collimation adjustment. Point the telescope towards any terrestrial object at least a mile distant, so that its focus shall be the same as that of a star, and note the exact point covered by the central hair. Then reverse the telescope, by raising it out of its *Vs* and turning its axis end for end. Point the telescope in the same direction as before and see if the central hair coincides with its former position. If not, move the ring carrying the cross-hairs, sideways over half this distance, and repeat until the adjustment is exact. As it often is not convenient to use a distant terrestrial object, a vessel of mercury may instead be placed under the telescope and the latter pointed down vertically towards it. A collimating eyepiece is now employed in which light is thrown down the tube of the telescope through a hole in the side of the eyepiece by a mirror inside. A simple substitute for this is to gum a little piece of mica or glass to the eyepiece so as to reflect the light of a lamp down the tube. On looking through the telescope an image of the cross-hairs will be seen reflected in the mercury, and coinciding with

the hairs themselves if the adjustment is exact. Since the mercury surface is always perfectly level, this adjustment, if the transit is reversed, serves also to show whether the axis is horizontal.

The transit is brought nearly into the meridian, by pointing it towards the pole star at its culmination. This is shown by its right ascension, the longitude and the time as given by a common watch; or more roughly, by noticing when the star ζ *Ursæ Majoris*, in the middle of the handle of the *Dipper*, lies in the same vertical plane as the pole star. A slight deviation from the meridian will be quite imperceptible for stars near the zenith, and the transit of a zenith star may therefore now be observed with precision. Wait until some known star culminates near the zenith and pointing the telescope towards it, count seconds with the clock or chronometer as the star approaches the first thread. Note mentally the position at the beginning of the second preceding and that following its transit and divide the interval into tenths by the eye. This gives the time to tenths of a second. Do the same with the other threads and take their mean. The difference between this time and the star's right ascension gives the error of the clock which should be set to the nearest minute. Next, observe the transit of the pole star and as the time approaches, as given by the clock, move the transit horizontally by the screws, moving one of the *Vs* so as to follow the star until the time is the same as its right ascension.

Before proceding further, we must determine the relative positions and distances apart of the threads or vertical cross-hairs. Since a star crosses the meridian but once in twelve hours, to increase the number of observations, several threads are used, and the time of transit over each observed. Instead of reducing them to the middle thread, an imaginary thread called the mean thread is used, corresponding in position with the mean of the real threads, and, therefore, very nearly coinciding with the central thread. To measure the position of the threads, observe the time of transit of a star over each, and the mean of all gives the time of transit over the mean thread. Subtract from each of the transits that of the mean thread, and divide the differences by the cosine of the star's declination. This is to reduce the interval to that which it would be if the star was on the equator, and is called the equato-

rial interval of the thread. The best results are obtained with a star near the pole, since, in this case, the intervals become large, and hence may be measured more accurately, but in this case a correction must be applied for the curvature of the path. This is readily done by dividing the sine of the observed interval by the cosine of the declination, which will give the sine of the required equatorial interval. If in any case we fail to obtain transits over all the wires, the mean of the observations may be obtained and corrected by subtracting from this mean the equatorial intervals of the wires used, divided by the cosine of the declination.

Owing to unequal expansion by changes of temperature and to other causes, it is impossible to keep a transit in perfect adjustment. It is therefore found to be better to adjust it once as nearly as possible, and afterwards measure its deviations and apply corrections. These are three in number, for azimuth, for level, and for collimation. The correction for azimuth may be found by observing the transit of the pole star at its upper and lower culmination. If the transit is precisely in the meridian, the difference in time should be exactly twelve hours. If the time differs from this by an amount d it may be shown that the deviation in azimuth $a = \frac{1}{2}d \sec L \cot D$, in which L equals the latitude, and D the declination of the pole star. A second method is to observe the transit of two stars differing considerably in declination, when the difference in time should equal the difference in right ascension. Calling d, as before, the error in time, $a = d \frac{\cos D \cos D'}{\cos L \sin (D' - D)}$. One of the stars should be near the pole, the other at some distance from it, and it is more convenient to select two stars differing but little in right ascension; 51 *Cephei* and δ *Ursæ Minoris* are well adapted to this purpose, only, in this case, as they culminate on opposite sides of the pole, we must give D a negative sign in the above formula, so that we shall have in the denominator $D' + D$ instead of $D' - D$.

The error in level is found by observing the bubble of the spirit-level in its two positions, when turned end for end; multiplying the mean deviation from the centre by the value of one division, gives the angular deviation b, in seconds.

Reverse the telescope and repeat, and if a different result is attained, it shows that the two pivots are unequal, the error equalling one half the difference in the readings. As the pivots may be irregular in shape, readings of the level should be taken with the telescope turned 10° at a time on each side of the zenith; a correction may then be applied for any given position of the telescope.

The error in collimation may be found directly with the spider-line micrometer, first measuring the angular magnitude for one turn of the screw as described in Experiment 180. Direct the telescope towards any well defined object and measure its distance from the central thread; then reverse the telescope, and measure again. The difference will equal twice the error of collimation. A more accurate method is to observe the transit of the pole star over two of the wires, then reverse the telescope and observe the transit over the same two wires, which will now be on the other side of the field. Reduce each to the mean thread, when the results will differ by twice the error. Multiplying this by the cosine of the declination, gives c. Another method is to direct the telescope towards its reflection in the vessel of mercury. Bring the movable wire to coincide with its reflection, or with that of the central thread and divide by two, and the distance from the central thread, correcting for level, gives c. The error in level may also be found by reversing the transit, when in one case the interval between the thread and its image will equal the sum, and in the other the difference of the errors of collimation and level. If the pivots are unequal, the error must be determined by the level, and a correction be applied. If the movable thread is parallel to the horizontal wire a measurement may still be made by forming a small square by the vertical thread, its reflection, the horizontal thread and the movable thread, as a slight deviation from equality in the sides is readily detected by the eye. If collimators are provided, their cross-hairs are used like the distant object in the first method. Two are employed to avoid reversing the telescope. Their cross-hairs are brought to coincide, after removing the transit telescope. To avoid the difficulty of superposing two vertical hairs, one collimator may have two parallel threads very near together, the other, two threads inclined at an angle.

Having thus found the values of a, b and c, we may determine the right ascension $R.A.$ of any body by the following formula, in which T is the time as given by the clock, E the error of the latter, L the latitude of the place of observation, Z the zenith distance of the object observed, and D its declination. Z is readily obtained from the latitude and declination,

$$R.A. = T + E + a \sin Z \sec D + b \cos Z \sec D + c \sec D.$$

To adjust the finding circle, set the telescope vertical, by viewing its reflection in the mercury surface, set the index at the latitude of the place and move the level of the finder by the adjusting screws until the bubble is in the middle. The index will then mark the declination of any object to which the telescope is pointed. If we wish to have the finder give zenith distances, the index should be clamped at 0° instead of at the latitude. If preferred, the telescope may be pointed towards any star whose declination is known, and the finder set to correspond, after correcting for refraction.

To find the time by the transit instrument it is only necessary to observe the transit of any known star, preferably one not too near the pole, and the mean of all the wires, after applying the above corrections, gives the sidereal time. The difference between this and the time as given by the clock gives its error. This should be determined frequently, and the error and rate thus deduced. The mean time may be deduced from the sidereal time and the sun's right ascension, or it may be observed at noon by observing the transits of both edges of the sun. Correcting this by the amount that the sun is slow or fast, as given in the Nautical Almanac, gives the mean time directly.

179. Latitude by Transit.

Apparatus. A transit instrument which may be set with its axis north and south, and a clock giving sidereal time.

Experiment. One of the best methods of determining the latitude of a place is by a transit set in the prime vertical, that is, in a vertical plane at right angles to the meridian, or with its axis north and south. To adjust it in this position, after setting the axis nearly north and south, it is levelled and the central cross-

hair brought into the plane of collimation as in the last Experiment. It is then brought into the proper azimuth by observing the transit of a star near the horizon, that is, one whose declination is small. The time of transit is first computed by the triangle ZPS formed by the zenith, star and pole, in which the angle at Z equals 90°, ZP equals $90° - L$, where L is the approximate latitude, and PS equals $90°—D$, or the star's north polar distance. The hour angle ZPS or H, is given by the formula, $\cos H = \text{tang } D \cot L$, and subtracting or adding this to the star's right ascension, according as the observation is towards the west or east, gives the sidereal time of transit of the star. At this instant bring the middle cross-hair to coincide with the star, and if the other adjustments have not been disturbed the instrument will be in position. Generally the axis will be no longer horizontal, and it is therefore necessary to repeat with a second star. If the telescope has a horizontal circle like an altitude and azimuth instrument, it is most easily adjusted by placing it in the meridian and then turning it exactly 90°. To find the latitude it is now only necessary to observe the two transits of a star which culminates a little south of the zenith, and calling one half of this time, or the hour angle, H, we have in the ZPS triangle, $PZS = 90°$, $PS = 90° - D$ and $ZPS = H$, whence we deduce the latitude $L = 90° - PZ$ by the equation $\text{tang } L = \text{tang } D \sec H$. The advantage of this method is that, if the star culminates near the zenith, a small error in H will make an almost imperceptible error in the latitude. If the only error of adjustment is that the axis is not horizontal, a correction is simply applied by adding to the latitude the inclination, if the north end is highest, and subtracting it, if lowest. The error in azimuth is found by observing the east and west transits of the same star, not too near the zenith, and the mean of the two times, after correcting for error of the clock, minus the right ascension of the star, equals the error in azimuth a. To correct for this error, we must multiply the second member of the equation given above by $\cos a$, or write $\text{tang } L = \text{tang } D \sec H \cos a$. If the telescope is reversed, the values of the latitude will in one case be too great and in the other too small by an amount equal to the error of collimation. Hence, if the same star is observed on two successive nights with the telescope re-

versed, the mean result will eliminate this error. The latitude may be determined from two stars observed on the same night with the telescope in reversed positions, if their declinations are known with precision. Of course in practice a large number of stars should be observed, and the mean result employed, as no single observation should ever be relied upon.

180. Transit Circle.

Apparatus. A transit eircle, cloek, and vessel of mereury.

Experiment. A transit eirele eonsists of an transit instrument to which is attaehed a large, finely graduated, vertical circle. Two or more reading microscopes serve to read the position of the cirele, and to show the altitude of the objeet to which the telescope is pointed. Each microscope contains a spider-line micrometer, and the distanee of their objectives from the spider-lines and from the graduated limb should be such that one revolution of the screws shall equal one minute. If the head is divided into sixty parts the reading may be made to single seconds, or by estimation, .to tenths of a second. As it is diffieult to keep the microscopes at precisely the right distances, the magnitude of one division of the eircle should be measured by each micrometer occasionally, and the readings correeted if neeessary. The eccentricity should also be examined as explained in Vol. I, Experiment 7.

A number of parallel vertical threads, a fixed horizontal thread, and three equidistant horizontal threads moved by a screw and forming a spider-line micrometer are inserted in the eye end of the telescope. We must now determine with precision the angular magnitude of one turn of the screw. This may be done in several ways; first, by measuring the distance from the optical centre of the lens, or its foeal distance F, and the piteh of the serew p in eentimetres or inches; then $a = \frac{p}{F}\ 206265$, in whieh a is the required angular magnitude in seconds. Secondly, measure any known angular magnitude, as the diameter of the sun, and divide the diameter, as given in the Nautical Almanae, by the number of turns. Thirdly, turn the threads around 90° so that they shall eoincide with the meridian, and note the time of transit

of a star over them when they are moved across the field by a known amount. If the screw has been turned n times and the star has a declination D and occupies a time t in traversing this distance, we must have $na = t \cos D$. The best results are obtained with the pole star, in which case, owing to the curvature of its path, we must write $\sin na = t \cos D$. Irregularities in the screw may thus be detected. Again, the telescope may be directed towards any well defined distant terrestrial object, the latter bisected by the cross-hairs, and the reading of the circle and of the micrometer observed. Move the telescope slightly and again bring the cross-hairs to coincide with the object, when the change in reading of the circle and micrometer serves to compare them; the cross-hairs of a collimator form an excellent object in this case. Finally, the telescope may be directed toward a theodolite and the angular distance corresponding to n turns of the screw measured directly.

The instrument is first adjusted precisely like a transit, and may in fact be used like it to determine time and right ascensions. In determining the error of collimation, the movable thread is brought into such a position as to form a square with the central vertical thread, its reflection and the fixed horizontal thread. Then move the thread over its reflection, so as again to form a perfect square, when the distance it has been moved will equal twice the interval between the vertical thread and its image, or four times the error in collimation. Since the square should always be very small, its sides may be rendered equal by the eye with great precision.

The zero point of the circle must next be found by setting the micrometer at zero, and moving the telescope until the thread coincides with its reflection in the vessel of mercury placed beneath it. The reading of the circle then gives the position of the *nadir* or point 180° distant from the zenith. The horizontal points may also be determined by observing a star and its reflection in a vessel of mercury, and bisecting the angle between them. Unless the star is near the pole, its motion will be too rapid to enable the circle to be read during its transit across the field of view. The circle should therefore be set approximately in the right position and read beforehand, and the star as soon as it appears, bisected

and the micrometer read two or three times. The telescope is then directed towards the image in the mercury, clamped, and the micrometer again read after turning it on the star. The second position of the circle may then be read at leisure. If a spirit-level finder is attached to the telescope, still more time may be saved by setting it beforehand so that the telescope can be set by simply turning it until the bubble moves from end to end. The mean of the readings of the star and its reflection gives the horizontal point of the instrument, and should differ by 90° from the nadir found above.

The advantage of this method is, that by using different stars, we can obtain various independent determinations of the zero point.

The apparent altitude of any star when on the meridian is determined directly from the graduated circle and micrometer. The circle may be set so that the star shall transit across the field of the telescope, clamped, and the circle read by the microscopes. When the star enters the field it is bisected by the micrometer wire and several readings taken. From the mean of these, the reading of the circle and the magnitude of the micrometer divisions, we deduce the corrected reading of the circle, and subtracting from this the zero point as found above, we obtain the apparent altitude. The true altitude equals the apparent altitude minus the refraction, and in this case the simple formula $r = 57''$ tang $(Z - 3r)$ given in Experiment 174, is not sufficiently exact.

Recourse must therefore be had to Tables, of which those of Bessel agree best with observation. These are based on the formula, $r = ab^m c^n \cot A$, in which r is the refraction, and A the apparent altitude; a, m, and n vary slowly with A and their values are accordingly given in a table with A as an argument; b depends on the pressure of the air, and is equal to the product of two factors, one dependent on the height of the mercury column, the other on its temperature; finally, c depends on the temperature of the air.

The latitude is readily found by this instrument by observing the altitude of any star at its upper and lower culmination. Evidently the first will equal the sum, and the second the difference of the altitude of the pole or the latitude, and the north polar

distance. The mean of the two altitudes will therefore give the latitude, and this method has the advantage that it is wholly independent of all previous determinations of the position of the star, depending only on the accurate graduation and adjustment of the instrument. The principal use of the transit circle is, however, the measurement of the exact position of the stars. Their right ascensions are found from the times of transit, as with the transit instrument, and their declinations from the latitude and altitudes.

The fixed stars, if carefully observed, will be found, their name notwithstanding, constantly changing their position. These motions are due in part to changes in position of the axis of the earth (precession and nutation), and to the velocity of light (aberration), also partly to their real motions with regard to the sun, or their proper motion. The position of two hundred of the brightest stars is given for every ten days in the Nautical Almanac. For the others, a star catalogue must be consulted, which gives not only their right ascensions and declinations at a given time or epoch, but also for each star certain constants for computing their position at any future time. Let t be the time expressed decimally in years after that for which the catalogue is computed, k the correction to be applied to the given right ascension, and k' the correction in declination; p and p' the proper motion in right ascension and declination, and a, b, c, d, and a', b', c', d', constants dependent on the position of the star, given by their logarithms in the catalogue. A, B, C, D and E are constants dependent on the time, and are given in the Nautical Almanac. E can generally be neglected, as it never exceeds .05″. Then the values of k' and k may be computed by the formulas,

$$k' = tp' + Aa' + Bb' + Cc' + Dd', \text{ and}$$
$$k = tp + Aa + Bb + Cc + Dd + E.$$

If a, b, c, etc., are not given, they may be computed trigonometrically, or the change in position determined from six other so-called independent constants.

181. Zenith Telescope.

Apparatus. A zenith telescope, or, if this is not available, a transit, or an altitude and azimuth instrument may be used, if a micrometer and sensitive level are attached, as described below.

Experiment. A zenith telescope consists of a telescope mounted so that it can turn either around a horizontal or a vertical axis and supported on a tripod with levelling screws. A very delicate level is attached to the telescope, and may be turned around an axis coinciding with, or parallel to, the horizontal axis of the instrument. A small graduated circle is commonly attached, like the finder of the transit instrument, to show the angle between the level and telescope, or the inclination of the latter. The eyepiece has a spider-line micrometer like that of the transit circle, and some fixed equidistant vertical hairs are also usually added for observing transits. Two stops are commonly attached to the horizontal circle so that the instrument can be turned in azimuth just 180°.

This instrument is intended to determine the latitude, which can thus be obtained with an accuracy at least equal to that of any other method. Two stars are selected differing but little in right ascension so that they shall culminate within a few minutes of each other, and with declinations such that they shall culminate, one north and the other south of the zenith by nearly equal angles, that is, so that the mean of their declinations shall nearly equal the assumed latitude of the place. Having selected several suitable pairs of stars, the instrument is placed with one of its legs to the north and the other two east and west. The level attached to the stand is now placed east and west, and the bubble brought to the centre, then turned north and south, and again levelled by the north screw, and this operation repeated until the bubble remains in the centre, while the telescope is turned completely around. If there is no level, except that attached to the telescope, it is set so that, when levelled and turned horizontally 180°, the bubble will remain nearly in the middle. The level is now perpendicular to the vertical axis and the instrument may be adjusted as before. The magnitude of the divisions of the level and of the micrometer must next be determined in angular measure, and the telescope then brought nearly into the meridian by turning it towards any known star, and bringing the central vertical cross-hair to coincide with it at the computed time of transit. The stops are set on the horizontal circle so that the telescope may be quickly set in the meridian, and the finding circle set to one half

the difference of the declination of the first two stars to be observed. The telescope is then turned to the north or south according to which star culminates first, and, as the star approaches the meridian, the telescope directed towards it. The level is then clamped to the telescope, the bubble is brought nearly to the centre and the reading of each end taken. The micrometer wire is now made to cover the star and bisect it at the instant of transit as given by the clock. Or, if this is missed, to bisect it at a known time after transit. The micrometer reading is then taken and the telescope turned 180°. When the second star enters the field, the micrometer wire is brought over it and a second bisection made at the instant of transit. The position of the ends of the bubble of the level is also taken. It may be remarked that while it will not do to alter the angle between the telescope and level during the observation, there is no objection to moving both together, if the vertical axis of the instrument is not properly adjusted. If then, on reversing the telescope, the bubble moves to the end of the tube, it may be brought back by moving the telescope. The difference in altitude of the two stars will now equal the difference in the micrometer readings, after adding or subtracting the error of level. Calling D the mean of the declinations of the two stars, a their difference in altitude, the latitude $L = D \pm \frac{1}{2} a$, using the plus sign if the altitude of the northern star is greatest, and the negative sign if it is the least.

The great advantage of this method is that it is so free from almost all instrumental errors, and depends only on the rigid connection of the telescope and level, and on the correctness of the micrometer screw. It is also, in a great measure, independent of refraction, since both stars, having about the same altitude, are affected nearly alike. To still further reduce this error, stars should always be selected culminating within 25° of the zenith. Evidently, any error in the position of the stars will affect the latitude, and it is therefore essential to use a number of pairs of stars, selecting by preference the brighter ones, since the position of these is more accurately known.

Since the measurement depends wholly on the rigid connection of the telescope and level, evidently a transit, or altitude and azimuth instrument may be used almost precisely like a zenith tel-

escope, and nearly the same directions apply to all. It is only necessary that a delicate level should be attached to the finding circle, or otherwise connected with the telescope, so that it can be set at any angle.

182. Altitude and Azimuth Instrument.

Apparatus. An altitude and azimuth instrument, or a surveyor's transit, a chronometer and an artificial horizon.

Experiment. An altitude and azimuth instrument or an altazimuth, resembles a surveyor's transit enlarged, and is used to measure simultaneously both horizontal and vertical angles. It is not much used in Observatories, as it is difficult to attain as accurate results with it, as with the simpler instruments, each additional complication being a new source of error. It has, however, been used for special purposes, as for studying the refraction, and for observing the moon when not on the meridian. In the field, on the other hand, the observer is much more likely to be able to procure a surveyor's transit than any astronomical instrument, and it seems therefore desirable to show how the latitude and time may be determined by it.

The instrument, if large, is mounted on three legs with levelling screws resting on a point, line and plane as described in Vol. I, Experiment 77. One leg is turned to the north, the other two east and west, and the instrument is levelled as described in the last Experiment. Smaller instruments, such as are used by surveyors, are supported on tripods and are levelled by four screws. The horizontal circular plate is turned until one of the levels is parallel to two of the screws, diagonally opposite each other, and the bubble brought to the centre by turning them in opposite directions, i.e., turning both thumbs in, or both out. The other screws are then turned until the other level is horizontal. By a little practice the screws will be turned together so that the instrument shall be neither loose nor wedged too tightly. Before using the instrument it should be carefully adjusted as follows. First, see that the plane of the levels is perpendicular to the axis of the instrument. Clamp the plate carrying the graduated circle, and, after levelling the instrument as directed above, turn the

upper plate carrying the cross-hairs 180°, and see if the bubbles remain in the centre of the tubes. If not, turn the screws supporting the levels so as to bring the bubble half way back, level the instrument again, reverse it, and repeat until the bubble remains in the centre in every position of the plate. Now clamp the upper plate and turn the plate carrying the graduated circle. If the bubble now moves from side to side, the axes of the two plates do not coincide, a defect not easily remedied. If however, the instrument is used as described below, keeping the lower plate clamped, no error is introduced by this deviation of the two axes. The telescope may now be adjusted for level, collimation and azimuth like a transit. In making the last adjustment, or bringing it into the meridian, the vernier should be set at zero, the lower plate tightly clamped, and the adjustment effected by its tangent screw. The time may now be found at night by observing the transit of any known star, or at noon by observing the transit of both limbs of the sun as described in Experiment 178. Turning the instrument so that the vernier shall be at 90°, the telescope will be in the prime vertical and the latitude may be determined as in Experiment 179. By making the proper adjustments the altzimuth may also be used as a transit circle as described in Experiment 180°, though in this case a spider-line micrometer should be inserted in the eye-piece. By adding a sufficiently delicate level to the vertical circle, it may be used as a zenith telescope, Experiment 181. From these varied uses the altazimuth is sometimes called the universal instrument.

Altitudes and azimuths may now be observed directly, the former by the vertical, and the latter by the horizontal circle. An interesting application of this instrument is to finding a star in the day time. The altitude and azimuth are first computed in advance allowing time for the accurate adjustment of the instrument. In the ZPS triangle, $PZ = 90° - L$, the co-latitude, $PS = 90° - D$, or the star's north polar distance, and $ZPS = H$ its hour angle. The latter is found by subtracting the sidereal time from the right ascension of the star. If the mean solar time only is given, it must be reduced to sidereal time as in Experiment 174. PZ, or the zenith distance, and the azimuth PZS are then computed as in Experiment 177, and the former corrected for

refraction. Set the telescope to this altitude and azimuth, and on looking through it at the proper time, the star should be seen at the intersection of the cross-hairs. Move the telescope if necessary, so as to bisect it at the given instant and reading the altitude and azimuth, determine the error. The star most readily seen in the day time is the planet Venus, if not too near the sun. Its position must be determined from the Nautical Almanac, and corrected for parallax. By looking along the telescope, so as to obtain the right direction, it can often be seen by the naked eye. Of the fixed stars, of course Sirius, as the brightest, is most easily found; but when the sky is dark blue any first magnitude star is readily seen with a common surveyor's transit.

It frequently happens, when portable instruments are used, that great accuracy is not needed, and that the time is limited. In this case the direction of the meridian may be determined by a single observation of any known star, since in the ZPS triangle we have given ZP, PS and ZPS. We can therefore compute PZS, and turning the telescope horizontally through this angle brings it into the meridian. The star best suited to this observation is the pole star, and, if the hour-angle is about six hours, a slight error in the clock will not introduce any appreciable error in the result. The meridian may also be determined, without observing the time, by setting the telescope on any star, observing the horizontal angle, and when the star again attains the same altitude on the other side of the meridian, observing its azimuth again. The mean of the two azimuths gives the direction of the meridian. This method may be applied in the day time to the sun, if we correct for its change in right ascension, but it is generally better to observe the position of the sun at noon by computing the time when the westerly limb will cross the meridian, and bringing the telescope to coincide with it. The result may be verified by observing the transit of the other limb of the sun.

The latitude may also be determined by an altazimuth precisely as with a sextant, from the altitude of the pole star, or of the sun at noon.

After finding the direction of the meridian it is well to measure the azimuth of some distant terrestrial object so that the meridian can be again determined from it at any time. A more convenient

method, in some cases, is to fasten a plane glass mirror to the wall, and placing the instrument opposite it, turn it until its cross-hairs coincide with their reflection. This gives the angle between the normal to the mirror and the meridian.

183. Longitude.

Apparatus. A transit and chronometer at the two points whose longitude is to be compared. A telegraph connecting them and a chronograph is also desirable, though not essential.

Experiment. The correct determination of the longitude is a matter of much greater difficulty then the corresponding measurement of the latitude. By the longitude of a place is meant the angle between its meridian and that of some other place assumed as a starting point, for which the Observatory at Greenwich is commonly employed. To determine the difference of longitude of two stations, it is only necessary to observe the time of occurrence of the same event at each, when the difference in the times will equal the difference in the longitude. For short distances we may use the flash of a cannon, the explosion of a rocket, or other similar effect, but generally in such cases the difference in longitude can be much more accurately determined trigonometrically. For more distant places we require some event visible over a large part of the earth, and must seek for this among the heavenly bodies. The entrance of the moon into the shadow of the earth during a lunar eclipse would satisfy these conditions, but unfortunately, owing to the refraction of the earth's atmosphere and the penumbra caused by the large angular dimensions of the sun, this effect is not sharply defined, so that its exact time cannot be observed. The motions of Jupiter's satellites are better adapted to the purpose and may be observed with any telescope of moderate power. The phenomena to be observed consist of the eclipses, or passages of the satellites into the shadow of the planet, transits or passages of the shadows of the satellites across the face of the planet, and occultations, or disappearances of the satellites behind the planet. The necessity of a telescope, however, precludes their observation at sea, and unfortunately the times are not sufficiently instantaneous for accurate observation. They differ more-

over, as seen with telescopes of different sizes and powers. Approximate results may, however, be attained by merely subtracting the observed local time from that given in the Nautical Almanac. From the ease of the observation and calculation, this method, is sometimes valuable to travellers.

The longitude may be determined with precision from the instants of contact during eclipses of the sun, from transits of Mercury or Venus, or from the instant of occultation of stars by the moon, but the rarity of these events and the difficulty of the computation involved, render it undesirable to discuss them here.

The motion of the moon in right ascension is so rapid that it may be used to determine the longitude. Its time of transit and that of some known star are observed, and the time at which its right ascension at Greenwich would have been the same, is then determined by interpolation from the Nautical Almanac. The great objection to this method is that an error in the position of the moon as computed or as observed is increased about twenty-seven times in the final result. Hence it is impossible by this method to obtain the longitude nearer then within about one second, however frequently the observations are repeated.

A better method of determining the longitude, and the one generally employed until within a few years, is by the transportation of chronometers. If, at the first station, we observe the transit of several stars by means of a chronometer giving sidereal time, we obtain its error directly by subtracting their right ascension. If this is repeated on several days we obtain its rate. Now carry the chronometer to the second station whose difference of longitude is to be determined, and observe the transit of the same stars there. Evidently the difference in time of the chronometer, after allowing for its changed error, will equal the longitude. To avoid accidental errors and the change in rate when the chronometer is carried from one point to the other, it should be sent back and forth several times, or better, the comparison should be made by several chronometers. Generally, instead of observing the transits directly by the chronometer it is compared at each station with the observatory clock and the longitude thus determined, after allowing for the errors. To compare a chronometer with a clock or with another chronometer, the minutes and seconds are read off

directly, and the fraction of a second between their ticks estimated. If the chronometers have very different rates, or better, if one gives sidereal and the other solar time, the difference can be determined with much greater accuracy. Since 367 sidereal seconds equal very nearly 366 solar seconds, it follows that in every 3 m. 3 s., the solar chronometer will gain half a second or one beat on the other. Accordingly every three minutes their ticks will coincide, and, observing by the ear their time of coincidence, the difference between them may be determined within about .05 of a second.

The best method of determining the longitude is, however, by means of the electric telegraph and chronograph as described in Vol. I, p. 17. The principal error in this case is the personal equation of the observers, or interval between the instant of transit of the star and the depression of the finger key. To eliminate it, the observers should change places, or determine their personal equations directly. The absolute personal equation may be found by observing the transit of an artificial star which records its correct time of transit automatically on the chronograph. The difference between the observed and true times equals the personal equation. The difference in personal equation of two observers may be found by letting one observe a transit over three or four wires of a transit instrument and the other over the remaining wires. Do this with twenty or thirty stars and reduce each to the mean thread by multiplying the equatorial interval by the secant of the star's declination. The difference between the mean of their results will equal their personal equation. To avoid the difficulty of the second observer being hurried in taking the place of the first, which may affect his personal equation, each may observe the clock error by several well known stars, and the difference will equal their personal equation. The determination of the personal equation is of even more importance in ascertaining longitudes by chronometers, unless the same observer determines the clock error at both stations.

184. Equatorial Telescope.

Apparatus. A telescope mounted equatorially with a spider-line and position micrometer. For class purposes, and when the

most perfect results are not needed, the siderostat will form a most convenient substitute for an equatorial mounting. A sidereal clock, a lantern, a good stellar map or globe, and Webb's Celestial Objects, or some similar book, are also needed.

Experiment. A great difficulty in the observation of celestial objects with large telescopes, especially with high powers, is that, owing to the motion of the earth, they move rapidly out of the field of the telescope. To avoid this difficulty, telescopes intended especially to study the physical aspects of the stars are mounted equatorially, as it is called, so that this motion is readily followed. One axis, called the polar axis, is directed toward the pole, that is, set in the meridian and inclined by an angle equal to the latitude of the place. At right angles to this is placed a second axis, called the declination axis. At one end of the latter, and at right angles to it, is placed the telescope, counterpoised by a weight at the other end. The amount that each axis turns is shown by a graduated circle and vernier. If the telescope is directed towards a star and slowly turned around the polar axis it will evidently describe the same path as the star, and may be made to follow it readily by hand. If clockwork is attached so as to make the axis turn once in twenty-four sidereal hours, the telescope will remain directed towards the star and will follow it indefinitely. Thus the hour angle of the star may be read off directly from the circle attached to the polar axis, and the north polar distance or declination, from the circle on the declination axis. Hence there are two positions of the telescope obtained by turning it around each axis 180°, in which it can be directed to any part of the heavens.

To adjust the equatorial, we must first bring the polar axis into the meridian. Set the declination axis horizontal and move the telescope and stand until it is directed to a star at the instant of culmination, as given by its right ascension and the clock. To incline the polar axis by the right amount, direct the telescope to any known star when near the meridian and read the polar distance by the declination circle. Reverse the telescope and read again. The mean of the two readings, corrected for refraction, gives the north polar distance, and this should be rendered the same as the true north polar distance, by raising or lowering the

axis. The difference in these readings divided by two gives the error of the vernier of the declination circle. To find the error in position of the vernier of the hour circle, set the telescope near the meridian and observe the transit of any known star not too near the pole. The difference between the hour angle as given by the clock and that given by the hour circle, shows the error in position of the latter. To see if the axis of the telescope is at right angles to the declination axis, observe the transit of an equatorial star, reverse the telescope and observe its transit again, when the difference between the interval as given by the clock and as given by the hour circle equals twice the error of collimation.

To the eye end of the telescope is attached a spider-line micrometer free to turn around an axis coincident with that of the telescope, forming what is called a position micrometer. The angle is measured by a graduated circle and vernier. A small telescope called a finder is attached to the side of the large telescope, and is set parallel to it. It carries cross-hairs in its focus, by which minute objects are more readily brought into the field of the larger instrument. To adjust the finder direct the large telescope towards any convenient object and, bringing it to the centre of the field, move the cross-hairs of the finder until they cover it.

The siderostat consists of a plane silvered-glass mirror mounted on two axes at right angles to each other, one being brought parallel to the axis of the earth, like an equatorial. The telescope is fixed parallel to the earth's axis and is directed down towards the mirror. Evidently if the mirror is turned so as to reflect a star into the telescope it will follow it in its motion like an equatorial. The advantages of this instrument for certain purposes are very great, especially where, as in the next Experiment, much apparatus is to be attached to the eye end. As the latter is fixed, the observations may also be made much more conveniently, especially where an object is to be shown to a class, since the observer always remains in the same position. The difficulties of a dome, otherwise necessary, are also avoided. The objections to this arrangement are the difficulty of making, and keeping, a surface perfectly plane, the loss of light, and the inconvenience in finding objects.

To find an object when its right ascension and declination are given, set and clamp the telescope to the proper declination by the declination circle. Subtract the right ascension of the object from the sidereal time, and set and clamp the hour circle to the difference, or the hour angle. Now on looking through the finder the object should be in the field, and may be brought by the tangent screws to the intersection of the cross-hairs. It will then be seen in the field of the larger telescope. No correction is here made for refraction owing to which the star will appear above, or since the telescope inverts, apparently below its predicted place. Unless the altitude is small, however, this will give little trouble. If the object is to be measured or observed for some time, the clock should be started so that it may be followed continuously. This is almost indispensable when high powers are used. Try finding some of the brighter stars in the day time, and see how nearly the predicted and observed places agree. Evidently the inverse method of pointing the telescope towards any unknown object, reading the two circles and observing the time, furnishes an easy means of determining its right ascension and declination after correcting for refraction, but it will not compare in accuracy with the methods of Experiment 183 and 185.

The position of an object is determined by the micrometer as follows. Light the lamp illuminating the cross-hairs, put on a low power eyepiece and turn the telescope so that two or three known stars shall be in the field at the same time. Regard one of them as unknown and measure its distance and direction from each of the others. To determine the direction, clamp the telescope so that it shall not be carried by the clock work and turn the micrometer until when the wire which is parallel to the screw is brought over the star, the latter will remain bisected as by the earth's motion it passes out of the field. The index of the position circle should now read zero. Connect the telescope with the clock, and turn the wire until parallel to, or rather until it covers, both stars. The angle through which it is moved is known as the position angle. Now determine the distance of the stars as described in Vol. I, Experiment 77. To eliminate the error due to the wires not coinciding, when the screw is set at zero, it is well to take two readings, one with the movable wire on each side of the fixed one,

and employ their mean. The difference in declination of the two stars will equal the reading of the micrometer, reduced to angular interval as in Experiment 181, multiplied by the sine of the position angle. The difference in right ascension will in like manner equal the distance multiplied by the cosine of the mean declination of the two stars. These formulas neglect the curvatures of the heavens between the two stars, and, if the distance is considerable, or great accuracy is required, a more accurate value must be deduced by spherical trigonometry. The position of any object is fixed by measurement from a single star, but greater accuracy is attained by comparison with two or three. To test the work, measure also the difference in right ascension and declination by the following method, which has the advantage that clockwork moving the telescope is not needed. Make the position angle 0°, so that the movable wire is parallel to the star's path, and, clamping the telescope a little to the west of both stars, observe the difference in the time of their transits over the vertical wires of the micrometer. Move the telescope again in advance of the stars and observe the transits a second time. The interval of the time will equal the interval of right ascension. The difference in declination may also be found by bringing the movable wire to coincide first with one star and then with the other, when the difference in reading will give the difference in declination.

Let us now consider, in order, the principal objects to be observed.

The Sun. To moderate the intense heat when the telescope is turned directly towards the sun, a cap with an aperture is sometimes placed over the object glass. The spherical and chromatic aberration is also thus diminished, but the aperture must not be too far reduced, or owing to diffraction the definition will be injured. A plate of smoked or colored glass is interposed between the eye and eye lens to cut off the light, and in this position its irregularities do not affect the image. It will be noticed that the centre of the sun's disk is much brighter than the edge, and that generally a number of dark spaces, or spots, are visible. The larger spots are surrounded by areas less bright than the surrounding surface which are called *penumbræ.* Spaces brighter than the disk called *faculæ* are also commonly seen. If the sur-

face is carefully examined with a good telescope, it is seen to be covered with a multitude of objects, resembling willow leaves or rice grains in shape. The spots are more frequent in the equatorial portions of the sun, or at least in latitudes 10° to 30°, and if watched from day to day will be seen to alter their shape and gradually move from west to east owing to the sun's rotation.

During an eclipse of the sun, the principal phenomena to be looked for are the following. Approaching disk of the moon before contact, instant of first contact, measurement of obscured portion of sun at known times during first few minutes after contact. Appearance of sun's surface adjacent to moon's limb which would be altered if any lunar atmosphere existed. Time of moon's limb reaching any marked spots. Breadth of crescent at observed times when most of the disk is eclipsed. If the eclipse is annular, the time of formation and rupture of the ring should be observed, and the appearances when the narrow line of light breaks. If the eclipse is total, too great care cannot be taken in preparation for its accurate observation. The colored glass should be arranged so that it may be instantly removed, as it will not then be needed during totality nor by the unaided eye for a few minutes before or after. Care should be taken that the eye is not dazzled by looking too much at the sun before totality, or, if the weather is cool, that the hands do not become numb during the chill accompanying the cutting off of the sun's heat. The appearance during totality is indescribably grand, and the aspect, both of the sun and of surrounding objects, is extremely difficult to depict. Observations may be made on the approach of the shadow by looking towards the western horizon, and, during totality, of the form, structure and polarization of the corona. The protuberances can be well observed when there is no eclipse, and therefore much time should not be spent on them. The light will be, roughly speaking, about that of twilight when the sun is 5° below the horizon, or that of a candle distant about fifty cms. as shown by the photometer, Vol. I, Experiment 68. Beside these observations others may be made with the spectroscope as will be described in the next Experiment. As the eclipse passes off, similar observations may be made. During the eclipse, observations may be also made with advantage on the variation of the light, on the temperature of the air, on the radiant

heat from the sun, and on the changes in polarization of the sky. Since the direct light of the sun is too intense to be observed directly by the photometer, it may be reduced by covering the front portion by a box and allowing only a small part to pass through a lens of short focus.

The Moon. As seen through a small telescope, no heavenly body shows to greater advantage than the Moon, both on account of the number of objects to be examined and the changes they undergo with variations of illumination, or when the Moon is in different phases. A good map is essential, of which the best is that of Beer and Mädler, its reduced copy in Webb's Celestial Objects, or the elaborately illustrated work of Carpenter and Nasmyth. The photographs of Rutherford and De la Rue may also be used for comparison. As one side only of the Moon is always directed towards the Earth, lunar objects always retain the same apparent relative positions and vary only with changes in illumination. The best effect is generally obtained when the light falls obliquely, or when the irregular edge of the illuminated portion called the *terminator* is near the object to be studied. If the Moon is observed when full, it is seen to consist of light and dark portions, the latter being designated by the Latin prefix *Mare*, as they were formerly supposed to be seas. Many parts of the surface are covered with circular rings of various sizes, which are supposed to be the craters of extinct volcanoes. Near the southern portion, which, since the telescope inverts, will be at the top, is a marked circle called *Tycho*, which seems to form the centre of the most disturbed portion of the moon's surface. Various lines extend from this crater several hundred miles in length, one of them passing over the centre, reaching nearly across the disk and bisecting the dark space known as *Mare Serenitatis*. In the northern portion is a dark elliptical spot about sixty miles in diameter, surrounded with walls nearly 4,000 high, known as *Plato*. South of this about quarter way to *Tycho* is another conspicuous crater, *Archimedes*, south east of this is *Copernicus*, and a little south of west from the latter is *Kepler*. Returning now to the north and a little to the east we reach *Aristarchus*, the brightest point of the moon's surface. A long precipitous range of mountains extending from *Mare Serenitatis* toward *Copernicus* is called the *Appenines* and

is a beautiful object when the moon is at the quarter. The small dark patch in the north eastern part of the moon's disk is called *Mare Crisium.* Near its centre will be noticed a curious pair of volcanic craters.

Mercury. On account of the short distance of Mercury from the sun, it can never be observed when the latter is far below the horizon, and may sometimes be seen by the naked eye, as a star near the horizon soon after sunset. The distortion due to the atmosphere is then so great and the intrinsic brightness of its disk so considerable, that it is best observed before sunset, pointing the telescope in the right direction from its right ascension and declination as given in the Nautical Almanac. It will then be seen to undergo the same changes of phase as the moon, according to its position with regard to the sun and earth. Its apparent diameter varies from 4″ to 12″, and its greatest distance from the sun never exceeds 30°.

Venus. The same remarks apply, though with less force, to Venus, which is best observed about sunset. With even a moderate magnifying power it will look larger than the moon to the naked eye, though it is difficult to convince one's self that this is the case unless the moon is near and seen with one eye, while the other is directed towards Venus. The angular diameter of Venus varies from 9″ to 62″ and its greatest elongation from the sun is 47°.

Mars. As seen through a telescope, Mars presents a nearly circular disk of a reddish or ruddy hue, having a diameter of from 3″ to 18″, or occasionally 23″. Certain markings may be seen upon its surface, and a whiteness is seen at the polar portion extending as the pole is turned from the sun, and which has been supposed to be snow. When Mars is not in the same line as the earth and sun, it assumes a gibbous form like the moon, the ratio of the two diameters amounting in some cases to 10 : 9.

Asteroids. Any of the larger Asteroids are easily found from their right ascensions and declinations as given in the German Nautical Almanac. They appear precisely like minute stars, but are readily distinguishable by their motion which is perceptible in the course of a few hours by a spider-line micrometer.

Jupiter. The largest of the planets, and the brightest, with the exception of Venus, is Jupiter, which in the telescope presents an elliptical disk, the ratio of the equatorial to the polar diameter being about as 17 : 16. The angular diameter varies from 30″ to 46″. It is accompanied by four satellites or moons, which appear as minute stars, readily seen in a small telescope or even with a large opera-glass. They frequently are eclipsed in traversing the shadow of the planet or occulted in passing behind its disk. When passing in front of it, or transiting across the planet, they are visible as bright spots, and the transits of their shadows, causing eclipses of the sun on Jupiter, are also visible as black spots. The times of all these phenomena are recorded in the Nautical Almanac. The satellites are generally seen in a line approximately coinciding with the path of Jupiter and the greatest angular elongation of the furthest satellite is about 10′. When Jupiter's disk is carefully examined it is seen to be traversed by two or three dark lines or belts, nearly parallel to its equator.

Saturn. The second in size of the planets is Saturn, which presents a disk of about 14″ to 21″ diameter. Its most remarkable peculiarity is the ring with which it is surrounded and which consists of three concentric annular disks, of which the two outer only are visible with ordinary telescopes. The plane of the rings is inclined 28° 11′ to the plane of the orbit of Saturn, and hence, twice during Saturn's revolution they are seen edgewise and disappear. This will be the case in 1877 and 1892.

Eight satellites surround Saturn, one, Titan, being of considerable size and appearing as an eighth magnitude star. The next largest is the outer satellite, Iapetus, whose light is, however, variable, probably owing to its surface being spotted. The third, fourth and fifth from the centre are visible with a good telescope, but the others can only be seen by the largest instruments. The greatest elongation of the outer satellite is about 10′, which, with their motion, serves to distinguish them from stars.

Uranus. This planet may be seen by the naked eye as a sixth magnitude star, and in the telescope presents a dim disk 4″ in diameter. Its satellites are beyond the reach of ordinary telescopes. Its place is given in the Nautical Almanac, and during

the remainder of the century it will be well situated for observation in the evening in the spring and summer.

Neptune. But little can be seen of Neptune except as a dim ill defined eighth magnitude star. Its position during the rest of the century will be in *Aries* and *Taurus*, and it may be most favorably observed during the winter.

Comets. In the observation of comets, low powers only can be used, and the tail can generally be seen better with the naked eye or with an opera-glass, than with a telescope. If the position is not accurately known, it may sometimes be found by *sweeping*. Point the telescope above and to one side of the supposed place, swing the telescope horizontally, then lower it a distance about equal to half the breadth of the field, and thus go on until the comet is found or the limits of its possible position passed. The lowest power should be used, to secure the greatest light and the largest field.

When a very minute or faint object is to be observed it may sometimes be seen more readily by looking to one side of its supposed place, so that its image shall be formed on one side of the centre of the retina.

Double Stars. The real angular diameter of a fixed star is so small, probably only a few hundredths or thousandths of a second, that their true shape is never perceptible, even in the most powerful telescopes. With the best instruments they present small circular outlines called spurious disks, due to diffraction, whose diameter increases as the aperture of the telescope diminishes. If we examine a large number of stars we find that many of them are double or consist of two very near together, and the proportion is much greater than could be accounted for by mere accidental juxtaposition. Where the interval is small, the pair are found to revolve around their centre of gravity in accordance with the law of gravitation. This cannot be verified in the case of the more distant components, on account of the immense interval of time which would be required to produce a perceptible motion. There are several examples among stars visible to the naked eye of the close approach of two, and presenting, therefore, the same appearance as close double stars seen through a telescope. Of these may be mentioned the two components of α *Capricorni*, of

γ *Leonis*, and of ε *Lyræ*. The distances in the three cases are 373″, 337″ and 207″. The star Alcor, ζ *Ursæ Majoris*, in the middle of the handle of the *Dipper*, has a small star 690″ distant from it, and resembling a double star with components of different sizes. The star ε *Lyræ* is valuable as a test of the eye. The vision is not perfect if the two stars cannot be distinguished. They are easily found as forming a small equilateral triangle with another star of equal brightness and with α *Lyræ*, the very bright star in the zenith in summer. A list of the more easily resolved double stars is given in Appendix B, Table 19. Find a number of them and measure their distance apart and position-angle by the micrometer.

Many of the stars when viewed with a sufficiently powerful telescope are seen to consist of three or more components. They are then called triple or multiple stars. As examples of these objects may be mentioned 14 *Can. Min.*, θ^2 *Sagit.*, o and η *Cyg.*, 90 *Leo.*, 11 *Monoc.*, 65 *Urs. Maj.*, ξ, ι, o and θ *Orionis*.

The color of the fixed stars varies very greatly, and this is especially noticeable with the double stars, since the two components are readily compared with each other. Besides these there is a small class of bright red stars of which those located in XVIII h. 57 m., *S* 5° 53′, XX h. 10 m., *S* 21° 45′, and XXIII h. 51 m. *N* 50° 40′ are good specimens. Several other red stars will also be noticed in *Persus*, *Auriga* and *Cygnus*. The brightness of many of the fixed stars has been observed to alter periodically. Among the most remarkable of these variable stars are o *Ceti*, β *Persei* and η *Argûs*.

Clusters and Nebulæ. A further examination of the heavens shows a tendency of the stars to collect in groups, of which, among stars visible to the naked eye, the *Pleiades* are the best example. Next to these is a misty whitish spot in *Cancer* known as *Præsepe*, which, with a telescope or even an opera-glass, is seen to consist of a group of small stars. Another example is in the so-called sword handle of *Perseus* which, under the slightest optical aid, is in a great measure resolved into stars. The *Milky Way*, a white cloudiness extending completely around the heavens, also under a sufficient magnifying power is resolved into stars. Besides these there are many objects presenting a similar appearance in the tele-

scope which are known as nebulæ. As a large part of them are resolved into clusters of stars in the most powerful instruments, and as a bright cluster in a small telescope looks exactly like a faint nebula in a large instrument, it was at one time supposed that all nebulæ might be resolved if sufficient optical power could be brought to bear on them. Later observations with the spectroscope have, however, proved that some of them consist of gaseous matter, and can never be resolved into stars.

A list of the most remarkable clusters and nebulæ is given in Table 20. Prominent among them is the great nebula of *Orion*, visible to the naked eye as a misty star in the middle of the sword handle. As seen through the telescope this star is resolved into four, surrounded with a bright hazy luminosity. This is a real nebula as shown by the spectroscope. The nebula in *Andromeda* almost equally bright, is large and oval, and, though not yet satisfactorily resolved, doubtless consists of stars, as it gives a continuous spectrum. The cluster in *Hercules* is also very bright, and in a large telescope is a superb object. Nebulæ often assume certain definite forms, as a ring, of which the only example accessible to small telescopes is that in *Lyra*, nearly midway between β and γ. Another form is the planetary nebula, which presents a small sharply defined circular disk, like a planet; the largest of these is 97*M* in *Ursa Major*, which has a diameter of $2' \, 40''$. Others are spiral in structure, of which the most remarkable is 51*M.* in *Canes Venatici.*

185. Spectrum Telescope.

Apparatus. The equatorial or siderostat, described in the last Experiment, and a spectroscope to replace its eyepiece, which for observations on the sun should have a very great dispersion, either by using a large number of prisms, or better by a diffraction grating. For other purposes, a direct vision spectroscope or other form giving a small dispersion is preferable. Very good results may be attained with a large cosmorama lens as an objective, since sharp definition is not needed, but a large amount of light is indispensable.

Experiment. When the object to be observed has a considerable angular aperture, good results are attained by directing a simple one prism chemical spectroscope towards it without using

the telescope. This is the case in studying the spectrum of the sky, of clouds, of the aurora borealis and of the zodiacal light. In all cases the wave-length must first be determined in terms of the scale-reading as described in Vol. I, Experiment 77. A direct-vision spectroscope may be used if a scale is inserted in its eye-piece, but with faint objects the loss of light is an objection.

The solar spectrum may be observed as described in Vol. I, Experiment 77. No limit has yet been reached to the dispersion which may be used with advantage. The best results seem to have been attained with a diffraction grating formed of fine lines ruled with a diamond on speculum metal or silvered glass. By observing a spectrum of a higher order an enormous dispersion may be obtained. The plate should be set at right angles to the observing telescope. The light of the sun being reflected directly by means of a mirror, we obtain rays from all parts of the sun's disk. If, however, a carefully corrected lens is interposed at a distance from the slit equal to its focal length, a well defined image of the sun will be formed upon the slit, and accordingly the spectrum of any portion of the sun's disk may be observed.

Carrying this a step further by enlarging the lens, we have the more common arrangement of a telescope with the eyepiece replaced by a spectroscope. If the spectrum of the space just beyond the edge of the sun is observed with a high dispersive power, the image of the sun being carefully focussed, the lines C and F will sometimes appear bright instead of dark, owing to the presence of protuberances such as are described in the last Experiment in connection with eclipses of the sun. If now the slit is gradually opened, the true shape of the protuberance will be seen in red or green on a dark background. Under favorable circumstances many other lines have been seen reversed, but the line C is most favorable for ordinary observation. The reason that the protuberances are thus rendered visible is that by dispersion the light of the sun may be indefinitely diminished, while that of the protuberances, consisting of a limited number of monochromatic rays, retains nearly its original brilliancy. Accordingly, with a sufficient dispersion, the light of the protuberance becomes brighter than that of the sun itself.

During an eclipse of the sun, many of the phenomena may be

14

studied to advantage with the spectroscope. If directed to a protuberance near the point of first contact, the approach of the moon is marked by the gradual covering up of the protuberance. The instant of first contact is thus well observed. During the partial phase the change, if any, along the edge of the moon may be looked for, and during totality the spectrum of the corona may be observed. This is best seen with a moderate dispersion or by a simple chemical spectroscope with no lens in front of the slit. The spectrum of the whole light around the sun is in this case observed.

The spectrum of the stars is best seen with a spectroscope consisting of but one or two prisms, as the light is generally too feeble for a greater dispersion. As the image of a star is a minute point, if allowed to fall directly on the slit its spectrum would be a narrow luminous line in which it would be difficult to distinguish the dark lines. To remedy this inconvenience a cylindrical lens is interposed in such a position as to form a line of light on the slit. A spectrum of less brilliancy but greater breadth is thus formed. The slit may be either parallel or perpendicular to the axis of the lens according to the distance at which it is placed. The latter position is, however, generally preferable, since, if the two are parallel, the spectrum will not be of equal breadth throughout, owing to the aberration of the lens. The fixed stars give spectra resembling that of the sun, and consisting of continuous spectra crossed by dark lines. Clusters of stars give similar spectra, though they are so faint that the lines are not visible, and the spectra appear continuous. Nebulæ, on the other hand, give spectra composed of three or four bright lines, a characteristic property of luminous gases, consequently such nebulæ can never be resolved into stars. This is the best if not the only certain means of distinguishing between faint clusters and nebulæ.

Much interest has been excited by the observation of the motion of the stars by the spectroscope. If the latter is rapidly approaching a luminous body the waves of light fall upon its slit at shorter intervals, and hence the wave-length appears to diminish. In the same way an increase in distance of the light and slit appears to increase the wave-length. Owing to this action the lines in a spectrum would move towards the red end by an amount

proportional to the velocity of recession of the light observed, or towards the violet end if the distance was diminishing. This method has the advantage that it measures the velocity of motion quite independently of the distance of the object, but the velocity of light is so great that it is only capable of measuring velocities amounting to several miles per second. On observing the spectrum of Sirius with a large dispersion, Mr. Huggins noticed that the F line in its spectrum was a little less refrangible than that given by a Geissler tube containing hydrogen. From this he inferred that Sirius was receding at the rate of 41 miles per second, or allowing for the motion of the earth 30 miles per second. Similar observations have been made on many other bright stars, but to attain accuracy a telescope of the largest size is indispensable on account of the feeble light. The velocity is readily computed as follows. Determine by a micrometer the change in wave-length l of the hydrogen line F compared with that of a Geissler tube. Then we have the proportion $\lambda : l = V : v$, in which λ is the wave length $= .0000004861$, for the F line, V the velocity of light $= 300400000$, and v the required velocity, all being given in metres. From this we deduce, $v = 618000000000000\ l$, or if l is given in millionths of a millimetre and v in kilometres, $v = 618l$. This method of measuring velocity is not, however, universally admitted, as certain considerations, both theoretical and experimental, seem to show that the motion of the light may have no effect on its wave-length.

LANTERN PROJECTIONS.

During the past ten years a new era has arisen in the illustration of lectures, by the general introduction of the magic lantern as a means of demonstration. Not only in science, but in the mechanic arts, in architecture, and in fact in any subject susceptible of illustration by engravings or photographs, a few glass plates, which may be carried in the hand, will interest and instruct an audience more than the finest diagrams, which are, moreover, far more cumbrous and expensive. It is, therefore, desirable that every one who may have occasion to address an audience should be able to manage a lantern and to project photographs on the screen. Again, especially in physical experiments, many objects are so minute that they cannot well be shown to a large number of persons, and an enlarged image of these may often be thrown on the screen and thus be seen by hundreds at a time.

The method employed is to illuminate the photograph or other object as strongly as possible by a very brilliant light, and then interpose a convex lens at such a distance that an image shall be formed at its conjugate focus on a large white screen stretched over the opposite wall. The sources of light most commonly employed are sunlight, the electric light, the magnesium light and the calcium light, which will be considered in turn.

186. Sunlight.

Apparatus. A window facing to the south is desirable, which may be closed by a shutter with a circular aperture. The other windows should be provided with shutters or opaque curtains so as to exclude all light. A *porte-lumière* may be fitted into the hole in the shutter or a heliostat may be placed on a shelf, outside

so as to reflect a ray of light into the room. The hole in the shutter is commonly closed by a board, which may be removed and the *porte-lumière* fastened in its place by screws or buttons.

Experiment. If always available, no source of light could compare with the sun for almost all projections. Its advantages are steadiness and great intensity, especially when a parallel beam is required. The *porte-lumière* consists of a mirror which may be turned around either of two axes at right angles to each other. These motions can be effected by handles inside the room so that the mirror may be turned in any desired direction. This is necessary, as, owing to the apparent motion of the sun, the direction of the light is constantly changing, and the mirror must be moved at intervals to correspond.

A simple form of *porte-lumière* may be made by passing a tin tube about four inches in diameter and six inches long through the shutter, making it free to turn, but held in place by friction. The lower end of the mirror is hinged to this tube, and the upper end is held by a cord which passes through the tube around a violin peg attached to the tube, inside the shutter. The string is kept tight by the weight of the mirror, and the latter may be raised or lowered by turning the peg. Turning the tube gives it a second motion at right angles to the first. Sometimes the proper motion is given to the mirror by clock-work, forming the instrument known as the heliostat. The apparent motion of the sun is a circle with the north pole of the heavens as a centre, or around the axis of the earth, and with a radius varying from 67° in summer to 90° in spring or autumn and to 113° in winter. Suppose now we place a rod parallel to the axis of the earth, that is, running north and south and inclined to the horizon by an angle equal to the latitude of the place, and that we cause it to revolve uniformly once every twenty-four hours, by clock-work. Every point of this will retain the same relative position with regard to the sun during the day; or, if an arm is attached pointing towards the sun it will follow the latter in its motion. Now suppose a mirror attached to the rod and turned through such an angle as to reflect the light in the direction of the rod, or parallel to the earth's axis. As the sun moves, the mirror will turn with it, and always throw the light in the same direction. The direction of the beam will

thus be fixed, and by a second mirror may now be turned in any desired direction. This is known as Fahrenheit's heliostat. To avoid using two mirrors a more complex arrangement is sometimes employed. In Silbermann's heliostat the mirror is attached at right angles to the diagonal of a parallelogram, one side of which is by clock-work kept turned towards the sun, while the other is fastened in the direction in which the light is to be thrown. The mirror is thus kept equally inclined to this direction and to the sun, the required condition. The principal objection to this instrument is that the joints give a jerking instead of a steady motion to the mirror. Foucault's heliostat consists of a rod turned by clock-work so that it shall always point towards the sun, with one end attached to the edge of the mirror and the other to a rod normal to the surface. The mirror is mounted on a universal joint and may be placed at any desired angle with the revolving rod. Its direction, and the revolving rod, will now always be equally inclined to the normal to the mirror, which is the required condition that the light shall always be thrown in the same direction as long as the rod is made to follow the sun.

To obtain a beam of sunlight, remove the board from the shutter and replace it by the *porte-lumière*, which may then be fastened in place. Turn the mirror until the light reflected from its surface falls upon the opposite wall, where, if the aperture is reduced, it will form a bright circular image of the sun. If the two surfaces of the mirror are not exactly parallel, a series of images is seen, their number and distance apart increasing with the angle of incidence. The first image formed by the front surface of the glass is generally fainter than the second formed by its rear surface, the others are due to continued internal reflection. If the position of these images is noticed, it will be seen that they are moving slowly over the wall, so that the mirror must be turned occasionally to keep them near the same point. If the heliostat is used, it must be adjusted so that its axis is parallel to the earth's axis. It should be fastened permanently at an angle equal to the latitude, and then turned into the plane of the meridian. The mirror is then moved until the light is reflected in the proper direction, and the clock-work started.

Great care must be taken always to bring the *porte-lumière* or heliostat inside after using, as exposure to the weather for even a single night may cause serious injury.

187. Electric Light.

Apparatus. A magneto-electric machine or a powerful battery, connecting wires and an electric light regulator. If the current is to be generated by a magneto machine, an engine is needed as a motor. The power and speed required will depend on the strength of current desired and the kind of machine used. Generally an engine of at least three or four horse-power is needed, and a speed of five to fifteen hundred turns per minute. The machine should be driven in the usual way by a belt and pulley. If a battery is employed it should be set up in an adjoining room with the windows open, or under a hood, to carry off the fumes.

Experiment. The brightest light that can be obtained artificially is the electric light, and but for its expense and the trouble of production, it would probably supersede other sources for projections. It is generated by passing a powerful current of electricity between two carbon points, which when separated by a short distance become heated to incandescence and give out an intense white light. Two methods are employed for producing the current, a magneto-electric machine and a galvanic battery. In the first of these an electro-magnet is caused to revolve rapidly past the poles of a permanent magnet, and the current thus generated excites a second much larger electro-magnet. A greatly increased current is now obtained by revolving another electro-magnetic armature in front of the latter. This is the principle employed in the Wilde machine, but in later forms no permanent magnet is used, the current being produced in the first magnet by induction or otherwise, and then maintained by the current itself. The current is rendered continuous in the form proposed by Gramme, in which a large number of coils are used in the revolving armature, and with which extraordinary effects are produced. A steam engine is required to drive these machines, but notwithstanding their large first cost, they form the cheapest source of powerful currents of electricity, and are now coming into general use for industrial purposes.

The more common method of producing the electric light is by means of a battery of from 40 to 60 large Bunsen or Grove cells. These are mounted as described on page 1, and give a current of sufficient power to generate an excellent light. The advantage over the previous method is that no engine is required, and the first cost is comparatively small; but the labor of amalgamating the plates and mounting the battery is very considerable, the consumption of acid great, the current rapidly grows feebler, and the fumes require a separate, well ventilated battery room.

The current thus generated and having an electromotive force of 50 to 100 volts, is passed between two terminals of gas coke which are then separated by a small amount. If this distance becomes too great the light flickers and is liable to go out, the current then ceasing and not flowing again until the carbons have been brought in contact; if it becomes too small the light is also enfeebled. Owing to the intense heat, both carbons are gradually volatilized, and, as the distance thus increases, a regulator is required to keep this distance constant. The positive terminal, which should be placed uppermost, wears away more rapidly than the other, in the ratio of about two to one. Much ingenuity has been expended on the regulators designed to render this distance constant. The form in most common use is that of Foucault. In this the current passes around an electro-magnet whose armature is adjusted against a spring, so that if it is drawn towards the magnet it releases a train of clock-work which separates the carbons, and if the armature recedes from the magnet, a second train of clock-work makes the carbons approach. If the current is passing the carbons wear away, and the resistance increases with the distance between them; as the current by Ohm's law becomes feebler the magnet is weakened, the spring overcomes the attraction on the armature, the latter recedes and the clock-work brings the carbons nearer together. If the light from any cause is extinguished, the same action goes on until the carbons are in contact. The current then passes with its full strength, the armature is drawn down, releases the other train of clock-work, and the action proceeds as before. A tolerable degree of steadiness is thus attained, and if extinguished the lamp will relight itself. An objection to this regulator is that, if the electromotive

force of the current is insufficient, it is liable to take on an oscillating motion; the carbons separate so far as to break the circuit, then rush together and again separate. This is especially objectionable with the magneto machines, as in the best forms the work required to drive them is small when the current is broken, and accordingly each change in the current produces a violent strain on the engine.

A much simpler regulator is that of Browning, in which the upper carbon is attached to an electro-magnet traversed by the current and free to slide down a vertical rod. As long as the current is strong the magnet attracts its armature, which acts as a brake and prevents its sliding down, but as soon as the current is weakened by the wearing away of the carbons the magnet descends and the carbons approach. A screw serves to regulate the position of the lower carbon.

To produce the light, start the engine, if the magneto machine is to be employed, or set up the battery, and connect the terminals with the binding screws on the regulator. If the carbons are not in contact the circuit will be broken. If the Foucault regulator is employed, it should first be wound up and the pointer in front turned from "Arret" to "Marche." The carbons will then slowly approach until they touch, when the armature will be drawn down and they will separate, and if all is right a brilliant light will be produced between them. To obtain the best effect, the spring regulating the force with which the armature is held in place must be carefully adjusted by the screw near the base of the regulator. When the carbons are consumed they are removed by unscrewing the carbon holder by a small wrench.

188. Magnesium Light.

Apparatus. A magnesium lamp and cloth chimney by which the smoke may be carried upwards out of the window or into a flue.

Experiment. The metal magnesium when ignited burns with intense heat, raising to whiteness the oxide formed. The simplest way of employing it as a light is to pass the metal in a finely divided state through the flame of a spirit lamp, when it emits an intense white light in burning. The more common method, how-

ever, is to burn it in the form of ribbons. Two coils of this are placed on reels on top of the lamp; their ends are drawn between rollers turned by clock-work, and two rollers placed below serve to cut off the ends of the burnt magnesium. Care should be taken in putting on the ribbons to pass them through the rollers in such a way that the ends shall curl outwards, that is, from each other. The brillancy and steadiness of the light depend in a great measure on the proper supply of air. To effect this, a chimney is provided carrying off the burnt magnesium which otherwise would soon fill the room with a white impalpable powder consisting of calcined magnesia. The chimney is commonly made of a cloth tube distended by a flat helical wire and should be carried upwards into the open air or into a flue. The upward direction is essential or the draught will be checked. Sometimes the tube terminates in a large bag which allows the air to pass and retains the magnesia, but the air-currents are thus checked. To light the lamp, open the damper in the back part and turn the detent which releases the clock-work until about two inches of the magnesium protrudes below the rollers, then stop it and light the ends by holding under them a match, or better an alcohol lamp, and as soon as they light start the clock-work again. If the wire is fed too slowly the flame will burn too high. This may be remedied by moving the vanes of the clock-work, or by keeping the latter well wound up, turning the key a little every few minutes. The opposite plan is to be adopted if the clock-work goes too rapidly. The position of the light may be viewed either through colored glass, or, if the lamp is used in a lantern, by observing from behind the reflection of the flame in the lens in front.

The advantage of this light is its portability, and that it is always ready at a moment's notice. The objections are its expense, which is considerable, the variability and insufficient intensity of the light.

189. Calcium Light.

Apparatus. Two holders, bags or reservoirs to contain the gases, a burner and some lime cylinders. If the oxygen is to be made, a retort, gas furnace, some black oxide of manganese, chlorate of potash, caustic potash and a wash-bottle are required.

Experiment. The light in most common use for lantern projections, on account of its cheapness and convenience, is that obtained by inserting a cylinder of lime in the flame of the oxy-hydrogen blowpipe.

The gases may be kept in holders over water, a greater or less pressure being produced by weights. Great care must be taken that they do not get mixed or there is danger of a most violent explosion on the approach of a flame. They should, therefore, never be interchanged, and care must be taken on first admitting the hydrogen that no air remains in the holder. Rubber bags are often used instead of holders as they are less expensive and much more portable, but they are more liable to leak, and therefore the hydrogen should never be kept long, if it can be avoided; it is also best to subject it to a small pressure that the leak, if any, may be outwards. Copper cylinders are now, however, frequently employed, in which the gases are confined under great pressure and thus preserved indefinitely and burned at a moment's notice. The current expense is comparatively small, as the gases are manufactured and compressed by a company and sent by express to any part of the country. It is better that the pressure should be the same for both gases, and it may be readily tested by attaching a common steam pressure gauge.

Pure hydrogen is now seldom used for the calcium light, coal gas being much cheaper and sufficiently good for the purpose. For any special purpose hydrogen may be made in large quantities from iron filings and dilute sulphuric acid much more cheaply than from zinc. Where coal gas is not easily obtained, the hydrogen may be replaced by an alcohol lamp, though the flame of which the oxygen is blown as in a common blowpipe. The light thus formed is called the Bude light.

As many persons prefer to make their own oxygen, the details are given below somewhat fully. A mixture of binoxide of manganese and chlorate of potash is heated in a flask which may be of copper, cast iron or sheet iron. The first is the most expensive and burns through in time, the second is the most durable but requires a furnace or stove, and hence the third is generally the most convenient. To render the joints gas-tight when first used, a little thin luting clay or plaster of Paris should be poured in and

the flask then heated. The cover may be screwed on or held in place with a gallows-screw connection and luting clay. The tightness of the joint may be tested by attaching a short rubber tube to the outlet, blowing into it, pinching it with the fingers and then seeing if the pressure is maintained. Oxygen may be made by heating either binoxide of manganese or chlorate of potash alone, but the former requires a high temperature and leaves a very disagreeable black mass, which is not easily removed, and the latter is very dangerous if heated too rapidly. It is therefore better to mix them, when the manganese seems to modify the decomposition of the chlorate so as to render the action more uniform. The proportion may be varied very greatly, equal parts may be used at first, and one part of manganese to two or four of potash when the experimenter is familiar with the process. Great care should be taken that the chlorate is pure and that no dust or organic matter is present in the flask, or it may cause a violent explosion. It is safest always to test the potash and manganese by heating a little in a glass tube. The greatest danger is that sulphide of antimony may be mistaken for the manganese, which it greatly resembles.

The oxide and chlorate should be finely powdered and well mixed by rolling them in a piece of paper or shaking them together in a bottle. It is better to mix them shortly before using, rather than in large quantities at a time, as in the latter case there is a little liability to clog and form lumps, from which the gas is given off with too great rapidity. About a pound of the mixture is placed in the flask, the cover fastened on and rendered air-tight as described above. It is then placed over a gas-stove, such as is used for cooking purposes, and connected by a short rubber tube with the wash-bottle. This consists of a large bottle closed by a cork or rubber stopper through which pass three tubes, one from the flask passing nearly to the bottom, through which the gas enters, a second ending just below the cork and a third, or safety tube, reaching nearly to the bottom of the bottle and with a rubber tube connected above and bent over into a vessel to catch the liquid if thrown out. The bottle is then about one-third filled with strong caustic soda or potash. The gas from the flask will now pass through the soda which will absorb any chlorine or other

impurity, and the remaining pure oxygen may then pass through a long rubber tube to the holder or bag. The object of having the rubber tube connecting the flask and wash-bottle short, is that if heated by the oxygen it may be decomposed, and the hydrocarbon vapors given off uniting with the oxygen cause an explosion, as accidents have occurred which seemed due to this cause. If, therefore, the tube becomes too hot, it is well to cool it with a wet cloth. The tube connecting the retort and wash bottle, and the outlet of the latter, should be as large as possible, as one of the most common causes of accident is the stoppage of these tubes during a violent formation of the gas, by the manganese or soda collecting in them.

Everything being in readiness, the gas may be lighted and turned on to the full. The flow is rather more uniform in this case than if heated up slowly, as the whole mass then reaches the point of decomposition at about the same time, and the gas is liable to be given off suddenly in great quantity, while if the heat is strong at first, the lower part of the mixture is decomposed before the upper part has been strongly heated. Owing to the expansion of the air, bubbles will appear in the wash-bottle almost immediately, which will increase in number as the gas is given off. The first portions should be allowed to escape, and then the tube should be connected with the holder or bag and the gas will pour into them. With a little supervision the process will now go on of itself, but it should be watched, or accidents may happen. The liquid will rise in the safety tube to a height dependent on the pressure in the holder, and the resistance of the tubes. If there is any stoppage, the safety tube will fill and run over, emptying the wash-bottle in a few seconds. This is avoided by instantly breaking the connection between the flask and bottle by the rubber tube. This should be done, in fact, in case of any accident. If the gas is generated too rapidly, the burner should be turned down, but as this will not produce an effect immediately, the bubbles should be watched so as to anticipate too violent an action. The danger from too rapid a flow of gas, is that the liquid in the wash-bottle will be thrown up into the outlet tube, and, running down into the rubber tube, close it and stop the flow of gas. The burner should not be extinguished until the wash-

bottle is disconnected, or the liquid in the latter may be drawn back into the retort, converted into steam and produce an explosion. A pound of the chlorate should generate about four cubic feet of gas, and when nearly this amount has been given off, the bubbles begin to come more slowly, the wash-bottle should then be disconnected from the flask, and the burner extinguished. When the flask is cold, the cover should be taken off and cold water poured in. After some time the water softens the potash and manganese, and they may then be easily removed.

When the gas is required in larger quantities the following method is more convenient. A common cast iron retort is supplied with two tubes, an outlet for the gas and an inlet terminating above in a large tin funnel in which is placed a quantity of chlorate of potash. To prevent the latter from passing directly into the retort, a stop-cock is interposed whose plug is not perforated, but has a cavity in it so that on turning it, a little of the chlorate passes each time into the retort. The latter being heated nearly to redness the plug is turned, when the gas is instantly liberated and passes over into the holder; this operation is repeated until a large quantity of gas has been generated. Instead of the funnel, a conical hopper closed above may be used, and the chlorate supplied by a revolving fan-wheel. Another method of generating the gas is by Edgerton's cylinders, which consist of wrought iron cylinders in each of which are placed a pound of chlorate of potash and four ounces of binoxide of manganese. The cylinder is then heated over a stove, taking care that it does not become red hot. The oxygen is thus generated under pressure and may be used as soon as the cylinder is cool, or kept indefinitely. A second cylinder lined with vulcanized rubber serves to prepare the hydrogen from sulphuric acid and zinc.

To burn the gases, they are brought by rubber tubes to the jet, which is made of various forms. The simplest and most efficient method would be to mix the gases in bulk and then burn them from a simple tube like a blow-pipe. But this method is never employed on account of its danger. The mixed gases in any considerable quantity explode with extreme violence, and the flame is liable to travel back through even a small tube. Formerly safety tubes were much used, consisting of tubes filled with

fine wires, inserted between the burner and holder, but these are not always effective, and the consequences of an explosion are so disastrous as never to justify mixing the gases in bulk.

The same effect may be obtained by making the jet terminate it a small copper chamber in which the gases mix and are burnt through a small hole in the end. The cap of a copper cartridge is sometimes used for this purpose. The hydrogen should be turned on first and lighted, and then the oxygen, taking care that there is not too much of the latter, as in that case, the flame is extinguished with a loud snap due to the explosion of the little mass of gas in the copper chamber. The blue part of the flame shortens when the oxygen is in excess, and just before the explosion, draws back till it reaches the orifice, so that with a little care, if the pressure is constant, this snapping is easily foreseen and avoided.

Another form of burner is made like a common blast lamp, the hydrogen being burnt through a large orifiee, while the oxygen is supplied through a small tube opening in its centre. This form has the advantage of perfect safety, as it is almost impossible for the two gases to be mixed, and there can therefore be no explosion. The light, however, is not as intense, since the gases do not unite as completely, and if the oxygen is delivered under too great a pressure it is liable to cool the lime, forming a dark spot in the centre of the light. A third and still simpler form consists of a large orifice for the hydrogen, and a small one for the oxygen, arranged like a common blow-pipe, so that the oxygen is blown through the large flame of the hydrogen. The last two forms of burner do not require a high pressure for either gas, but in the first form a pressure of a foot or more is needed or the gases may snap.

The oxy-hydrogen flame, although eolorless, has a most intense heat. This may be seen by holding a pieee of steel, as a wateh spring, in it, which will be burnt off with a shower of sparks, or by a piece of platinum wire which will be heated to whiteness, and then melted. If now a piece of quick-lime is placed in the flame, it is heated to sueh intense whiteness as to afford a brilliant light. The limes, as they are called, are made in the form of eylinders, and are either held in small eups, or sometimes have holes bored

through them, and are mounted on vertical wires. They tend to absorb moisture and when not in use should, therefore, be kept in glass stoppered bottles containing quick-lime, otherwise they are liable to crack and fall to pieces. When the flame has played for a considerable time on the same point of the lime, the light becomes less intense, and the lime should, therefore, be turned. To obtain a very uniform light, clock-work is sometimes attached, by which the lime is caused to revolve slowly so as continually to expose a fresh portion of its surface.

To avoid the difficulty of the wearing away of the limes and their hygroscopic nature, cylinders of zirconia have been employed but the light they emit is less brilliant.

To produce the light, therefore, take a lime from the stoppered bottle in which it is contained, and place it in its cup. Wind up and start the clock-work, if any is used. Close the stop-cocks for both oxygen and hydrogen at the burner and open them at the holders. To make sure that there are no mixed gases, it is well to allow a little of each gas to escape through its tube before igniting them. A considerable weight must be placed on each holder or bag to produce the requisite pressure; thus to obtain a pressure of a foot a weight of sixty-two pounds per square foot is required. If two bags are used they should be placed between three boards hinged so as to form a *Z*. One bag is placed on each side of the inclined board, and the weights on top. Moving the weights to one side or the other any desired pressure can be exerted on either bag. If the gas is contained in cylinders under pressure, it must be regulated only by the valves on them, and the stopcocks on the burner kept wide open all the time, or the connecting tubes will be burst by the pressure. The cylinders are closed by conical valves operated by a long handle which may be very exactly adjusted by gentle blows with the hand. Turn on the hydrogen and light it, and after the lime has become somewhat warmed admit the oxygen carefully. A rustling, hissing sound is produced if the latter is in excess, or with the first kind of burner a violent snap extinguishing the light. The appearance of the flame when the gases are in the right proportion is soon learned. The brilliancy of the light also affords an excellent test of the correct proportion of the two gases. If the light becomes dim

see if the pressure of either gas has altered so that they are not in the right proportions and try turning the lime so as to expose a fresh surface.

Finally, the advantages of the calcium light are its cheapness and its great steadiness, in which latter respect it has a great advantage over both the electric and magnesium lights.

Although the above description has been confined to the brighter lights, yet for many purposes excellent effects are obtainable with an oil lamp, especially in a small room. The flame should be as intense as possible and not very large. For this reason a kerosene lamp placed edgewise gives good results.

190. Lantern.

Apparatus. For the lantern a simple wooden box about a foot on a side may be employed. It should be blackened inside and have a hole in the top to allow the hot air to escape. In front is a circular hole four or five inches in diameter, and on each side and behind larger apertures closed by curtains or doors. If the latter are used, that in the rear should be hinged above, and the others hinged in front, so that, when open, the light will not fall on the screen. The whole is placed on a long board on which may be placed the lenses and other apparatus employed, or a stand like the bed-plate of a lathe may be placed in front of the lantern and the various instruments attached to this. For the simple projection of pictures, however, it is more convenient to have the condensers and projecting lenses attached directly to the lantern, and between them and close to the condensers, a place for inserting the picture-holder. The construction of these will be given below. The projecting lenses are movable and may be slid in or out by the hand or better by a rack and pinion.

The pictures may be projected directly on a white wall, or, if this is not available, upon a screen of white cloth. If the latter is to be used permanently it should be moistened, stretched and held in place by tacks and then painted. Good results may also be obtained with a white curtain. Cloth may be obtained eleven feet wide, and, therefore, a seam is unnecessary except in large halls, where if well made, it is not likely to be very noticeable.

Experiment. The method of projecting objects on the screen is shown in Fig. 106. A represents the source of light and BB' a lens at a distance from it equal to its focal distance, so that the emergent rays shall be parallel. CC' is a second lens which

brings the rays to a focus at D where the projecting lens is placed. The object is placed at FF' at such a distance from D that its conjugate focus shall fall upon the screen EE'.

The lenses BB' and CC' are called the condensers, and D the projecting lenses. Evidently the diameter of the circle of light upon the screen will be EE', and that of the field or of the largest object that can be projected, $F\,F'$. Since the image of F is formed at E' and of F' at E, the upper part of the object will appear at the lower part of the screen and *vice versa*. By similar triangles it follows that $FF' : EE' = DF : DE'$ or since DE' is generally very large compared with FD, the latter will nearly equal the focal distance of the lens D. Hence it follows that the size of the object : size of the image = focal distance of D : distance of the screen. The ratio of the diameter of the circle on the screen to the distance of the latter should not exceed one to two, and should generally be one to three or four. The advantage of the latter ratio is that the aberration is diminished, but it renders it necessary that the lantern should be placed at a greater distance from the screen. A convenient arrangement is to place the lantern behind the spectators, and throw the light over their heads. An imperfect image of the light A will be formed at D, whose size will bear the same ratio to that of A as their distances from the condenser. As the aberration will still further increase the size of the image, it is evident that if the screen is placed a long way from the lantern, to have a field and image of the same size, the focal length of D and its diameter must both be increased, or part of the light will be lost by not passing through it.

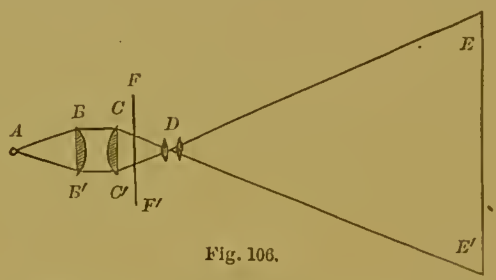


Fig. 106.

As the object of D is simply to form an image of FF', it must be carefully corrected both for spherical and chromatic aberration. Other defects, however, such as striæ, dust on the surface or even cracks, do little harm except so far as the small loss of light is

concerned. The condensers, on the other hand, are designed simply to collect as much light as possible so that it shall pass through the lens *D*. For ordinary projections, therefore the spherical and chromatic aberration are unimportant so long as they are not sufficient to throw the light outside of the lens *D*. Any striæ, dust or cracks are, however, liable to show upon the screen, especially if, as is commonly the case, the field of view FF' lies near the condenser. On the other hand if this distance is considerable, the field of view is reduced, or larger condensers are required.

In the most perfect instruments a large part of the light is lost. Let *A* be a luminous point radiating light equally in all directions, and let the distance of BB' equal one third of its diameter. Then a little more than a fifth of the whole light, or 22.3 per cent will fall on it. The condensers consist of from two to five lenses, the projecting lenses of from two to six, and the object ordinarily consists of one or two plates of glass. Therefore the light has to traverse from four to thirteen pieces of glass. Taking six as the most common number, and recollecting that .90 only is transmitted by a clear glass plate, we see that $.90^6 = .53$ of 22.3 or 11.8 per cent only is received on the screen. A portion of this light might be saved by a reflector, and this is sometimes done with the electric light, but with the calcium light the front portion only of the lime is illuminated. The reflector should be spherical with the light at the centre, and in this case were there no loss the light would be doubled. In practice, however, the gain is much less.

The most obvious way to increase the light is to bring the condensers nearer the light. But the limit is soon reached, owing to the heat, if this distance is reduced below three inches. On this account a piece of glass is sometimes interposed between the light and the first condenser. If cracked it is easily replaced and the lens is protected. This is especially desirable with the electric light since minute pieces of incandescent carbon are liable to snap off and fuse into the glass.

Condensers are made of various forms; the simplest is that shown in Fig. 106, and consists of two plano-convex lenses with their curved surfaces turned towards each other. In this position

the aberration is least, according to the rule that the most curved face should be turned towards the least convergent beam. Formerly condensers were made of three equal double convex lenses, but the increased loss of light was an objection. A form much used at present is the Cresson condenser formed of four lenses, three designed to render the rays parallel, and the fourth to converge them to D. The first lens is 4.5 inches in diameter, the others 5 inches. The radii of curvature of the first six surfaces are ∞, 4.5, — 30, 6, 52, 8.75 inches respectively. Evidently the first lens is plano-convex, the second a meniscus and the third a double convex lens. They are placed near together, and united are equivalent to a single lens with but little aberration, having a focus only three inches distant from the plane surface of the first lens. The light being placed at this point the rays emerge very nearly parallel. They are concentrated by a double convex lens whose two surfaces should have curvatures in the ratio of one to six, since this form gives the least aberration. For use in large halls the Morton condenser is preferable to the Cresson. It consists of three plano-convex lenses with radii 4.5, 3.5 and 4 inches, and diameters 4.5, 5 and 5 inches. Its focal distance is only 2 inches, so that it takes in nearly twice as much light as the Cresson and produces a proportionately brilliant image, but the aberration is also much greater.

To project large objects, large condensers are needed. These have the advantage when a very intense light is used that they are less liable to crack, since they must be placed farther from the light. A simple and quite efficient arrangement is to bring BB' Fig. 106, somewhat nearer to A, so that the light shall diverge after passing it. CC' is then replaced by a plano-convex lens seven or eight inches in diameter placed at such a distance that it shall be wholly within the cone of rays. A projecting lens of suitable focus is then placed at D and an object as large as the lens replacing CC' may thus be projected. By this arrangement but one large lens is required. For most purposes, however, condensers four or five inches in diameter are most convenient, as, when greater than this the image at D is enlarged, and consequently a larger projecting lens is required.

The conditions of excellence in the projecting lenses are nearly

the same as those for the portrait lenses of a photographic camera, and hence such lenses are much used for projections. A diaphragm cannot, however, be used, on account of the loss of light.

When sunlight is used the conditions are much simplified, since the beam is already parallel. The condenser may consist of a simple convex lens of diameter a little greater than that of the object to be projected, and of focal length as much less than the distance of the screen as the diameter of the lens is less than that of the desired circle of light. The light will form a cone at the apex of which is placed the projecting lens, which may consist of a simple convex lens of small diameter with a focus somewhat less than that of the condenser. The aberration will be small, since all the light will pass very near the centre of the projecting lens, and excellent projections may accordingly be obtained by very simple means.

In a lecture room accommodating a hundred persons, a screen of eight or ten feet square is most convenient. The lantern should be placed opposite the centre of this, on a table three feet high, the whole being raised on a platform so that the light shall pass entirely over the heads of the spectators. The lenses being placed in position in front of the lantern, the room is darkened and the calcium or other burner inserted in the lantern and lighted. A circle of light will now be formed on the screen which may be rendered concentric with it by inclining or turning the lantern. To see if the light is at the proper distance from the condenser, remove the front lens of the latter and the projecting lens and move the light until the emergent rays are approximately parallel, or until an inverted image of the light is formed upon the screen. Replace the front lens of the condenser and, where the emergent cone of light has the smallest cross-section, insert the projecting lenses. If now any object, as a pencil, is interposed near the condenser, an enlarged inverted image of it will be formed on the screen. This marks where the objects should be placed. As the latter are commonly photographs on glass, a wooden holder is convenient in which they may be held and brought into position by sliding them along a groove in front of the condensers. Having once adjusted the apparatus, the light is brought into position, without altering the lenses, by moving it

until the circle on the screen is bright and uniform. The presence of irregular bluish patches generally denotes that the light is too near the condensers, but if, on removing it, the top of the circle remains dark the light is too high and should be lowered, and raised if the upper part is dark. If the right hand portion is dark the light must be moved to the right, and *vice versa.* If the light is too far, the field will be bordered with a reddish fringe. Finally, the picture being placed in position, it is carefully focussed by moving the projecting lenses until the image on the screen is as distinct as possible.

By observing a few simple precautions the effect of the pictures may be greatly improved. Thus in the presence of an audience the bright circle of light should not be formed on the screen before showing a picture, or the latter will look dark by contrast. Never allow the light to shine directly on the audience, or their eyes will be dazzled and the effect of the pictures diminished. Two picture-holders are needed so that one picture may be put in place while the other is being exhibited. Holders are sometimes made with places for two pictures and slide back and forth, each picture after it is shown being replaced by another without removing the holder. Sometimes the pictures slide in a groove and are pushed in, one in front of the other.

Before the exhibition, the pictures should be arranged in order, all turned in the same way, so that they may readily be placed in the holder, inserted in the lantern upside down, and turned so that the right side shall be towards the light. If the last condition is not fulfilled, letters and numbers will be read backwards, and views will appear turned end for end. The rule is that the lettering must read correctly as seen from the light. The first picture to be shown should be tried in the lantern beforehand, the position of the light adjusted and the projecting lens very carefully focussed. The light is then extinguished and a black cloth thrown over the projecting lens. When the picture is to be shown, the light is produced, the room darkened and the cloth then removed. Place the second picture in its plate-holder and when the first has been seen long enough, replace it quickly by the other. It is a good plan to cover the projecting lens with the black cloth when changing the pictures, so that the audience shall

not see them move. The brightness also appears greater, by contrast with the intervening darkness. If the light is very bright it will be found to be more agreeable and much less trying to the eyes not to render the room entirely dark.

Sometimes the lantern, instead of being placed in front of the screen, is placed at an equal distance on the other side of it. The screen should be free from seams, and moderately transparent. There is always, however, a great loss of light, and an assistant is needed to manage the lantern. Another objection is, that, if the spectators are directly in front, they will see the light through the screen as a brilliant spot of light. The advantage, however, is that the lantern is out of the way and does not obstruct the view or otherwise disturb the spectators.

Dissolving views are rarely employed in scientific work, except in showing some of the phenomena of color. They are produced by two lanterns adjusted so as to throw similar pictures, as a summer and winter view of the same landscape, on the same part of the screen. The light is wholly cut off from one lantern, and this is gradually uncovered while the other is obscured, so that one picture gradually fades into the other. The change may be effected by two sets of wedge-like points attached to a rod which may be moved laterally. In one position the light is completely cut off where the bases of the wedges come together, and, as the rod is moved, more and more light passes between the points until they are entirely removed. Another form of screen is circular, with an aperture formed of two circles whose centres do not coincide. As this screen is turned, the light is admitted in varying amounts from one or the other lantern. Sometimes a single lantern is used with two condensers at right angles to each other, the two beams being rendered parallel by reflection from mirrors.

The phantasmagoria might have application in scientific work, though now only used as a toy. The lantern in this case is placed behind the screen, and the pictures are commonly painted in bright colors on a dark background. The lantern is first placed close to the screen forming a very minute picture, which enlarges and therefore seems to approach as the lantern is withdrawn. The light should be much reduced at first and gradually increased,

and the focus altered as the lantern is moved. Both these adjustments may be effected automatically.

191. Objects for Projection.

Apparatus. The lantern and the various objects suitable for projection described below.

Experiment. The most common objects for the lantern are photographs of the form known as glass positives described Vol. I, p. 187. Almost any known object may thus be shown to an audience, for instance, landscapes, buildings, sculpture, machinery, and especially engravings and woodcuts. The latter form an excellent substitute for diagrams, and by collecting the best illustrated books and selecting cuts from them, a set of excellent diagrams is obtained at a trifling expense, with the advantage of furnishing perfect fac-similes of the originals. In copying woodcuts they must be very sharply focussed and not exposed too long, so as to give a density between an ambrotype and a negative. The size of plate commonly used is $3\frac{1}{4}''$ by $4\frac{1}{4}''$ known to photographers as quarter plates. Sometimes double this or "half size" is employed. To protect the negative, it is sometimes covered by a second plate of glass held in place by strips of paper around the edges. If, however, the plates are varnished they are not very liable to injury, especially if kept in boxes with saw-cuts in the sides to prevent their surfaces from touching. If thick paper is pasted on the side on which the photograph is taken, they may be laid upon each other without danger. This also renders it easier to label and number them. If the picture does not fill the plate, the blank edges must be covered either with paper or black paint, as a broad white border will make the picture look much fainter.

Many other objects may be shown in the same way, for instance, acoustic curves on smoked glass or collodion, or any thin object whose outline is characteristic. A great field is opened by using the screen as a blackboard. For this purpose a common smoked glass may be employed, the drawing being prepared beforehand or even in the presence of the audience. An excellent surface is obtained by preparing a plate for a photograph, exposing to a strong light, developing and fixing; drawings of great delicacy may be

made on this by removing the film with a sharp point. As it is a little difficult in this case to draw so well on a vertical surface, the vertical lantern described below is often more convenient. To draw objects in dark lines on a bright ground, thin sheet gelatine may be employed or a simple sheet of glass, first covering it with a thin coating of gum by pouring dilute gum water over it, and then letting it dry. Glass perfectly free from grease may also be employed, but less conveniently. India ink should be used, as it adheres better than common ink.

A great variety of models may be well shown by projection if care is taken to make them of suitable size and so that they will lie in one plane. For instance clock escapements, electro-magnetic engines and various forms of telegraph. A thermometer is another good object, and small variations in temperature are thus readily shown.

Many of the laws of animal electricity may be shown to an audience by projecting a frog's legs on the screen, and showing the twitching caused by electrical excitations. If large condensers are employed, the whole frog may be shown, thus rendering the experiment much more intelligible.

An interesting experiment, especially with the electric and magnesium lights, is to project the light itself upon the screen. This is easily accomplished by removing the condensing lenses and bringing the projecting lenses nearer, until a distinct inverted image of the light is seen. With the electric light the wearing away of the carbons, and the action of the regulator is clearly shown. When sunlight is used, the sun itself may be projected on the screen if the mirror of the *porte-lumière* is plane. The best effect is obtained by placing a telescope horizontally as if we wished to look at the image in the mirror, and drawing out the eyepiece somewhat beyond the position of distinct vision for parallel rays. The image formed at the focus of the objective will then be projected on the screen. The sun spots may thus be well shown, and during a partial eclipse the phenomona may be watched by a large number at a time.

Some interesting effects may be obtained with the direct light, removing both condensing and projecting lenses. Thus if any large object is interposed, its shadow will be projected very

sharply on the screen. If now a plate of ground glass is inserted, the shadow becomes hazy, the change increasing with the distance of the glass from the light. The difference between a penumbra and shadow is thus well shown. With sunlight a condensing lens should be interposed to bring the parallel rays to a focus.

The effect of a mirage may be shown on the screen by inserting as an object a metallic plate with a small hole in it, or better, removing the condensers, interposing the plate, and projecting the aperture on the screen as when projecting the image of the light. Now interpose in the beam of light a nearly horizontal plate of metal heated below by a gas burner. The hot air in contact with the plate will reflect a portion of the light forming an irregular image of the aperture above the other image and readily distinguished from it by its irregular waving motion.

Another interesting class of phenomena are those of phosphorescence and fluorescence. Many solids, especially the phosphides, if exposed to a strong light, continue to shine in the dark. Sets of tubes are sometimes prepared with substances which emit various colors after such an exposure. The electric and magnesium lights are particularly adapted to produce these effects, owing to the predominance of the more refrangible rays. It is only necessary to expose the tubes to the light for a few seconds, taking care to interpose a screen to cut off the light from the eyes of the audience; then holding the tubes up they will shine for some time quite brightly. The fluorescence of sulphate of quinine and other substances is well shown by painting a flower or other object with their strong solutions on a sheet of white paper. It will be almost invisible by ordinary light, or, if hung against the screen, by the circle of light from the lantern. If, however, a piece of violet glass or even blue cobalt glass is interposed, the portions covered with the fluorescent substance will shine brightly.

The curious change of color of some cobalt salts with heat is well shown by covering a piece of glass with a strong solution of chloride of cobalt and gelatine. If projected on the screen it will give a pink tint to the light which will gradually change to deep blue under the influence of the heat. The pink color will be restored if the plate is left in a cool, moist place.

The chromatrope consists of a disk of glass mounted so that it

can be made to revolve rapidly in its own plane, and used as an object in the lantern. In the best form, the glass is ground to a circular disk and made to revolve on wheels covered with rubber. By using circles covered with glass or gelatine of different colors the effect of their combination is readily shown to an audience. Among the best effects are those of yellow and blue glass which produce white when combined, and red and green which produce a yellow circle on the screen, when the disk revolves rapidly. Colors may also be combined by partially covering the projecting lens with pieces of glass of various colors taking care that they do not overlap. Chinese fireworks are formed of two disks of glass painted in various colors, and so mounted that they shall turn in opposite directions. The motion should be comparatively slow, and by varying the colors, forms and positions of the figures, great variety is attainable. Two sets of circles with centres on one side of the centre of motion give good results. Replacing the colored glases by perforated cards, wire gauze, or lace, curious interference figures are obtained.

The formation of crystals are among the most beautiful of objects for the lantern. Prepare a hot saturated solution of the salt to be shown, pour it on a plate of glass and insert in the lantern. As the water evaporates the crystals will be formed covering the screen with forms of great beauty and variety. The rapidity of formation may be increased by using alcohol instead of water. Almost any crystallizable salt may be employed, but among the best objects may be mentioned urea, which forms beautiful needles, also oxalate and chloride of ammonium and nitrate of potash.

The effects of a kaleidoscope may be obtained on a screen by inserting the instrument between the condensers and projecting lenses after removing the plate of ground glass. The great difficulty is the loss of light and unequal brightness of the reflected image.

192. TANKS.

Apparatus. The lantern and two or three tanks, each formed of two pieces of plate glass held together by four clamps and separated by a strip of rubber half an inch thick. The rubber is cut straight and bent so as to form the bottom and two sides of

the tank. Notches should be cut in one side that it may bend more easily. Liquids may be added drop by drop by pipettes, of which the most convenient form consists of pointed glass tubes terminating above in elastic rubber balls. A galvanic battery of sufficient power to decompose water is also needed, and the following chemicals; litmus, cochineal, red cabbage, alcohol, some aniline color, sulphuric acid, ammonia, sulphate of copper, acetate of lead, ferrocyanide of potassium, perchloride of tin and lime water. In the bottom of one tank a small coil of platinum wire is placed which may be heated by the battery, and in another tank are two platinum electrodes. Some small test tubes, U tubes and glass rods are also needed. Various small living animals as minnows, larvæ, etc., may also be shown on a large scale upon the screen. To erect the image a right angled or erecting prism is required.

Experiment. A great variety of chemical and electrical experiments may be shown by the use of tanks. Place one of these in the lantern as an object and half fill it with water. The inversion of the image is here an objection, as the water will appear on the upper part of the screen and the air below. It may be obviated by placing the right angled prism in front of the projecting lens, with its hypothenuse horizontal. The image will then be reflected from the latter and being inverted a second time will represent the object as erect, as is shown in Fig. 107. It is evident that the ray A will be reflected to A', and B to B', *i.e.*, the image inverted. Zentmayer's erecting prism differs from this in having the angles 27°, 27° and 126°, as shown by the broken lines, so that the rays will pass through the prism parallel to its faces. It has the advantage that no glass is wasted and with glass of given thickness a broader beam is transmited in the ratio of 1.44 to 1.

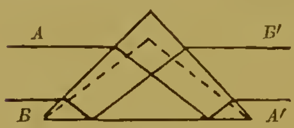


Fig. 107.

The convection currents produced by heat are well shown by filling the tank containing the platinum coil with water, and connecting it with the galvanic battery. Immediately a current of warm water will rise from the platinum and spreading over the surface descend on the sides. These currents are rendered much more visible by adding to the water a little strong solution of cochineal which will at once fall to the bottom of the tank and be raised by the heat to the surface. A similar convection due to

their difference of density is well shown by adding to a tank containing water a little perchloride of tin. Quite a different effect is obtained by adding to a tank filled with alcohol a drop at a time of one of Judson's aniline colors which, as it falls, will divide up into root-like threads.

The rise of water in capillary tubes is readily shown on the screen by dipping a fine glass tube into the tank, and the hyperbolic curve between two plates, by holding a plate against one side of the tank. The liquid in this case should be colored.

Three methods are available for showing chemical decompositions on the screen. First, mixing the substances directly in the tank. This is generally the best method, but, since the tank must be washed out after each decomposition, much time is required. In the second method the decompositions are effected in test tubes immersed in the water of the tank. They may, therefore, be changed rapidly and the appearance more nearly resembles a real chemical analysis. The third method will be described in connection with Experiment 194. Fill a tank with water and color it blue with a little litmus. Add a few drops of acid and stirring with a glass rod the color will change to red. The blue color may be restored by a little ammonia, and the effect repeated indefinitely. If cochineal is used instead of litmus the acid will turn it yellow, and ammonia, purple. A solution of red cabbage in boiling water is blue, but will change to red with acids, to green with alkalies, and to purple with alum. Wash out the tank and fill with a dilute solution of sulphate of copper, which is a pale blue. A little ammonia produces a white opaque precipitate appearing as a black cloud, which, on adding more ammonia, dissolves into a clear deep blue liquid. A little acid makes the precipitate reappear. To the clear liquid add a drop of ferrocyanide of potassium. Instantly clouds of the dark brown ferrocyanide of copper appear. Place a test tube in the tank after refilling with water; it will look like a polished metallic rod, owing to total reflection. On filling it with water, however, it will nearly disappear. Chemical reactions in such a tube may be shown to an audience very much as they are ordinarily seen by a single individual, and a complete course in qualitative analysis may be thus illustrated. The great difficulty is, that all opaque precipitates, whatever their color, will appear

black. Thus sulphate of baryta and oxalate of lime appear black instead of white. The generation of hydrogen or carbonic acid is well shown on the screen, also the turbidity of lime water on adding carbonic acid. To effect the latter, it is only necessary to fill the test tube with lime water and blow through a tube immersed in it, when the carbonic acid of the breath will precipitate the lime.

Electrical decompositions may also be shown in two or three ways. Place in the lantern the tank having two platinum electrodes, fill it with dilute sulphuric acid and connect the battery. A torrent of bubbles will at once ascend from each electrode, the hydrogen being given off from the negative, the oxygen from the positive terminal. They may be distinguished by the greater volume of the hydrogen which, when the current is reversed will appear at the other electrode. If the negative electrode is palladium instead of platinum, the hydrogen will be absorbed instead of set free. If then the current is reversed, the gas will be set free tumultuously, the palladium twisting and turning like a serpent. The other decompositions described in Experiment 95 may be shown similarly. That of acetate of lead is especially beautiful, the deposited lead resembling a tree and growing rapidly on the screen while bubbles of oxygen are set free at the other pole. When the current is reversed the tree appears to wither, and is gradually absorbed or dissolved, no bubbles appearing until all the lead has disappeared. Meanwhile a second tree is forming on the other platinum terminal. The other decompositions are better shown in a U tube immersed in a tank of water, one electrode being placed in each arm of the tube.

In lectures on natural history a great variety of minute marine animals may be projected living on the screen by placing them in tanks containing water.

193. Stroboscope.

Apparatus. The lantern and a circular disk of tin perforated with several equidistant holes, and mounted so that it may be turned uniformly by some small motor, or by hand. Its weight should be considerable that it may run uniformly like a fly-wheel. As objects we may use a tuning fork whose vibrations are sustained electrically, an electro-magnetic engine or a fan wheel

regulating clockwork. One of the best objects is a large wooden wheel painted in radial stripes, which may be turned uniformly, but almost any moving objects may be employed.

Experiment. The effect of the stroboscope depends on the persistence of vision, or the fact that the image of an object on the retina, even if seen but for an instant, will remain for quite a fraction of a second. Accordingly if the object is seen at very short intervals, it will appear to be visible continuously. Project the circle of light on the screen and interpose the edge of the tin disk at the point where the cone of rays is smallest. As this point coincides with the projecting lens, the latter should be altered, since freedom from aberration is not very important in this experiment, while if the disk is not properly placed the edges of the objects shown will appear indistinct. If now the disk is turned, the screen will appear alternately light and dark, but if the motion is rapid the light will appear to be continuous. When, however, any moving body is introduced in the beam of light, a number of images are seen. This is well shown by letting a person walk in front of the screen or by shaking the hand near the projecting lens. In the latter case, the number of fingers will seem to be enormously increased. To study the effect more carefully, remove the tin disk, place the large wheel against the screen, and set it in motion. It will now appear as a circle of uniform grayish color. Replacing the tin disk and turning it, a certain speed will be found at which the large wheel will turn through an angular distance just equal to that between its spokes while the tin wheel passes from one aperture to the next. The large wheel will then appear to be at rest, although really revolving very rapidly. If the motion of either wheel is altered, the large wheel may be made to appear to turn slowly in either direction, according as the interval between the flashes of light is a little greater or a little less than that between the passage of one spoke of the large wheel to the place occupied by the next preceding. This explanation may be tested by attaching a piece of white paper to one spoke of the wheel, when it will be seen to be in rapid motion although the wheel may appear to be at rest. By increasing the speed of the tin disk exactly two or three times, the number of spokes will appear to be increased in the same ratio. This effect

is best seen if the wooden wheel has but few spokes. The tuning-fork or electro-magnetic engine form excellent objects for the stroboscope. If large, they may be placed near the screen, and in this case the projecting lenses should be dispensed with, but if small they are best seen when projected in the usual way. Any other object, whose motion is too rapid to be easily followed by the eye, may be shown in the same manner.

Excellent stroboscopic effects may also be obtained by a Holtz' machine or an induction coil and condenser, with the advantage that, as the spark is instantaneous, the image of the moving object is perfectly distinct.

194. Vertical Lantern.

Apparatus. A vertical lantern and various objects for projection, several plates of glass, some camphor and essential oils, a small magnet, iron filings, a sieve, a magnetic needle, which may be balanced on a needle point fastened to a plate of glass by a drop of sealing-wax, and a tank formed by cementing a ring of glass to a glass plate are required.

Experiment. Many objects may be brought into a horizontal plane much more conveniently than into a vertical plane, and these may be projected by the arrangement represented in Fig. 108 and known as the vertical lantern. A is the source of light, BB' the first lens of the condenser or portion rendering the rays parallel, and GG' a plane glass mirror which reflects the parallel beam of light vertically. CC' is the front lens of the condenser which brings the light to a focus at D where the projecting lens is placed. It is then reflected horizontally to the screen by the right angled prism or plane mirror E. Although the prism reflects more light than the mirror, yet it is open to the objection that, unless the screen is much above it, the lower portion of the circle of light will be darker than the rest, because the light is not totally reflected; the two parts will be separated by a colored curved line, and the portion not totally reflected will, if not intercepted, form an

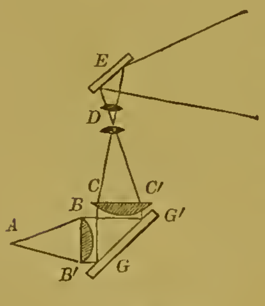


Fig. 108.

image on the ceiling. The mirror has the further advantage that its size may be increased much more readily than that of the prism.

Any object may now be projected by merely laying it on CC', focussing by raising or lowering D, and bringing the circle to the centre of the screen by moving E. Since E inverts the image, the object should be so placed that the top shall be towards the light, and that lettering shall read correctly when seen from above. Project in this way some photographs and other objects. The principal objection to this arrangement is that there is a considerable loss of light by the two reflections, and that, consequently, the projections are less bright. It is, however, particularly convenient for making drawings or writing on the screen, since the prepared surface is horizontal, and the letters need not be written backwards.

The simpler laws of magnetism may be admirably shown in the vertical lantern. Placing the compass needle on its pivot in the field, it will point north and south, and approaching a second magnet or needle, their attractions and repulsions are well shown. Bringing a second suspended needle into the field near the first, and setting one of them swinging, their complex mutual action is well seen. The formation of magnetic curves, described in Experiment 116, is admirably adapted to the vertical lantern. It is only necessary to form them on a plate of glass instead of on card-board. The attraction and repulsion of parallel currents, of solenoids, and the effect of the wire conducting a current on a magnetic needle, all form excellent objects for the vertical lantern.

Chemical and electrical decompositions may be well exhibited in the vertical lantern. The simplest way is to pour a few drops on a plate of glass or into a watch glass, when effects similar to those of Experiment 19 may be obtained, with the advantage that the plates may be almost instantly cleaned. Almost any delicate experiment can be better shown in the vertical lantern, since where careful manipulation is required, it is generally much easier to work with the object in a horizontal, then in a vertical plane. Living objects may also be thus projected more conveniently. When we wish to employ a considerable quantity of water, the tank is used. The motion of fragments of camphor on the sur-

face of water and Tomlinson's cohesion figures may also be best presented to an audience by the vertical lantern. The tank, must be perfectly clean, washed with potash, then with distilled water and allowed to dry, but not be wiped. Fill it with water and add a drop of almost any of the essential oils, as cinnamon or coriander, when various curious forms are obtained as it spreads over the surface.

The transmission and interference of waves are well shown by such a tank filled with alcohol, in which the motion is slower than in water. The waves may be excited by touching the surface with a sharp point, or allowing drops of liquid to fall upon it. The foci of an elliptical tank are shown by immersing an elliptical diaphragm within the glass circle. The best effect is obtained by placing the lens at double the distance required to produce an image of the surface of the liquid on the screen, or more properly by using a lens of one half the usual focal length. Wave motion may also be shown by reflection from the surface of mercury. A fine tube is placed nearly in contact with the surface and connected with a cavity covered with a piece of sheet rubber. Tapping on the latter sends a puff of air upon the mercury, generating a wave. A series of waves may be maintained by a current of air passed through the tube, which should then be made to touch the liquid.

Waves may be admirably shown by Crova's apparatus, which depends, however, on a wholly different principle. A number of curves are drawn on a circular disk of blackened glass and a diaphragm interposed with a slit in it. Projecting this as an object, a series of dots appear which alter their position as the disk is turned. By employing suitable curves various forms of wave motion may thus be shown. Transverse vibrations are also illustrated by a disk with a series of parallel slits through which different parts of the curve are projected.

195. Lantern Polariscope.

Apparatus. The usual method of projecting the phenomena of polarized light is readily understood from Fig. 108, if we slightly modify it and regard it as a horizontal, instead of a vertical section. The light being rendered parallel by *BB,'* falls on *GG'*

which now represents a bundle of thin plates of glass inclined at an angle of 55° instead of 45° so that the light will be turned 110° and then fall on the lens CC' which is placed at right angles to the reflected beam. The light is brought to a focus at D where it passes through a large Nicol's prism, the projecting lens being placed between it and CC'. The reflector E is not used. A large Nicol's prism forms a better polarizer than the bundle of plates, as it gives a brighter image, but the expense is much greater.

Experiment. Turn the lantern nearly at right angles to the screen, so that when the plates of glass are set at an angle of 55° to the incident light the reflected beam shall be thrown on the screen. Then interpose the lens CC' and the projecting lens, when a circle should be formed on the screen as usual, only somewhat less bright. To see if the light is totally polarized place the Nicol's prism in front of the projecting lenses, taking care that it shall be at the point where the beam of light has the smallest cross-section. This is essential to save as much light as possible. The projecting lenses may be of larger size than usual, a simple plano-convex lens giving good results. On turning the prism, the brightness of the circle of light on the screen will vary, and it will disappear completely if the polarization is total. If this is not the case, the angle of the plates must be altered until this condition is fulfilled. The simple laws of polarized light may now be demonstrated; for instance, using as an object a diaphragm with a small hole in it, and inserting a double image prism, two images will be formed whose intensities will vary as the prism is turned. The effect of other analyzers, as a tourmaline or bundle of glass plates, may also be tested.

Any of the phenomena of polarized light requiring a parallel beam may next be shown by placing the object near the front lens of the condenser and forming an image of it on the screen with the projecting lens, when, if doubly refracting, it will appear of a color dependent on the position of the Nicol's prism and its own thickness. If the object is too small to be shown well in this way, a projecting lens of shorter focus may be used. Project in like manner some selenite figures, compressed and bent glass and unannealed glass. Interposing a plate of mica near the projecting lens, it will render the light circularly or elliptically polarized. Rotary polarization, Babinet's wedges and a bi-quartz may be

similarly projected. Generally the most striking effect is obtained by crossing the analyzer and polarizer so that the field shall be perfectly dark until the object is inserted.

Objects requiring a converging beam are readily projected by placing them near the analyzer, and the light will be increased by removing the projecting lens, though the circle on the screen will no longer have a distinct border. Try in this way objects 1 to 8 in Vol. I, Experiment 92. The rings will appear colored, since the light is not monochromatic. If the lime light is used, remove the lime cylinder and replace it with a glass rod or a stick dipped in salt as described in Experiment 200, when the rings will appear in vastly increased number and extent. They will be alternately yellow and black.

The field of view is not very large in this case, that is, the angle between the extreme rays passing through the crystal is comparatively small. To remedy this difficulty, two very convex lenses are sometimes inserted, in the place of the projecting lens, at a distance apart equal to the sum of their focal distances, and the objects interposed between them. The two systems of rings of a biaxal crystal may thus be shown, even if the angle between the axes equals 60°.

196. Lantern Microscope.

Apparatus. The lantern with a projecting lens, an objective of short focus, and a stand for the object by which its position may be carefully adjusted. A tank of water or solution of alum is needed to cut off the heat, and should be placed near the condensers. Any objects, if not too small, may be employed, but the best are preparations of whole insects, injected tissues, and minute living animals.

Experiment. The conditions for success in projecting minute objects on the screen are very simple. The light should be very bright but small, and the condensers as free from spherical aberration as possible, so as to concentrate the light at the projecting lens into a very small space. The diameter of the objective should be large, so that it may take in as much of the light as possible, and on this account microscope objectives, except for very low powers, seldom give satisfactory results. An objective of much shorter focus than an inch is rarely desirable, on account of

the loss of light; with sunlight, however, where the rays are already very nearly parallel, the aberration is small, and magnificent effects are obtainable with common microscope objectives. The light after leaving the condenser should pass through a tank of water or alum which absorbs almost all of the heat, otherwise the object is liable to be injured by the Canada balsam becoming softened.

The magnifying power is much less than is ordinarily supposed; the angular enlargement, as seen from a distance equal to that of the lantern, being only about eight or ten times with a one inch objective. Hence, but little more can be shown than is visible to a single observer with a common pocket magnifying glass. The linear enlargement is much greater than this, being equal to the ratio of the distance of the lantern from the screen, divided by the focal distance of the objective. Thus, with a one inch objective, and a screen twenty-five feet distant, the linear enlargement is three hundred diameters. But if we approach within a few inches of the screen to get the full effect of this enlargement, the irregularities of the surface and aberrations neutralize in a great measure the advantage thus gained. With sunlight, however, much higher powers may be used.

To project an object, see that everything is in proper adjustment, the light burning its brightest, all the surfaces of the lenses clean, and that the liquid in the tank is transparent. After some time the bubbles that collect in the water should be removed, as they reduce the light. The objective must be so placed that as much light as possible shall pass through it, and the object placed at the proper distance and carefully focussed. Objects of considerable size will then show to great advantage, but those of greater delicacy cannot be shown satisfactorily.

Effects of great beauty may be obtained, without reducing the light too much, by combining the polariscope and microscope if large objects are employed

197. Opaque Objects.

Apparatus. A large lantern in which the doors are replaced by curtains, or two smaller lanterns without projecting lenses, by which the light is directed upon the object which is placed in a

small dark box. An image is then thrown on the screen by a projecting lens.

Experiment. Very many small objects cannot be projected on the screen on account of their opacity, and the fact that their shadows are not sufficiently characteristic. Accordingly various plans have been tried to project objects on the screen as we ordinarily see them, that is, by reflected light. The great trouble is the want of sufficient brightness, and, but for this difficulty, this method would doubtless be very largely used.

The simplest method of projecting opaque objects on the screen is to remove the projecting lenses and condensers, and replace the latter by a large, short focus, convex lens. The light is then turned around, withdrawn to one side, and shaded so that it shall not shine on the lens. If now any object, as the hand, is introduced into the lantern near the focus of the lens, it will be strongly illuminated by the light and an enlarged inverted image of it will be thrown on the screen. The best objects for such projections are plaster-casts, jewelry, a watch open and closed, glass ware, flowers, and any bright or sparkling objects. Unfortunately objects are turned right for left so that print reads backwards. Paper photographs do not show to great advantage, probably because we involuntarily compare them with the far more brilliant glass transparencies.

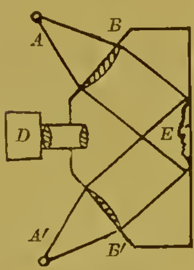


Fig. 109.

Another method of projecting opaque objects is shown in Fig. 109. A, A_1 are two calcium or other lights, and B, B' condensers rendering their rays parallel. E is the object to be projected which is thus strongly illuminated. D is a projecting lens by which an image of E is formed on the screen. The advantage of this method is, that two lights are used instead of one, thus producing a stronger illumination, but the cross light thus thrown reduces the shadows and thus destroys in a great measure the effect of relief.

198. Lantern Galvanometer.

Apparatus. The lantern and the various forms of galvanometers described below. For projecting the deflections of the mirror

galvanometers, a convex lens of about a foot focus, and a diaphragm with a small circular hole are required. The diaphragm should be so mounted that it may be inserted in the place of the condensers.

Experiment. A large part of the various measurements of electrical quantities depends on the deflection of a galvanometer needle. There are various ways in which this may be shown on the screen. A simple and convenient galvanometer is made by suspending a small piece of magnetized watch spring by a filament of silk from a plate of glass, attaching a fine wire as a pointer, and introducing it as an object in the vertical lantern. The upper plate may be sustained by a ring of wood, forming a circular box. Another plate of glass on which is photographed a graduated circle forms the bottom of the box. The difference in level is so slight that if the circle is focussed on the screen the image of the index is still distinctly visible. Two coils of covered wire are now wound on each side of the box just outside of the graduated circle and a galvanometer is thus obtained of considerable delicacy, while at the same time, owing to the short length of the magnet, it will follow very nearly the law of the tangents. It is, therefore, possible to make quantitative measurements with considerable precision from readings on the screen. Various devices may be employed to obviate the difference in focus of the needle and graduated circle. Thus, two lanterns may be employed, one projecting the needle and the other the circle so that they shall be concentric. Again, for the mirror of the vertical lantern a piece of plate glass may be used, and part of the light passing upwards will form an image of the needle, while the remainder and larger portion will pass through, and may be concentrated by an additional lens and an image of the circle projected like any other object. The difficulty of want of delicacy from the distance of the coils is sometimes obviated by drilling a hole through the lens beneath the graduated circle and the mirror, and attaching to the index a vertical wire, to the lower end of which and below the mirror, is attached a second needle. The galvanometer is thus rendered astatic and great delicacy may be attained, since the coil surrounding the needle may be brought very close to it and may be made of great length. The vibrations of the needle may also

be checked by attaching an air vane, or by allowing the lower end of the wire to dip in a liquid.

The greatest delicacy is, however, attainable with the mirror galvanometer, and it becomes desirable to consider how the deflections of such an instrument may be projected on the screen, since it is equally applicable to electrometers, the horizontal pendulum, and various other instruments designed to measure minute forces or alterations in form.

Turn the lantern away from the screen, remove the condensers and projecting lenses and insert the diaphragm. Place the galvanometer in line so that the light shall shine through the circular aperture upon its mirror. Interpose the lens and form a distinct image of the circular aperture on the wall. A slight motion of the mirror will now produce a large deviation of the spot of light. If quantitative measurements are to be made, a large scale must be hung on the screen or projected by a second lantern. But generally the direction of the deviation is all that is needed, and this is shown by merely resting a pointer against the wall under the spot when no current is passing.

That the spot may be distinct and have the greatest brilliancy, several precautions are necessary. The light is commonly first reduced to a parallel beam by the condensers, and the diaphragm then interposed; but this arrangement seems to have no advantages if the aperture is small, while there is considerable loss from the reflection and absorption by the condensers. If a large aperture is used the condensers become necessary to secure a uniform illumination of the image.

199. Projection of Lissajous' Curves.

Apparatus. The lantern, a diaphragm with holes of various sizes which may replace the condenser, a convex lens of about two feet focus and two tuning forks with adjustable weights and with mirrors attached to the ends of their prongs, are needed. The curves may also be projected by the other instruments described below.

Experiment. The curves of Lissajous are described in full in Vol. I, Experiment 65, and their importance and beauty render it desirable that they may be shown to many persons at a time.

Remove the condenser and place the diaphragm near the light, so that it shall shine through a small circular aperture. Form a bright image of this on the screen by means of the lens. Then interpose the fork, so that the light shall fall on one mirror, be reflected from this to the other mirror, and thence upon the screen. There it will form a bright spot, and, vibrating one or both of the forks, the various curves will be produced. With sunlight the curves will be very bright, but with the calcium light it is quite difficult to obtain satisfactory effects. The mirrors should be near each other and near the lens to give the best results. The size of the curve will be proportional to the distance of the screen from the mirrors and to the amplitude of the vibrations of the fork. On the other hand, the spot, which should be as bright and small as possible, will have a diameter proportional to the distance of the screen. The diameter may be diminished by using a long focus lens, but the mirrors must then be further from the lantern and the size of the curves thereby diminished.

The curves of Lissajous may be shown on the screen on a much larger scale by various mechanical devices. One of the best of these consists of two pendulums vibrating in planes at right angles to each other, their lengths being adjustable by raising or lowering the weights forming their bobs. To one is attached a plate of smoked glass which is placed as an object in the vertical lantern, and to the other a metallic point which by its motion scratches a line on the smoked surface. If now both are set in motion together, a line resembling the curves of Lissajous will be obtained, except that as the amplitude of the vibrations diminish the curve will continually approach the central point. This difficulty may be avoided by maintaining the motion of the pendulums by clock-work or electricity.

Another arrangement consists of two sets of strong clock-work by which two wheels can be driven at any desired speed. The latter is varied by partially winding up the spring, if this is used as a motor, or by altering the driving weights. On the face of each wheel a pin projects, one of which moves in a horizontal, the other in a vertical slit. These pins form two opposite corners of a parallelogram of which the third corner is fixed, and the fourth carries a plate pierced with a small hole which forms the object in

the ordinary lantern. Evidently this hole will rise and fall with one wheel and move backward and forward with the other. By varying the rate of motion of the two wheels all the curves of Lissajous may be shown on a large scale, and the changes may be made to take place continuously so that in a few minutes we may show all the possible notes in the gamut. Two plates with horizontal and vertical slits may also be used as an object, and one raised and lowered, the other moved horizontally by two pins attached to revolving wheels.

200. Projection of Spectra.

Apparatus. The lantern, a convex lens, and two large flint glass or bisulphide of carbon prisms, or an Eaton's prism, various metals whose spectra are to be shown, solutions of the chlorides of the alkalies, some sodium, a platinum spoon, and some sticks about quarter of an inch square are needed.

Experiment. To project a spectrum on the screen, the condensers are replaced by the diaphragm with adjustable slit, and an image of this formed on the wall with a lens. A prism is then introduced in the path of the rays when a spectrum will at once be formed to one side. Various precautions must be taken to secure satisfactory results. The breadth, or distance from the top to the bottom, of the spectrum will bear nearly the same proportion to the length of the slit, that its distance bears to the focal distance of the lens. If this breadth is considerable, the condenser should be placed between the slit and light, otherwise the edges of the spectrum will be indistinct, the slit not being uniformly illuminated. With a long slit the colors will be curved owing to the light passing obliquely through the prism. This is best remedied by a curved slit turned the other way. The form should be parabolic and the amount of curvature should be the same as that of the colors. As the prism is turned, the spectrum will attain a minimum of deviation and the red end will move towards the violet whichever way the prism is turned from this. It will be noticed, however, that while when the edge of the prism is turned away from the light the spectrum will shorten, when turned toward the light the spectrum will increase greatly in length. This method of lengthening the spectrum must be employed with mod-

eration, as the light rapidly diminishes and the change in focus necessitates a change in position of the lens. This principle is made use of in Eaton's prism, in which the deviation of a bisulphide of carbon prism turned so as to give a very long spectrum is compensated by a crown glass prism turned the other way. A long spectrum is thus produced directly on the screen, while with an ordinary prism the lantern must be turned. Prisms are made of flint-glass and bisulphide of carbon, the latter having a decided advantage in the great dispersion they produce. Diffraction gratings do not give satisfactory results from the want of light.

Some persons prefer to use a condenser and converge the light upon the slit, inserting just in front of it a concave lens. This method, which would have great advantages were the source of light a point, loses much of its efficiency from the reflection, absorption and aberration of the lenses.

A convenient arrangement for imitating various spectra is to hang up a curtain of black lace and project the spectrum on it, taking care that no bright objects are behind the lace or they will be illuminated and rendered visible to the audience. Any spectrum may now be closely imitated by attaching strips of white paper to the lace in such positions that they shall be illuminated by light of the proper color. A long strip of paper will represent a continuous spectrum on which dark lines may be drawn if desired. Faint lines or bands may be represented by darker paper and very faint continuous spectra by white lace.

With sunlight, brilliant spectra may be obtained, but of course they always contain the solar lines. The latter are so fine that it is difficult to render them visible to an audience, and requires careful focussing. They are best seen with a lens having a focal length of two feet, or even more.

With the calcium light the spectra of the alkalies and alkaline earths may be shown on the screen by removing the lime and holding in its place in the flame a stick quarter of an inch square, previously soaked in a saturated solution of the chloride of the metal to be shown. A brilliant colored flame is thus produced, and the wood chars slowly if dipped frequently in the liquid. A rod of soda glass may be used in the same way to produce the

yellow line of soda, and this forms one of the best sources of intense monochromatic light.

The reversal of the sodium line may be shown by producing the continuous spectrum of the lime light, and interposing the flame of a Bunsen burner in which a platinum spoon is held containing a small piece of sodium. The latter bursts into flame, giving out dense yellow clouds of sodium vapor which cut off the yellow light, and form in the spectrum a black line in the yellow.

The spectra obtained by the electric arc are much brighter owing to its intensity. The heat is so great that not only the alkalies but many other metals are readily volatilized by it and their spectra shown, for instance, cadmium, zinc, copper and lead. The lower carbon is replaced by a number of carbon cups which may be brought in turn under the upper carbon. A little of the metals is placed in each cup and after adjusting the apparatus by the continuous spectrum of the carbons, the latter are somewhat more widely separated so as to throw the continuous spectrum out of the field. The spectrum of the incandescent metal vapor is then projected on the screen, in some cases with great brilliancy.

The soda spectrum may be reversed as with the calcium light, or less easily by placing some sodium on the carbons which are then brought near together. The sodium vapor surrounding the incandescent carbon cuts off the yellow light.

The absorption bands of dried blood, colored glass, or other bodies, are well shown by interposing these substances in the beam, when the continuous spectrum is formed on the screen. For liquids, as solutions of didymium salts and permaganate of potash, a wedge-shaped cell may be used. For gases and vapors, as nitrous fumes and iodine, a globe with plane glass faces is interposed in the same manner.

Appendix A.

ELECTRICITY.

Electrical phenomena may be explained according to various theories. One of the simplest, and for purposes of instruction, one of the most convenient, is that which regards electricity as a material substance devoid of weight, and infinitely more subtle than the most rarefied gas. All space is supposed to be filled with this substance, and electrical phenomena to be due to changes in its distribution. A prominent theory, that of Edlund, assumes that it is identical with the ether by which vibrations of light are transmitted. Electricity passes through some bodies much more readily than through others. The latter are said to have a much greater electrical resistance than the former. When the resistance is small, the body is said to be a good conductor, when large, a non-conductor or insulator. These terms are only relative, as there are no perfect conductors or perfect insulators; that is, the resistance is never either zero or infinity. When a body has more than its normal quantity of electricity it is said to be positively electrified, and when less, negatively electrified. It is by no means certain that these terms are not reversed, as we have no certain means as yet, of distinguishing which is which. As in the case of gases, the particles of electricity are supposed to be mutually repulsive; hence when two bodies, one electrified positively, the other negatively, are brought in contact, the electricity always tends to pass from the former into the latter. This tendency is said to be due to the difference in potential of the two bodies, or, in common language, difference in tension. The term electromotive force is used to denote the force which produces a difference in potential. The absolute potential of a body is not used, since we have no standard with which to compare it; and when a body is said to be electrified positively or negatively we mean with regard to the surrounding medium, or the part of the earth in which it is placed. The passage of electricity from one body to another having a less potential, is called a current, as in the case of gases or liquids. The discussion of the phenomena due to electricity when at rest, is called statical electricity, the phenomena of electrical currents, dynamical electricity. In the former, as in frictional electricity, we have commonly small quantities, but enormous differences in potential. In dynamic as in galvanic electricity, very large quantities, but slight differences of potential.

Statical Electricity. The law for the amount of electrical repulsion is similar to that of gravitation, being proportional to the product of the amounts of electricity in the two bodies, and inversely as the square of the distance.

If, now, two particles A and B are positively electrified with regard to the surrounding medium, they mutually repel each other, since the repulsion of the particles they contain is greater for each other than for the medium which they displace. In general, then, two positively electrified bodies repel each other. Now suppose the potential of the medium increased until it is equal to that of the particle B, which contains the least electricity of the two. Evidently now no action will take place, since the repulsion of A on the surrounding medium is for equal volumes precisely the same as that on B. The effect of A is composed of two parts, that on B, and that on the medium surrounding it. These two must be just equal and opposite, since their resultant is zero. Again, A's effect on B alone would be a repulsion, hence its effect on the surrounding medium would be to attract B. Suppose, now, the electricity in B is diminished, or it is electrified negatively with regard to the surrounding medium. The force of repulsion is thus diminished, while the attraction due to the surrounding medium is unchanged. The latter, therefore, is in excess, and the particles tend to approach, or attract each other. Two bodies, one positive the other negative, therefore attract each other. Next, suppose the potential of the medium equal to that of A, so that B, which has a less potential. appears negatively electrified. As before, the effect is zero, being composed of the two equal and opposite effects, the effect of B and the medium surrounding it on A, and that on the medium around A. But the first of these has been shown above to be an attraction, hence the second must be a repulsion. Now diminish the potential of A so that it will be less than that of the medium. Evidently the force of attraction will be diminished, while the repulsion will be unchanged. But now both A and B are negatively electrified, hence two negatively electrified bodies repel each other. These laws are briefly expressed by saying that bodies containing like kinds of electricity repel each other, while unlike, attract.

When B is a conductor of appreciable size at the same potential as the surrounding medium, and A is positively electrified, a different effect is produced. The electricity in the part of B nearest A is repelled and driven to the further end, so that the latter becomes positively electrified, the nearest end negatively. One end is therefore repelled, the other attracted, but the attraction is the greatest, since the distance is less. In the same way, if A is negatively electrified the electricity in B will rush to the part nearest it, being repelled by the surrounding medium least on that side. The part next A will therefore be electrified positively, the further parts negatively. As before, attraction will take place. If, then, a body electrified either positively or negatively is brought near a conductor at the same potential as the surrounding medium, attraction will take place, and the further end of the conductor will be electrified the same way as the first body, its near end oppositely. This phenomenon is known as induction.

Induced Currents. When currents of electricity flow in the same direction through two parallel conductors, attraction takes place; when in opposite directions, repulsion. It may be proved mathematically that this law follows from the above properties of electricity, if it is assumed that a certain time is required for the force of repulsion to act between two particles at a distance. If, when the currents are passing in the same direction, the conductors yield to the attraction and approach, a part of the energy of the currents is lost, and they become weaker. On the other hand, if suddenly separated, the currents will be strengthened, the work done being converted into electricity. If no current is passing through one of the conductors, suddenly withdrawing the other will create one, while suddenly

approaching it will produce a current flowing in the opposite direction. The more rapidly the conductor is approached or withdrawn the more marked the effect. Hence the best result is attained by suddenly making or breaking the circuit, as this has the same effect as instantly bringing the conductor from an infinite distance, and again removing it to infinity. The conductors are commonly wound in coils in order to obtain a great length in a small space. By making the second coil of fine wire of great length, a current may be induced in it of high potential, since each coil will add to the effect of the others.

Magnets. Ampère explained all the phenomena of magnets by supposing that an electric current flows around each particle of iron. In a magnet all these currents flow in the same direction, which is, at the south end, the same as the hands of a watch. The currents in the interior neutralize each other, those only on the exterior being perceptible. In soft iron the currents flow in all directions, but are easily brought into the same plane on the approach of a magnet. In hardened steel, on the other hand, this change takes place only with difficulty, but is permanent when the magnet is removed. When the opposite ends of two magnets are brought near each other the currents flow in the same direction, and hence attract. If the ends are alike the currents are in opposite directions, and hence repel. Other magnetic phenomena are simply explained in the same manner.

Electro-Magnetism. When a current traversing a conductor is brought near a magnet, it attracts its currents and tends to make them parallel to itself, in which case the magnet will assume a position with the line connecting its poles at right angles to the current. The side to which the north end will be diverted may be determined from the rule given above, or it may be remembered by the law given by Ampère, that if the observer imagines himself placed in the conductor facing the magnet and the current entering at his head, the north pole will always turn to the right. If soft iron is used instead of a magnet, all the currents will be turned in the same direction parallel to the conductor, and it will become magnetic. The conductor is commonly wound in a coil around the soft iron, and the poles are readily distinguished by recollecting that the end around which, when we face it, the current flows in the direction of the hands of a watch, is the south pole.

Magneto-Electricity. A current may be induced in a conductor by the currents of a magnet, precisely as by a current in a second coil. It is only necessary to insert the magnet in the coil, or withdraw it rapidly, that the distance between the conductors may be suddenly altered, and a current induced.

Electrical Measurement. In studying electrical phenomena, several distinct quantities present themselves for measurement. Prominent among these are quantity, resistance and potential, each of which require the accurate establishment of a unit. As in the English system of weights and measures, originally units were adopted having no simple relation to each other, the unit of quantity being the amount required to generate 1 $cm.^3$ of mixed oxygen and hydrogen from the decomposition of water. The unit of resistance was that opposed by a cylinder of mercury having a length of one metre and cross-section of one square millimetre. The failure of the first Atlantic Cable in 1858, was felt to be due in a great measure to the insufficient knowledge of the proper electrical conditions and insufficient means of accurate measurement. Recognizing this difficulty, a Committee was appointed by the British Association, of the most eminent electricians of England, with Prof. Williamson as chairman.

They devoted several years to the task, and as a result, proposed a system of electrical units which has been generally adopted. Two conditions were assumed by the Committee in selecting the units. First, that they should be absolute units, that is, dependent on no arbitrary conditions, but derived directly from the centimetre, gramme and second; and secondly, that they should be so connected together that, as in the metric system, reductions may be made from one to another without employing any other factor than unity.

The following equations show the relation these quantities bear to one another. Faraday proved that $Q = CT$ . . . (1), or the quantity transmitted by a conductor equals the product of the time by the strength of the current. Joule showed that $W = C^2RT$ . . . (2), or the work done by a current equals the product of the time by the resistance, by the square of the current. Finally, by Ohm's law, $C = \frac{E}{R}$, or $E = CR$ . . . (3), the electro-motive force equals the current multiplied by the resistance. If, now, either one of the units is determined, with the units of time and space, all the others can be deduced. Thus having given the unit of resistance, the unit of current is deduced from (2) by making W, R and T, equal to unity, and equals that required to do a unit of work per second in overcoming a resistance of unity. Again, (3) gives the unit of potential or electro-motive force, by making C and R equal to unity, and equals the electro-motive force required to force a unit current through a unit of resistance. Finally, the unit of quantity is given by (1) making C and T equal to unity, in which case the unit of quantity equals the amount transmitted by the unit of current per second. It therefore only remained to determine one of these units to define all the rest, and for this purpose the unit of resistance was selected as more easily determined, and more easily constructed in a permanent form.

Two systems of measurement may be employed to connect electrical units with those of time, space and mass. First, the electro-static system, in which the unit quantity would be defined as that required to produce a repulsion of unity between two particles at distance unity, and secondly the electro-magnetic system, in which the units are defined by the effect of a current on a magnet. The second of these systems is adopted as more convenient in practice, and the ratio between the two units of quantity is found to be equal to 28 billion centimetres, or the velocity of light within the limits of errors of observation. The two systems may also be compared as follows. In both, 1st, the unit current conveys the unit quantity per second; 2d, the unit current in a conductor opposing a unit's resistance will do a unit's work; and 3d, the unit current will be transmitted by a conductor opposing a resistance of unity, if the difference of potential of its two ends is unity. But in the electro-static system the unit quantity will repel a similar quantity at a unit's distance with unit force; while, in the electro-magnetic system, the unit current flowing through a conductor of unit length will create a unit force on a unit magnetic pole at a unit distance. By a unit pole is meant that which repels an equal pole at a distance unity with force unity.

The following method was employed to establish the relation between the unit of resistance and the units of time, space and mass. A coil of wire was caused to revolve with uniform velocity around a vertical axis. At the centre of the coil was placed a small magnet delicately suspended by a filament of silk, and carrying a mirror by which its motion could be measured by a telescope and scale. When the coil is turned a current is

induced in it by the magnetism of the earth, and the needle deviates from the magnetic meridian. From the dimension of the coil, its velocity, and the deviation of the needle, a relation is established between the resistance of the coil and the absolute unit. Measurements were made in this way in 1863 and 1864, and the probable error of the final result amounted to only .08 of one per cent. A number of copies of the standard unit were then made, formed of coils of wire of 1 part platinum and 2 parts silver, the whole imbedded in parafine, and enclosed in a thin copper case. The resistance alters about 3.2 per cent. between 0° and 100° C., and the temperature at which they are exact is marked on each. The error amounts to less than .01 of one per cent. To use them, they are immersed in water which is then brought to the required temperature.

When, as in the present case, quantities of very different orders of magnitudes are to be dealt with, either enormously great or exceedingly minute, instead of writing out a large number of ciphers before or after each, they may be replaced by writing 10^n, in which n is a whole number, either positive or negative, according as the quantity is very great or very small. Thus for a million we may write 10^6, for one millionth 10^{-6}. For brevity 10^n, when n is positive, is denoted by appending the cardinal number, and when n is negative, prefixing the ordinal number. For example, the velocity of light is about 3×10^{10} centimetres, or 3 centimetre-tens, the wave-length of yellow light about 5×10^{-5} centimetres, or 5 fifth-centimetres, and hence a yellow body vibrates 6×10^{14} times per second.

The absolute unit of resistance, as found above, is an exceedingly small quantity, hence it is multiplied by a billion to make it of a convenient size, or the adopted unit $= 10^9$ absolute units. In the same way the absolute unit of capacity is enormous, and the adopted unit $= 10^{-9}$ absolute units. The unit of quantity $= 10^{-1}$ absolute units, and the unit of potential $= 10^8$ absolute units.

These units are often called, from the name of the Committee, the B. A. Units; but since they depend only on the centimetre, gramme and second, the Committee recommend that they should be called the C. G. S. Units. Names are given to each, after the physicists who have distinguished themselves in the branch of electricity to which they relate. Thus the unit of quantity is called a veber, the unit of capacity a farad, the unit of resistance an ohm, and the unit of potential a volt, after Weber, Faraday, Ohm and Volta. To denote quantities very much greater or less than the units, the prefixes mega- and micro- are used, the former denoting a million times, the latter a millionth part; thus a megohm equals 1,000,000 ohms, a microfarad .000001 farad. The following examples will serve to show the magnitude of these various units. A piece of No. 16 copper wire (diameter .06 inches) 60 ft. long has a resistance of about 1 ohm. A Smee cell has an electro-motive force of about 0.25 volts, a Daniell cell about 1.1 volts, and a Grove or Bunsen about 1.8 volts. The capacity of the Atlantic cable is only 800 microfarads, and its resistance 8000 ohms.

Kirchhoff's Laws. The magnitude of the currents in a system of conductors is often readily determined by the two following laws discovered by Kirchhoff. 1st. When any number of conductors meet in a point the sum of the currents flowing towards it equals the sum of those flowing from it. This is obvious, as otherwise the quantity of electricity in the point would alter, and it would constantly become more and more positively or negatively electrified. 2d. In any closed circuit the sum of the products of the resistances by the currents equals the sum of the electro-motive forces in the circuit. Ohm's law follows as a special case of this.

Batteries. A most valuable application of Ohm's law is to determine the strength of the current which will flow through a given piece of apparatus with different forms of battery, thus enabling us to decide which is best adapted to our purpose. In the equation $C = \frac{E}{R}$, let E denote the electro-motive force of the battery, and R the total resistance of the circuit, which consists of two parts, the resistance of the battery B, and that of the instrument and connecting wire. The latter is constant, and may be called P, so that $C = \frac{E}{B + P}$. If, now, the battery consists of n cells, each having an electro-motive force E and resistance B, and they are connected for tension, or the zinc of one connected with the carbon of the other, evidently the total electro-motive force will be nE. The total resistance also will be nB, since all the electricity has to pass through each cell. The current then, $C = \frac{nE}{nB + P}$. Next, suppose the cells connected for quantity, or all the zincs connected together, and all the carbons connected. The electro-motive force will be only E, but the resistance will be much less than that of a single cell. It will in fact be only $B \div n$, since instead of one passage for the current, n are open. In this case, multiplying both numerator and denominator of the fraction by n, we deduce $C = \frac{nE}{B + nP}$. As a third case, suppose the battery divided into p sets of m cells, and that in each set all the zincs and all the carbons are connected together, while the sets are connected for tension, or one set of carbon with the next set of zinc. The battery is then said to be connected for quantity m and tension p, and is equivalent to p large cells of electro-motive force E and resistance $B \div m$. The current, therefore, $C = \frac{mpE}{pB + mP}$. It may be proved mathematically that with a given battery the strongest current is obtained when the resistance of the battery, or $\frac{pB}{m}$ is most nearly equal to P, or as it is commonly expressed, the resistances inside and outside the battery are equal. Generally the outside resistance is much the greatest, and therefore the best effect obtained when the battery is connected for tension.

Two special cases should here be considered. First, if the outside resistance P is very great, so that B can be neglected compared with it, the first equation becomes $C = \frac{nE}{P}$, or in this case, n cells connected for tension give n times the current of one cell. If connected for quantity, however, $C = \frac{nE}{nP} = \frac{E}{P}$, or there is no gain by increasing the number of cells, a hundred giving no greater current than one. Next, if P is very small the opposite result is obtained, the first equation becoming $C = \frac{nE}{nB} = \frac{E}{B}$, and the second $C = \frac{nE}{B}$. Hence with a small resistance the cells should be connected for quantity, for if connected for tension there is no stronger current than with a single cell.

The electro-motive force of a battery is wholly independent of the size of the plates, and depends only on the difference of the chemical action on them. The resistance of the battery, on the other hand, is nearly inversely as their cross-section, and proportional to their distance apart, and it is

only on account of the diminished resistance, that large cells are to be preferred to small. The consumption of zinc is proportional to the number of cells connected for tension, or to p in the above formula. A battery connected for quantity is therefore much less expensive than when connected for tension.

Shunts. Sometimes we wish to allow a portion only of a current to pass through a given instrument. This is particularly the case with galvanometers, which are often made so delicate that they would be easily injured if subjected to too powerful a current. In this case a second passage is opened to the current called a *shunt*, since it allows part of the electricity to shun the original circuit. In Fig. 110, let R' be the resistance of the galvanometer, or other instrument to be shunted, and R'' the resistance of the shunt. Then calling C, C' and C'' the currents in the circuit outside the shunt, in R', and in R'', we have by Kirchhoff's first law $C=C'+C''$, and by his second law $C'R'=C''R''$. Hence $C'=C.\frac{R''}{R'+R''}$, from which we see that by making R'' small enough we may reduce the current in R' as much as we please.

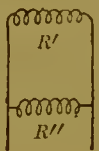


Fig. 110.

The combined resistance R, of R' and R'', or in any other case of a divided circuit, is found as follows. Let E be the difference of potential of the two junctions of R' and R'', then by Ohm's law $C' = \frac{E}{R'}$, $C'' = \frac{E}{R''}$, or the whole current $C' + C'' = E\left(\frac{1}{R'} + \frac{1}{R''}\right)$. Now R must have such a value that if it replace R' and R'' the current will be unchanged, or $C = \frac{E}{R} = E\left(\frac{1}{R'} + \frac{1}{R''}\right)$, hence $R = \frac{R'R''}{R' + R''}$.

Quantity. 1. *Voltameter.* Two platinum electrodes are immersed in a vessel containing dilute sulphuric acid, and a glass tube graduated to cubic centimetres is placed over them to collect the gases set free. The current is allowed to pass, and the volume of gas collected is then corrected for temperature, pressure and moisture. Then we have $Q = .17v$, in which Q is the required quantity in vebers, and v the corrected volume in cm^3. The principal objections to this method are the difficulty of determining the correct volume of the gases, and their solubility in the liquid.

2. *Deposition of Copper.* Two copper electrodes are placed in a beaker containing a saturated solution of sulphate of copper. They are inserted in the circuit, and the increase of weight of that attached to the negative or zinc pole of the battery is noted. Then $Q = .32w$, in which w is the increase of weight in milligrammes. This method is much to be preferred to the preceding.

Current. 1. *Tangent Galvanometer.* A compass needle is hung at the centre of a coil of insulated wire, whose radius is at least three times its length. Sometimes two parallel, vertical coils are used, wound so that their depth shall be to their breadth as 1 : .928 and separated by an interval equal to their radius. The instrument is so placed that the coils shall lie in the magnetic meridian, and the needle be parallel to them, or at zero. The current is then passed through them, when the needle will be acted on by two forces, H the horizontal component of the earth's magnetism, which tends to keep it parallel to the plane of the coils, and C the effect of the current tending to turn it at right angles to the coils, as shown in Fig. 111. For equilibrium, the needle

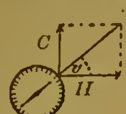


Fig. 111.

must coincide with the resultant of these forces, when a simple construction shows that $C' = H$ tang v, in which v is the angle of deviation of the needle. But the current C is proportional to C', or equals kC', in which k is the galvanometer constant, and depends only on the form of the instrument. Therefore $C = kH$ tang v, in which kH must be determined by the method of depositing copper, after which the instrument may be used directly for measuring currents. The galvanometer constant may also be determined by computation, from the dimensions of the coil. Let y be the radius of the coil, x the distance of its centre from the magnet, and l the length of the wire. Then $k = \frac{(x^2 + y^2)^{\frac{3}{2}}}{ly}$ from which k is readily computed.

2. *Sine Galvanometer.* In this instrument the coils may be of any desired form, and no graduation is needed for the needle, which is always brought to the same point, or to the zero. A graduated circle is, however, attached to the coils so that the angle through which they are rotated may be measured. The coils are first turned so that the needle points to zero, the current is then passed through them and they are again turned until the needle points to zero. Call v the angle through which they have been moved, then $C' = H$ sin v, since constructing the parallelogram of the forces acting on the needle, as in Fig. 112, we find that H is now the hypothenuse of a right-angled triangle, of which C' is the side opposite v. As before, $C = kC'$, and hence $C = kH$ sin v in which kH is determined as in the case of the tangent galvanometer.

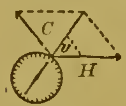


Fig. 112.

3. *Cosine Galvanometer.* If the coils of a tangent galvanometer are free to turn around a horizontal axis, their effect on the needle may be diminished at will. For since their effect is always equivalent to a force acting at right angles to their plane it may be decomposed into two; one, the vertical component equal to C' sin w, the other acting horizontally, equal to C' cos w, in which w is the angle of inclination of the coils. The first of these components tends only to incline the needle or to make it dip, and the second only to deviate it. The strength of the current therefore is measured by the equation $C = kC' = \frac{kH \text{ tang } v}{\cos w}$, in which by giving different values to v and w many readings may be obtained for the same current. Moreover it may be used on currents too powerful to give good results with the tangent galvanometer by merely making w nearly 90°, when v may be made as small as is desired.

Resistance. 1. *Differential Galvanometer.* The simplest method of measuring resistances is by a differential galvanometer, in which two equal coils are wound around the needle. If equal currents pass through these in opposite directions, the deviation of the needle will be nothing. To measure a resistance, the current of the battery is divided, so that part will pass through one coil and a set of resistance coils, or other arrangement for varying the resistance, and the remainder through the other coil, and the resistance to be measured. The variable resistance is then altered until the needle is brought to zero, when its amount equals the required resistance, since the two currents will be equal only when the two circuits oppose the same resistance.

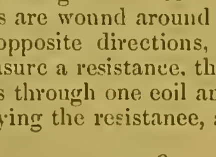


Fig. 113.

2. *Wheatstone's Bridge.* The principle of this most valuable instrument is shown in Fig. 113. Four resistance coils, M, N, O and P, are connected together end to end, and the opposite junctions connected with the battery B and galvan-

ometer G, as in the figure. The current from B divides, part going through M and O, and the remainder through N and P. If the resistances are so related that $M : N = O : P$, no current will pass through the galvanometer, since its two terminals will have the same potential. Accordingly having given three of the resistances, the fourth may be determined with great accuracy if a delicate galvanometer is used. If $M = N$, O will equal P, and thus a resistance may readily be copied.

The formula $M : N = O : P$, may be proved as follows. In Fig. 114 let abscissas represent resistances, and ordinates the excess of potential above that of the negative pole of the battery. Lay off four distances equal to M, N, O and P, and erect a perpendicular at each junction equal to its potential. This will be greatest at the junction MN, and zero at the junction OP. At any intermediate point it is found by drawing straight lines from the ends of O and P to the perpendicular at MN, since the potential will diminish continuously by an amount proportional to the change in resistance. From the figure it is obvious that the perpendiculars at the junctions MO and NP will be equal only when $M : N = O : P$; but when this is the case, no current will pass through the galvanometer, since its terminals will have the same potential.

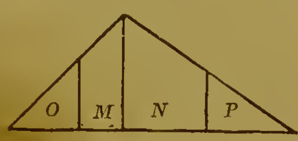


Fig. 114.

This same proposition may be proved by Kirchhoff's laws. Calling the current in M, C_M, the current in N, C_N, etc., we have by the first law $C_M + C_G - C_O = 0$, since M, G and O meet in a point; but $C_G = 0$, since no current passes through the galvanometer, hence $C_M = C_O$. In the same way, $C_N = C_P$. Now in the closed circuit MNG, we have by the second law, $C_M M + C_G G - C_N N = 0$, and in the circuit OPG we have, $C_O O - C_G G - C_P P = 0$, giving the negative sign when the current flows in the opposite direction. Dividing the first of these equations by the second, and recollecting that $C_G = 0$, we have $\frac{C_M M}{C_O O} = \frac{C_N N}{C_P P}$, or $\frac{M}{O} = \frac{N}{P}$, since $C_M = C_O$, and $C_N = C_P$.

Capacity. Condensers. Capacities are usually measured by condensers formed by separating two good conductors by a thin insulating film, as in the case of a Leyden jar. They are commonly made of alternate sheets of tin foil and oiled-silk or waxed paper, connecting the alternate sheets of foil together. A very important example in practice of a condenser, is a submarine cable, in which the insulating covering replaces the paper, and the core and outer covering, or water, the two conductors. The relative capacities of two condensers may be measured precisely like resistances, with a Wheatstone's bridge. They are inserted in the place of two of the resistances, as O and P. They are then charged by connecting the battery, when the needle will deviate unless their capacities bear the same ratio as M and N. They are next discharged by connecting their inner and outer surfaces together, or replacing the battery by a conductor, when the electricity flowing out of them will deviate the needle in the opposite direction. By changing M or N the ratio of the two capacities is readily found.

A second method is to use a differential galvanometer, connecting the two condensers with the two coils, and connecting a variable shunt with the coil to which the largest condenser is attached. A third method is, to charge the condensers in turn from the same battery, interposing a galvanometer, and noticing the swing of the needle in each case, as it shows the amount of electricity which must pass into the condenser to bring it to the same potential as the battery.

Potential. Electrometers. Electro-motive forces or differences of potential are measured by electrometers, of which the most perfect is Thomson's quadrant electrometer. In its simplest form this consists of four quadrants of sheet brass, over which hangs an aluminum needle connected with the interior of a Leyden jar. The latter is charged so that the needle is positively electrified. If, now, two opposite quadrants have a higher potential than the other two, the latter will attract the needle, and cause it to tend to become parallel to the line bisecting them. A mirror and scale serves to show the amount of the torsion. The quadrants are first connected with the poles of a standard battery, and then with the two bodies whose difference of potential is to be determined. The comparative deviations show the required difference. The whole instrument is covered with a glass shade and kept perfectly dry by sulphuric acid. In the more complete form of the instrument the Leyden jar is charged by a little replenisher, somewhat like a Holtz machine, until it is capable of exerting a known attraction, it therefore always gives constant results, and from it the potential is obtained directly.

Appendix B.

TABLES.

Tables 1 to **9** give the tabular numbers most commonly required in computation and are all arranged according to the same plan, so that the method of using them shall be as nearly as possible alike. Each right hand page should properly be placed immediately below that opposite it, but since this was impracticable they are placed side by side. To render the tables more legible the units when repeated are in some cases omitted, and given only for every fifth number and when they change. They may always be correctly inserted by taking the units just above.

Table 1 gives the Squares of numbers from 1.00 to 9.99, differing by hundredths of a unit. To find the square of any number, as 3.27, take the column headed 3 and follow it down to the number opposite .27 where we find 10.6929, the required square, retaining the 10 from the number above. If the second figure is less than 5 the result should be taken from the left hand page, otherwise, from the right hand page. Thus, the square of 7.89 is 62.2521. If the number contains more than three significant figures the result is obtained more accurately by interpolation. Generally first differences only need be used, and since the numbers follow each other in order vertically, the subtraction is readily made. Thus, to find the square 3.276; the square of 3.27 is 10.6929, of 3.28 is 10.7584, and their difference .0655; .6 of this equals .0393 which added to 10.6929 gives 10.7322. If the decimal point of the number is moved, that of the square must be moved twice as many places; thus, the square of 3.27 is 10.6929, of 327 is 106929, and of .0327 is .00106927.

This table may be used to extract square roots approximately. Thus, to find the square root of 2 or the number whose square is 2, we find by following down the columns that it is contained between 1.41 and 1.42. Moreover, the difference between their squares, or 2.0164 — 1.9881 =.0283, and dividing 2.— 1.9881 =.0129 by this, gives .4205, which multiplied by .01 and added to 1.41 gives 1.414205 as the square root of 2. Its true value is 1.414214. The square root of 200 is in like manner 14.142 and of .02 is .14142, moving the decimal point of the square root one half as far as that of the number. To find the square root of .2 move the decimal point two places, when the square root of 20 is given in the table as 4.47, and that of .2 is .447.

Table 2 gives in precisely the same manner the Cubes of numbers from 1.00 to 9.99. Thus, the cube of 8.32 is 575.930, of 8.324 is 576.76 and of

478 is 109.215. If the decimal point must be moved, that of the cube must be moved three times as far; thus the cube of 832 is approximately 575930000, of .832, .00057593. Similarly, cube roots may be extracted. The cube root of 3 is 1.44, or interpolating, 1.4422. The cube root of 3 is one tenth that of 300 or .6694, of .03, a tenth of the cube root of 30 or .3107.

Table 3 gives the Reciprocals of the same numbers from 1.00 to 9.99, and is used in the same way. Thus, the reciprocal of 1.28 is .78125, of 1.284, .7788. If the decimal point has been moved, that of the reciprocal must be moved an equal amount in the other direction; thus the reciprocal of 12.84 is .07788, of .1284, 7.788, of .01284, 77.88.

Table 4 gives various powers of a hundred numbers from 0 to 10, varying by tenths. These numbers are very useful in testing observations to determine the law connecting them. Thus, to see if one quantity varies inversely as the square of another we use the values of x^{-2}. Similarly the square root, cube root, inverse square, cube, square root, cube roots, fourth and fifth powers are given. Combining this with Tables 1, 2, and 3 we may find at once the value of x raised to the powers $-3, -2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 2, 3, 4$ or 5. If the decimal point is different, the powers may still be found approximately by changing the decimal point as in the previous Tables. The fraction $\frac{x}{1-x}$ and its square are frequently employed, the first, for instance, in the British Association Bridge and the second in the photometer. The corresponding values are accordingly computed in the last two columns. Thus, if the reading of the bridge is .38, the relative resistances are as 1 to 1.632. Again in the photometer, if the disk stands 72 inches from one of the lights, their ratio is 6.612.

Table 5 gives the logarithms of numbers from 1.00 to 9.99, to four places of decimals. The arrangement differs from the common tables since the tabular numbers follow each other vertically instead of horizontally, but it is believed that this is an undoubted improvement from the ease in interpolation, the diminished liability to error, and a uniformity with tables of the other functions. As in Table 1, take the column with the same heading as the left hand figure and follow it down to the number opposite the second and third figures. If the second figure exceeds 4, use the right hand page. If the number is contained between 1 and 10 its characteristic will be 0. Increase it by unity for each place to the right that the decimal point is moved, and if a fraction, or the decimal point moved to the left, call the characteristic 10 and diminish it by the same amount. Thus, $\log 438 = 0.6415$, $\log 4.387 = 0.6415 + .0007 = 0.6422$, $\log 438 = 2.6415$, $\log .0438 = 8.6415$.

Table 6 gives the Natural Sines and Cosines of angles for every tenth of a degree. Each column contains the sines for five degrees, the tens being given at the top of the page, and the units and tenths in the left hand column. When the units exceed four the right hand page must be used. Thus nat. $\sin 62°.7 = .8886$ and nat. $\sin 18°.3 = .3140$. If the angles are given in single minutes the sines may be obtained by dividing by six and, if necessary, interpolating. By the inverse process the angle corresponding to any sine is found; thus, $\sin^{-1} .2 = 11°32'$. Cosines are found in the same way, reading from the bottom and right hand column. Thus, $\cos 18°.6 = .9478$, $\cos 72°\ 28' = .3013$, $\cos^{-1} .9 = 25°\ 51'$.

Table 7 gives Natural Tangents and Cotangents of angles for every tenth of a degree and is used precisely like Table 6. Thus, tang 52°.4 = 1.2985, $\cot^{-1}$ .4376 = 66° 22′.

Table 8 gives Logarithmic Sines and Cosines of angles for every tenth of a degree and is used precisely like Table 6. Thus, log sin 3°.6 = 8.7979, log $\cos^{-1}$ 9.00 = 84° 16′.

Table 9 gives the Logarithmic Tangents and Cotangents of angles for every tenth of a degree and is used precisely like Table 6. Thus, log tang 22°.4 = 9.6151 log $\cot^{-1}$ 0.2368 = 59°.9.

Table 10 gives the numerical constants and ratios most used in physics. The first column defines the constant, the second gives its numerical value, the third gives its reciprocal and the fourth its logarithm. Thus the line 'Grain in grammes' shows that .0648 grammes make a grain, and 15.432 grains make a gramme. The logarithm is useful in reducing from one unit to another. In this, as in several of the following tables, the decimals are not carried as far as is customary, but all figures having any significance are here retained and those omitted are liable to mislead, as implying a greater accuracy that has really been obtained in their determination. Thus a metre is commonly stated to equal 39.37079 inches, but different measurements differ greatly in the last two places. Where the right hand figures are known to be zero they are retained, thus 1 inch equals 2.5400 cms. more nearly than 2.5399 or 2.5401.

Table 11 gives the Properties of the Metals whose names are in the first column. The next column gives their chemical symbols; the third column gives their atomic weights, and the fourth their specific gravities; the values given in this column are taken from *Clarke's Constants of Nature*. Only one place of decimals is retained, since the values commonly vary by at least one tenth in different specimens. As exceptions we might give, Hg = 13.596, Li = .58, Na = .97, K = .87. The next column gives the moduli of elasticity, or forces in kilogrammes required to double the length of a bar having a cross section of 1 mm. if the same law of elasticities continued to hold for such large extensions as for small. This modulus may also be defined as the ratio of stress to strain for moderate strains. Following this are the hardness according to Bottone (*Les Mondes*, xxxi, 720), and the specific heats, mostly taken from *Watts' Dict., Supplement* I, p. 665. The points of fusion follow, taken from *Clarke's Constants of Nature*. The coefficients of expansion are Fizeau's results (*Watts' Dict., Supplement* I, p. 680); they must be divided by 10^8 to give the coefficient per degree C. or they equal the change in length in ten millionths, per degree. Conduct. gives the conductivities according to Wiedemann and Franz (*Pogg. Ann.*, lxxxix, 497) reduced to absolute units. Electrical Resist. gives the resistance in ohms of a wire of the metal one metre in length, and having a cross section of one millimetre, according to the observations of Matthiessen, except for cadmium, palladium and thallium, where Benoit's results (*Bib. Univ.*, cciii. 284) are given. Thermo-Elect. gives the thermo-electric position of the metals at 20° C. per degree C. in microvolts compared with lead, according to the observations of Matthiessen. Thus, a pair composed of nickel and iron with its terminals differing 5° will give a electromotive force of 5 [11.4 — (— 17.5)] = 144.5 microvolts or .0001445 volts. The last column gives the refractive equivalents, or indices of refraction minus one, divided by the densities, according to Gladstone (*Phil. Trans.*, 1870, p. 9).

Table 12 gives the Properties of the most common Liquids whose names are given in the first column. The next column gives their chemical symbols; the next, their specific gravities; Capillarity gives the height to which the liquid will rise in a tube of diameter 1 mm. according to Frankenheim (*Pogg. Ann.*, lxx, 515). Compress. gives the compressibility multiplied by 10^6 or diminution in volume in millionths per atmosphere. The next column gives the velocity of sound in an unlimited mass of the liquid according to Wertheim (*Ann. Chim. Phys.*, III, xxiii, 434). Then follow the specific heat, the total expansion in heating the liquid from 0° to 100°, the boiling point, and the latent heat. The next two columns give the index of refraction for the sodium line, and the dispersion or difference in index of the red and violet rays. The last column gives the magnetic power compared with water, according to Faraday (*Bib. Univ.*, xxiii, 105).

Table 13 gives the Properties of the most common Gases whose names are given in the first column. In the next columns are given their chemical symbols, their molecular weights, and their densities or specific gravities, air being taken as unity. Then follow the weight in grammes per litre; the specific heat for equal weights; the specific heat for equal volumes; the boiling points or temperature necessary to reduce them to the liquid form; the velocity of transpiration according to Graham (*Phil. Trans.*, 1846, p. 573, 1849, p. 349); the velocity of sound according to Dulong; the index of refraction for the sodium-line minus one, multiplied by a thousand. Thus the index for air = 1.0002923. Where four places of decimals are given the results are those of Mascart (*Comptes Rendus*, lxxviii, 801), the others are those of Dulong (*Ann. Chim. et Phys.*, II, xxxi, 154). The last column gives the dispersion, or value of B in the formula of Cauchy, $n - 1 = A\left(1 + \frac{B}{\lambda^2}\right)$.

Table 14 serves to reduce various hydrometer readings. The first column gives the reading or point to which the hydrometer sinks in the liquid. The second column gives the corresponding specific gravity, if the hydrometer is graduated according to Baumé's scale for liquids heavier than water. The third column corresponds to Baumé's scale for liquids lighter than water; columns four and five give the similar readings on Beck's scale, column six on Cartier's, and column seven on Twaddell's. The latter may also be computed by the formula $g = 1 + .005\ r$ in which r is the reading and g the required specific gravity.

Table 15 gives the temperatures in Centigrade and Fahrenheit degrees of various phenomena. The first column describes the effect, the second gives the temperature on the Centigrade and the third that on the Fahrenheit scale.

Table 16 gives the pressure of Vapors according to the experiments of Regnault (*Memoirs of the French Acad.*, xxi, 624; xxvi, 374). The first column gives the temperature, the others the pressure in millimetres of the liquid whose name heads the column.

Table 17 furnishes the means of determining the amount of moisture in the air, from the readings of the Wet and Dry Bulb Thermometers. Column one gives the temperature of the air as given by the dry bulb

thermometer, and the other columns give the pressure of the aqueous vapor in millimetres corresponding to a difference in reading of the two thermometers, by an amount equal to the number heading the column. Thus, if the dry bulb reads 16° and the wet bulb 10°, or their difference 6°, we follow down the first column to the point 16° and then horizontally to the column headed 6° where we find the number 9.9 which equals the required amount of moisture in the air. The second column gives the amount of moisture if the difference in the two thermometers is zero, or the air saturated. It may therefore be used in connection with Table 16 for the pressure of steam at intermediate pressures. The relative humidity may be found by dividing the actual amount of moisture in the air, by that which would be required to saturate it. Thus in the above example at 16°, 13.5 mm. would saturate air, or the relative humidity is 9.9 divided by 13.5 or .73. The dew point also is found by noting the temperature at which the observed moisture would saturate the air, or from the first column the reading corresponding to a value of 9.9 in the second column. In the present case this lies between 10 and 12, or is about 11.1°C.

Table 18 gives the principal elements of the Solar System, assuming the solar parallax to be 8.94″. Most of these numbers are taken from *Lockyer's Astronomy*. They give the names of the planets, their symbols, distances from the sun in miles, distances compared with that of the earth, the times of revolution, the eccentricity of the orbit, its inclination to the ecliptic, the longitude of the ascending node, the diameter in miles, masses compared with that of the earth and specific gravities. Corresponding elements are also given for the sun and moon.

Table 19 gives the position of some of the most conspicuous of the Double Stars. Those are selected which are of sufficient size to be easily seen by the naked eye, that they may be observed by those whose telescopes have no equatorial mounting. For the same reason only those are given, both of whose components are readily seen with telescopes of moderate power. They are arranged in the order of their right ascensions, which are given in the first column. The declination is given next, then the constellation in which they are situated, their specific name or letter, the magnitude of the larger and then of the smaller component, their distance apart in seconds and their position angle in degrees and tenths. The last column gives their color, using the following abbreviations, p. pale, d. deep, bl. blue, gr. green, lil. lilac, pur. purple, r. red, vi. violet, and w. white. When no color is given the authorities differ. When the star has three components it is marked T., and B. denotes that it is binary. This Table and the following are compiled in a great measure from *Webb's Celestial Objects for Common Telescopes.*

Table 20 gives similarly a list of the more conspicuous Clusters and Nebulæ. The first column gives the number in the Catalogue of the British Association, the second the right ascension, the next the declination, then the Constellation and specific name; ♅ denotes the catalogue of the elder Herschel, and M that of Messier. The last column serves to describe the object; E., denotes that it is visible as a misty spot to the naked eye. O., that a small optical power only is needed, as a finder or opera glass, to recognize its place. C., denotes that the spectrum is continuous, or that the object is probably a cluster composed of stars, and G., that it is gaseous or a nebula. The other abbreviations are clust. and cl. for cluster, neb. for nebula. plan. for planetary, resolv. for resolvable, and diam. for diameter.

I. Squares.

n	1.	2.	3.	4.	5.	6.	7.	8.	9.
.00	1.0000	4.0000	9.0000	16.0000	25.0000	36.0000	49.0000	64.0000	81.0000
.01	.0201	.0401	.0601	.0801	.1001	.1201	.1401	.1601	.1801
.02	.0404	.0804	.1204	.1604	.2004	.2404	.2804	.3204	.3604
.03	.0609	.1209	.1809	.2409	.3009	.3609	.4209	.4809	.5409
.04	.0816	.1616	.2416	.3216	.4016	.4816	.5616	.6416	.7216
.05	1.1025	4.2025	9.3025	16.4025	25.5025	36.6025	49.7025	64.8025	81.9025
.06	.1236	.2436	.3636	.4836	.6036	.7236	.8436	.9636	82.0836
.07	.1449	.2849	.4249	.5649	.7049	.8449	.9849	65.1249	.2649
.08	.1664	.3264	.4864	.6464	.8064	.9664	50.1264	.2864	.4464
.09	.1881	.3681	.5481	.7281	.9081	37.0881	.2681	.4481	.6281
.10	1.2100	4.4100	9.6100	16.8100	26.0100	37.2100	50.4100	65.6100	82.8100
.11	.2321	.4521	.6721	.8921	.1121	.3321	.5521	.7721	.9921
.12	.2544	.4944	.7344	.9744	.2144	.4544	.6944	.9344	83.1744
.13	.2769	.5369	.7969	17.0569	.3169	.5769	.8369	66.0969	.3569
.14	.2996	.5796	.8596	.1396	.4196	.6996	.9796	.2596	.5396
.15	1.3225	4.6225	9.9225	17.2225	26.5225	37.8225	51.1225	66.4225	83.7225
.16	.3456	.6656	.9856	.3056	.6256	.9456	.2656	.5856	.9056
.17	.3689	.7089	10.0489	.3889	.7289	38.0689	.4089	.7489	84.0889
.18	.3924	.7524	.1124	.4724	.8324	.1924	.5524	.9124	.2724
.19	.4161	.7961	.1761	.5561	.9361	.3161	.6961	67.0761	.4561
.20	1.4400	4.8400	10.2400	17.6400	27.0400	38.4400	51.8400	67.2400	84.6400
.21	.4641	.8841	.3041	.7241	.1441	.5641	.9841	.4041	.8241
.22	.4884	.9284	.3684	.8084	.2484	.6884	52.1284	.5684	85.0084
.23	.5129	.9729	.4329	.8929	.3529	.8129	.2729	.7329	.1929
.24	.5376	5.0176	.4976	.9776	.4576	.9376	.4176	.8976	.3776
.25	1.5625	5.0625	10.5625	18.0625	27.5625	39.0625	52.5625	68.0625	85.5625
.26	.5876	.1076	.6276	.1476	.6676	.1876	.7076	.2276	.7476
.27	.6129	.1529	.6929	.2329	.7729	.3129	.8529	.3929	.9329
.28	.6384	.1984	.7584	.3184	.8784	.4384	.9984	.5584	86.1184
.29	.6641	.2441	.8241	.4041	.9841	.5641	53.1441	.7241	.3041
.30	1.6900	5.2900	10.8900	18.4900	28.0900	39.6900	53.2900	68.8900	86.4900
.31	.7161	.3361	.9561	.5761	.1961	.8161	.4361	69.0561	.6761
.32	.7424	.3824	11.0224	.6624	.3024	.9424	.5824	.2224	.8624
.33	.7689	.4289	.0889	.7489	.4089	40.0689	.7289	.3889	87.0489
.34	.7956	.4756	.1556	.8356	.5156	.1956	.8756	.5556	.2356
.35	1.8225	5.5225	11.2225	18.9225	28.6225	40.3225	54.0225	69.7225	87.4225
.36	.8496	.5696	.2896	19.0096	.7296	.4496	.1696	.8896	.6096
.37	.8769	.6169	.3569	.0969	.8369	.5769	.3169	70.0569	.7969
.38	.9044	.6644	.4244	.1844	.9444	.7044	.4644	.2244	.9844
.39	.9321	.7121	.4921	.2721	29.0521	.8321	.6121	.3921	88.1721
.40	1.9600	5.7600	11.5600	19.3600	29.1600	40.9600	54.7600	70.5600	88.3600
.41	.9881	.8081	.6281	.4481	.2681	41.0881	.9081	.7281	.5481
.42	2.0164	.8564	.6964	.5364	.3764	.2164	55.0564	.8964	.7364
.43	.0449	.9049	.7649	.6249	.4849	.3449	.2049	71.0649	.9249
.44	.0736	.9536	.8336	.7136	.5936	.4736	.3536	.2336	89.1136
.45	2.1025	6.0025	11.9025	19.8025	29.7025	41.6025	55.5025	71.4025	89.3025
.46	.1316	.0516	.9716	.8916	.8116	.7316	.6516	.5716	.4916
.47	.1609	.1009	12.0409	.9809	.9209	.8609	.8009	.7409	.6809
.48	.1904	.1504	.1104	20.0704	30.0304	.9904	.9504	.9104	.8704
.49	.2201	.2001	.1801	.1601	.1401	42.1201	56.1001	72.0801	90.0601

n	1.	2.	3.	4.	5.	6.	7.	8.	9.
.50	2.2500	6.2500	12.2500	20.2500	30.2500	42.2500	56.2500	72.2500	90.2500
.51	.2801	.3001	.3201	.3401	.3601	.3801	.4001	.4201	.4401
.52	.3104	3504	.3904	.4304	.4704	.5104	.5504	.5904	.6304
.53	.3409	.4009	.4609	.5209	.5809	.6409	.7009	.7609	.8209
.54	.3716	.4516	.5316	.6116	.6916	.7716	.8516	.9316	91.0116
.55	2.4025	6.5025	12.6025	20.7025	30.8025	42.9025	57.0025	73.1025	91.2025
.56	.4336	.5536	.6736	.7936	.9136	43.0336	.1536	.2736	.3936
.57	.4649	.6049	.7449	.8849	31.0249	.1649	.3049	.4449	.5849
.58	.4964	.6564	.8164	.9764	.1364	.2964	.4564	.6164	.7764
.59	.5281	.7081	8881	21.0681	.2481	.4281	.6081	.7881	.9681
.60	2.5600	6.7600	12.9600	21.1600	31.3600	43.5600	57.7600	73.9600	92.1600
.61	.5921	.8121	13.0321	.2521	.4721	.6921	.9121	74.1321	.3521
.62	.6244	.8644	.1044	.3444	.5844	.8244	58.0644	.3044	.5444
.63	.6569	.9169	.1769	.4369	.6969	.9569	.2169	.4769	.7369
.64	.6896	.9696	.2496	.5296	.8096	44.0896	.3696	.6496	.9296
.65	2.7225	7.0225	13.3225	21.6225	31.9225	44.2225	58.5225	74.8225	93.1225
.66	.7556	.0756	.3956	.7156	32.0356	.3556	.6756	.9956	.3156
.67	.7889	.1289	.4689	.8089	.1489	.4889	.8289	75.1689	.5089
.68	.8224	.1824	.5424	.9024	.2624	.6224	.9824	.3424	.7024
.69	.8561	.2361	.6161	.9961	.3761	.7561	59.1361	.5161	.8961
.70	2.8900	7.2900	13.6900	22.0900	32.4900	44.8900	59.2900	75.6900	94.0900
.71	.9241	.3441	.7641	.1841	.6041	45.0241	.4441	.8641	.2841
.72	.9584	.3984	.8384	.2784	.7184	.1584	.5984	76.0384	.4784
.73	.9929	.4529	.9129	.3729	.8329	.2929	.7529	.2129	.6729
.74	3.0276	.5076	.9876	.4676	.9476	.4276	.9076	.3876	.8676
.75	3.0625	7.5625	14.0625	22.5625	33.0625	45.5625	60.0625	76.5625	95.0625
.76	.0976	.6176	.1376	6576	.1776	.6976	.2176	.7376	.2576
.77	.1329	.6729	.2129	.7529	.2929	.8329	.3729	.9129	.4529
.78	.1684	.7284	.2884	.8484	4084	.9684	.5284	77.0884	.6484
.79	.2041	.7841	.3641	.9441	.5241	46.1041	.6841	.2641	.8441
.80	3.2400	7.8400	14.4400	23.0400	33.6400	46.2400	60.8400	77.4400	96.0400
.81	.2761	.8961	.5161	.1361	.7561	.3761	.9961	.6161	.2361
.82	.3124	.9524	.5924	.2324	.8724	.5124	61.1524	.7924	.4324
.83	.3489	8.0089	.6689	.3289	.9889	.6489	.3089	.9689	.6289
.84	.3856	.0656	.7456	.4256	34.1056	.7856	.4656	78.1456	.8256
.85	3.4225	8.1225	14.8225	23.5225	34.2225	46.9225	61.6225	78.3225	97.0225
.86	.4596	.1796	.8996	.6196	.3396	47.0596	.7796	.4996	.2196
.87	.4969	.2369	.9769	.7169	.4569	.1969	.9369	.6769	.4169
.88	.5344	.2944	15.0544	.8144	.5744	.3344	62.0944	.8544	.6144
.89	5721	.3521	.1321	.9121	.6921	.4721	.2521	79.0321	.8121
.90	3.6100	8.4100	15.2100	24.0100	34.8100	47.6100	62.4100	79.2100	98.0100
.91	.6481	.4681	.2881	.1081	.9281	.7481	.5681	.3881	.2081
.92	.6864	.5264	.3664	.2064	35.0464	.8864	.7264	.5664	.4064
.93	.7249	.5849	.4449	.3049	.1649	48.0249	.8849	.7449	.6049
.94	.7636	.6436	.5236	.4036	.2836	.1636	63.0436	.9236	.8036
.95	3.8025	8.7025	15.6025	24.5025	35.4025	48.3025	63.2025	80.1025	99.0025
.96	.8416	.7616	.6816	.6016	.5216	.4416	.3616	.2816	.2016
.97	.8809	.8209	.7609	.7009	.6409	.5809	.5209	.4609	.4009
.98	.9204	.8804	.8404	.8004	.7604	.7204	.6804	.6404	.6004
.99	.9601	.9401	.9201	.9001	.8801	.8601	.8401	.8201	.8001

x	1.	2.	3.	4.	5.	6.	7.	8.	9.
.00	1.000	8.000	27.000	64.000	125.000	216.000	343.000	512.000	729.000
.01	1.030	8.121	27.271	64.481	25.752	17.082	44.472	13.922	31.433
.02	1.061	8.242	27.544	64.965	26.506	18.167	45.948	15.850	33.871
.03	1.093	8.365	27.818	65.451	27.264	19.256	47.429	17.782	36.314
.04	1.125	8.490	28.094	65.939	28.024	20.349	48.914	19.718	38.763
.05	1.158	8.615	28.373	66.430	128.788	221.445	350.403	521.660	741.218
.06	1.191	8.742	28.653	66.923	29.554	22.545	51.896	23.607	43.677
.07	1.225	8.870	28.934	67.419	30.324	23.647	53.393	25.558	46.143
.08	1.260	8.999	29.218	67.917	31.097	24.756	54.896	27.514	48.613
.09	1.295	9.129	29.504	68.418	31.872	25.869	56.401	29.475	51.089
.10	1.331	9.261	29.791	68.921	132.651	226.981	357.911	531.441	753.571
.11	1.368	9.394	30.080	69.427	33.433	28.099	59.425	33.412	56.058
.12	1.405	9.528	30.371	69.935	34.218	29.221	60.944	35.387	58.551
.13	1.443	9.664	30.664	70.445	35.006	30.346	62.467	37.368	61.048
.14	1.482	9.800	30.959	70.958	35.797	31.476	63.994	39.353	63.552
.15	1.521	9.938	31.256	71.473	136.591	232.608	365.526	541.343	766.061
.16	1.561	10.078	31.554	71.991	37.388	33.745	67.062	43.338	68.575
.17	1.602	10.218	31.855	72.512	38.188	34.885	68.602	45.339	71.095
.18	1.643	10.360	32.157	73.035	38.992	36.029	70.146	47.343	73.621
.19	1.685	10.503	32.462	73.560	39.798	37.177	71.695	49.353	76.152
.20	1.728	10.648	32.768	74.088	140.608	238.328	373.248	551.368	778.688
.21	1.772	10.794	33.076	74.618	41.421	39.483	74.805	53.388	81.230
.22	1.816	10.941	33.386	75.151	42.237	40.642	76.367	55.412	83.777
.23	1.861	11.090	33.698	75.687	43.056	41.804	77.933	57.442	86.330
.24	1.907	11.239	34.012	76.225	43.878	42.971	79.503	59.476	88.889
.25	1.953	11.391	34.328	76.766	144.703	244.141	381.078	561.516	791.453
.26	2.000	11.543	34.646	77.309	45.532	45.314	82.657	63.560	94.022
.27	2.048	11.697	34.966	77.854	46.363	46.492	84.240	65.609	96.598
.28	2.097	11.852	35.288	78.403	47.198	47.673	85.828	67.664	99.179
.29	2.147	12.009	35.611	78.954	48.036	48.858	87.420	69.723	801.765
.30	2.197	12.167	35.937	79.507	148.877	250.047	389.017	571.787	804.357
.31	2.248	12.326	36.265	80.063	49.721	51.240	90.618	73.856	06.954
.32	2.300	12.487	36.594	80.622	50.569	52.436	92.223	75.930	09.557
.33	2.353	12.649	36.926	81.183	51.419	53.636	93.833	78.010	12.166
.34	2.406	12.813	37.260	81.747	52.273	54.840	95.447	80.093	14.780
.35	2.460	12.978	37.595	82.313	153.130	256.048	397.065	582.183	817.400
.36	2.515	13.144	37.933	82.882	53.991	57.259	98.688	84.277	20.026
.37	2.571	13.312	38.273	83.453	54.854	58.475	400.316	86.376	22.657
.38	2.628	13.481	38.614	84.028	55.721	59.694	01.947	88.480	25.294
.39	2.686	13.652	38.958	84.605	56.591	60.917	03.583	90.590	27.936
.40	2.744	13.824	39.304	85.184	157.464	262.144	405.224	592.704	830.584
.41	2.803	13.998	39.652	85.776	58.340	63.375	06.869	94.823	33.238
.42	2.863	14.172	40.002	86.351	59.220	64.609	08.518	96.948	35.897
.43	2.924	14.349	40.354	86.938	60.103	65.848	10.172	99.077	38.562
.44	2.986	14.527	40.708	87.528	60.989	67.090	11.831	601.212	41.232
.45	3.049	14.706	41.064	88.121	161.879	268.336	413.494	603.351	843.908
.46	3.112	14.887	41.422	88.717	62.771	69.586	15.161	05.496	46.590
.47	3.177	15.069	41.782	89.315	63.667	70.840	16.833	07.645	49.278
.48	3.242	15.253	42.144	89.915	64.567	72.098	18.509	09.800	51.971
.49	3.308	15.438	42.509	90.519	65.469	73.593	20.190	11.960	54.670

x.	1.	2.	3.	4.	5.	6.	7.	8.	9.
.50	3.375	15.625	42.875	91.125	166.375	274.625	421.875	614.125	857.375
.51	3.443	15.813	43.244	91.734	67.284	75.894	23.565	16.295	60.085
.52	3.512	16.003	43.614	92.345	68.197	77.168	25.259	18.470	62.801
.53	3.582	16.194	43.987	92.960	69.112	78.445	26.958	20.650	65.523
.54	3.652	16.387	44.362	93.577	70.031	79.726	28.661	22.836	68.251
.55	3.724	16.581	44.739	94.196	170.954	281.011	430.369	625.026	870.984
.56	3.796	16.777	45.118	94.819	71.880	82.300	32.081	27.222	73.723
.57	3.870	16.975	45.499	95.444	72.809	83.593	33.798	29.423	76.467
.58	3.944	17.174	45.883	96.072	73.741	84.890	35.520	31.629	79.218
.59	4.020	17.374	46.268	96.703	74.677	86.191	37.245	33.840	81.974
.60	4.096	17.576	46.656	97.336	175.616	287.496	438.976	636.056	884.736
.61	4.173	17.780	47.046	97.972	76.558	88.805	40.711	38.277	87.504
.62	4.252	17.985	47.438	98.611	77.504	90.118	42.451	40.504	90.277
.63	4.331	18.191	47.832	99.253	78.454	91.434	44.195	42.736	93.056
.64	4.411	18.400	48.229	99.897	79.406	92.755	45.944	44.973	95.841
.65	4.492	18.610	48.627	100.545	180.362	294.080	447.697	647.215	898.632
.66	4.574	18.821	49.028	01.195	81.321	95.408	49.455	49.462	901.429
.67	4.657	19.034	49.431	01.848	82.284	96.741	51.218	51.714	04.231
.68	4.742	19.249	49.836	02.503	83.250	98.078	52.985	53.972	07.039
.69	4.827	19.465	50.243	03.162	84.220	99.418	54.757	56.235	09.853
.70	4.913	19.683	50.653	103.823	185.193	300.763	456.533	658.503	912.673
.71	5.000	19.903	51.065	04.487	86.169	02.112	58.314	60.776	15.499
.72	5.088	20.124	51.479	05.154	87.149	03.464	60.100	63.055	18.330
.73	5.178	20.346	51.895	05.824	88.133	04.821	61.890	65.339	21.167
.74	5.268	20.571	52.314	06.496	89.119	06.182	63.685	67.628	24.010
.75	5.359	20.797	52.734	107.172	190.109	307.547	465.484	669.922	926.859
.76	5.452	21.025	53.157	07.850	91.103	08.916	67.289	72.221	29.714
.77	5.545	21.254	53.583	08.531	92.100	10.289	69.097	74.526	32.575
.78	5.640	21.485	54.010	09.215	93.101	11.666	70.911	76.836	35.441
.79	5.735	21.718	54.440	09.902	94.105	13.047	72.729	79.151	38.314
.80	5.832	21.952	54.872	110.592	195.112	314.432	474.552	681.472	941.192
.81	5.930	22.188	55.306	11.285	96.123	15.821	76.380	83.798	44.076
.82	6.029	22.426	55.743	11.980	97.137	17.215	78.212	86.129	46.966
.83	6.128	22.665	56.182	12.679	98.155	18.612	80.049	88.465	49.862
.84	6.230	22.906	56.623	13.380	99.177	20.014	81.890	90.807	52.764
.85	6.332	23.149	57.067	114.084	200.202	321.419	483.737	693.154	955.672
.86	6.435	23.394	57.512	14.791	01.230	22.829	85.588	95.506	58.585
.87	6.539	23.640	57.961	15.501	02.262	24.243	87.443	97.864	61.505
.88	6.645	23.888	58.411	16.214	03.297	25.661	89.304	700.227	64.430
.89	6.751	24.138	58.864	16.930	04.336	27.083	91.169	02.595	67.362
.90	6.859	24.389	59.319	117.649	205.379	328.509	493.039	704.969	970.299
.91	6.968	24.642	59.776	18.371	06.425	29.939	94.914	07.348	73.242
.92	7.078	24.897	60.236	19.095	07.475	31.374	96.793	09.732	76.191
.93	7.189	25.154	60.698	19.823	08.528	32.813	98.677	12.122	79.147
.94	7.301	25.412	61.163	20.554	09.585	34.255	500.566	14.517	82.108
.95	7.415	25.672	61.630	121.287	210.645	335.702	502.460	716.917	985.075
.96	7.530	25.934	62.099	22.024	11.709	37.154	04.358	19.323	88.048
.97	7.645	26.198	62.571	22.763	12.776	38.609	06.262	21.734	91.027
.98	7.762	26.464	63.045	23.506	13.847	40.068	08.170	24.151	94.012
.99	7.881	26.731	63.521	24.251	14.922	41.532	10.082	26.573	97.003

x.	1.	2.	3.	4.	5.	6.	7.	8.	9.
.00	1.00000	0.50000	0.33333	0.25000	0.20000	0.16667	0.14286	0.12500	0.11111
.01	0.99010	.49751	.33223	.24938	.19960	.16639	.14265	.12484	.11099
.02	.98039	.49505	.33113	.24876	.19920	.16611	.14245	.12469	.11086
.03	.97087	.49261	.33003	.24814	.19881	.16584	.14225	.12453	.11074
.04	.96154	.49020	.32895	.24752	.19841	.16556	.14205	.12438	.11062
.05	0.95238	0.48780	0.32787	0 24691	0.19802	0.16529	0.14184	0.12422	0.11050
.06	.94340	.48544	.32680	.24631	.19763	.16502	.14164	.12407	.11038
.07	.93458	.48309	.32573	.24570	.19724	.16474	.14144	.12392	.11025
.08	.92593	.48077	.32468	.24510	.19685	.16447	.14124	.12376	.11013
.09	.91743	.47847	.32362	.24450	.19646	.16420	.14104	.12361	.11001
.10	0.90909	0.47619	0.32258	0.24390	0.19608	0.16393	0.14085	0.12346	0.10989
.11	.90090	.47393	.32154	.24331	.19569	.16367	.14065	.12330	.10977
.12	.89286	.47170	.32051	.24272	.19531	.16340	.14045	.12315	.10965
.13	.88496	.46948	.31949	.24213	.19493	.16313	.14025	.12300	.10953
.14	.87719	.46729	.31847	.24155	.19455	.16287	.14006	.12285	.10941
.15	0.86957	0.46512	0.31746	0.24096	0.19417	0.16260	0.13986	0.12270	0.10929
.16	.86207	.46296	.31646	.24038	.19380	.16234	.13966	.12255	.10917
.17	.85470	.46083	.31546	.23981	.19342	.16207	.13947	.12240	.10905
.18	.84746	.45872	.31447	.23923	.19305	.16181	.13928	.12225	.10893
.19	.84034	.45662	.31348	.23866	.19268	.16155	.13908	.12210	.10881
.20	0.83333	0.45455	0.31250	0.23810	0.19231	0.16129	0.13889	0.12195	0.10870
.21	.82645	.45249	.31153	.23753	.19194	.16103	.13870	.12180	.10858
.22	.81967	.45045	.31056	.23697	.19157	.16077	.13850	.12165	.10846
.23	.81301	.44843	.30960	.23641	.19120	.16051	.13831	.12151	.10834
.24	.80645	.44643	.30864	.23585	.19084	.16026	.13812	.12136	.10823
.25	0.80000	0.44444	0.30769	0.23529	0.19048	0.16000	0.13793	0.12121	0.10811
.26	.79365	.44248	.30675	.23474	.19011	.15974	.13774	.12107	.10799
.27	.78740	.44053	.30581	.23419	.18975	.15949	.13755	.12092	.10787
.28	.78125	.43860	.30488	.23364	.18939	.15924	.13736	.12077	.10776
.29	.77519	.43668	.30395	.23310	.18904	.15898	.13717	.12063	.10764
.30	0.76923	0.43478	0.30303	0.23256	0.18868	0.15873	0.13699	0.12048	0.10753
.31	.76336	.43290	.30211	.23202	.18832	.15848	.13680	.12034	.10741
.32	.75758	.43103	.30120	.23148	.18797	.15823	.13661	.12019	.10730
.33	.75188	.42918	.30030	.23095	.18762	.15798	.13643	.12005	.10718
.34	.74627	.42735	.29940	.23041	.18727	.15773	.13624	.11990	.10707
.35	0.74074	0.42553	0.29851	0.22989	0.18692	0.15748	0.13605	0.11976	0.10695
.36	.73529	.42373	.29762	.22936	.18657	.15723	.13587	.11962	.10684
.37	.72993	.42194	.29674	.22883	.18622	.15699	.13569	.11947	.10672
.38	.72464	.42017	.29586	.22831	.18587	.15674	.13550	.11933	.10661
.39	.71942	.41841	.29499	.22779	.18553	.15649	.13532	.11919	.10650
.40	0.71429	0.41667	0.29412	0.22727	0.18519	0.15625	0.13514	0.11905	0.10638
.41	.70922	.41494	.29326	.22676	.18484	.15601	.13495	.11891	.10627
.42	.70423	.41322	.29240	.22624	.18450	.15576	.13477	.11876	.10616
.43	.69930	.41152	.29155	.22573	.18416	.15552	.13459	.11862	.10604
.44	.69444	.40984	.29070	.22523	.18382	.15528	.13441	.11848	.10593
.45	0.68966	0.40816	0.28986	0.22472	0.18349	0.15504	0.13423	0.11834	0.10582
.46	.68493	.40650	.28902	.22422	.18315	.15480	.13405	.11820	.10571
.47	.68027	.40486	.28818	.22371	.18282	.15456	.13387	.11806	.10560
.48	.67568	.40323	.28736	.22321	.18248	.15432	.13369	.11792	.10549
.49	.67114	.40161	.28653	.22272	.18215	.15408	.13351	.11779	.10537

x.	1.	2.	3.	4.	5.	6.	7.	8.	9.
.50	0.66667	0.40000	0.28571	0.22222	0.18182	0.15385	0.13333	0.11765	0.10526
.51	.66225	.39841	.28490	.22173	.18149	.15361	.13316	.11751	.10515
.52	.65789	.39683	.28409	.22124	.18116	.15337	.13298	.11737	.10504
.53	.65359	.39526	.28329	.22075	.18083	.15314	.13280	.11723	.10493
.54	.64935	.39370	.28249	.22026	.18051	.15291	.13263	.11710	.10482
.55	0.64516	0.39216	0.28169	0.21978	0.18018	0.15267	0.13245	0.11696	0.10471
.56	.64103	.39062	.28090	.21930	.17986	.15244	.13228	.11682	.10460
.57	.63694	.38911	.28011	.21882	.17953	.15221	.13210	.11669	.10449
.58	.63291	.38760	.27933	.21834	.17921	.15198	.13193	.11655	.10438
.59	.62893	.38610	.27855	.21786	.17889	.15175	.13175	.11641	.10428
.60	0.62500	0.38462	0.27778	0.21739	0.17857	0.15152	0.13158	0.11628	0.10417
.61	.62112	.38314	.27701	.21692	.17825	.15129	.13141	.11614	.10406
.62	.61728	.38168	.27624	.21645	.17794	.15106	.13123	.11601	.10395
.63	.61350	.38023	.27548	.21598	.17762	.15083	.13106	.11587	.10384
.64	.60976	.37879	.27473	.21552	.17730	.15060	.13089	.11574	.10373
.65	0.60606	0.37736	0.27397	0.21505	0.17699	0.15038	0.13072	0.11561	0.10363
.66	.60241	.37594	.27322	.21459	.17668	.15015	.13055	.11547	.10352
.67	.59880	.37453	.27248	.21413	.17637	.14992	.13038	.11534	.10341
.68	.59524	.37313	.27174	.21368	.17606	.14970	.13021	.11521	.10331
.69	.59172	.37175	.27100	.21322	.17575	.14948	.13004	.11507	.10320
.70	0.58824	0.37037	0.27027	0.21277	0.17544	0.14925	0.12987	0.11494	0.10309
.71	.58480	.36900	.26954	.21231	.17513	.14903	.12970	.11481	.10299
.72	.58140	.36765	.26882	.21186	.17483	.14881	.12953	.11468	.10288
.73	.57803	.36630	.26810	.21142	.17452	.14859	.12937	.11455	.10277
.74	.57471	.36496	.26738	.21097	.17422	.14837	.12920	.11442	.10267
.75	0.57143	0.36364	0.26667	0.21053	0.17391	0.14815	0.12903	0.11429	0.10256
.76	.56818	.36232	.26596	.21008	.17361	.14793	.12887	.11416	.10246
.77	.56497	.36101	.26525	.20964	.17331	.14771	.12870	.11403	.10235
.78	.56180	.35971	.26455	.20921	.17301	.14749	.12853	.11390	.10225
.79	.55866	.35842	.26385	.20877	.17271	.14728	.12837	.11377	.10215
.80	0.55556	0.35714	0.26316	0.20833	0.17241	0.14706	0.12821	0.11364	0.10204
.81	.55249	.35587	.26247	.20790	.17212	.14684	.12804	.11351	.10194
.82	.54945	.35461	.26178	.20747	.17182	.14663	.12788	.11338	.10183
.83	.54645	.35336	.26110	.20704	.17153	.14641	.12771	.11325	.10173
.84	.54348	.35211	.26042	.20661	.17123	.14620	.12755	.11312	.10163
.85	0.54054	0.35088	0.25974	0.20619	0.17094	0.14599	0.12739	0.11299	0.10152
.86	.53763	.34965	.25907	.20576	.17065	.14577	.12723	.11287	.10142
.87	.53476	.34843	.25840	.20534	.17036	.14556	.12706	.11274	.10132
.88	.53191	.34722	.25773	.20492	.17007	.14535	.12690	.11261	.10121
.89	.52910	.34602	.25707	.20450	.16978	.14514	.12674	.11249	.10111
.90	0.52632	0.34483	0.25641	0.20408	0.16949	0.14493	0.12658	0.11236	0.10101
.91	.52356	.34364	.25575	.20367	.16920	.14472	.12642	.11223	.10091
.92	.52083	.34247	.25510	.20325	.16892	.14451	.12626	.11211	.10081
.93	.51813	.34130	.25445	.20284	.16863	.14430	.12610	.11198	.10070
.94	.51546	.34014	.25381	.20243	.16835	.14409	.12594	.11186	.10060
.95	0.51282	0.33898	0.25316	0.20202	0.16807	0.14388	0.12579	0.11173	0.10050
.96	.51020	.33784	.25253	.20161	.16779	.14368	.12563	.11161	.10040
.97	.50761	.33670	.25189	.20121	.16750	.14347	.12547	.11148	.10030
.98	.50505	.33557	.25126	.20080	.16722	.14327	.12531	.11136	.10020
.99	.50251	.33445	.25063	.20040	.16694	.14306	.12516	.11123	.10010

4. Powers. $y = x^n$.

x	x^{-2}	x^{-3}	$x^{1/2}$	$x^{1/3}$	$x^{-1/2}$	$x^{-1/3}$	x^4	x^5	$\frac{x}{10-x}$	$\left(\frac{x}{10-x}\right)^2$
0.0	∞	∞	0.0000	0.0000	∞	∞	0.0000	.00000	.00000	0.0000
0.1	100.00	1000.0	.3162	.4642	3.1623	2.1544	.0001	.00001	.01010	.0001
0.2	25.00	125.0	.4472	.5848	2.2361	1.7100	.0016	.00032	.02041	.0004
0.3	11.11	37.0	.5477	.6694	1.8291	1.4938	.0081	.00243	.03093	.0009
0.4	6.25	15.6	.6325	.7368	1.5811	1.3572	.0256	.01024	.04167	.0017
0.5	4.0000	8.0000	0.7071	0.7937	1.4142	1.2599	0.0625	.03125	.05263	0.0028
0.6	2.7778	4.6296	.7746	.8434	1.2910	1.1856	.1296	.07776	.06383	.0041
0.7	2.0408	2.9155	.8367	.8879	1.1952	1.1262	.2401	.16807	.07527	.0057
0.8	1.5625	1.9531	.8944	.9283	1.1180	1.0772	.4096	.32768	.08696	.0076
0.9	1.2346	1.3717	.9487	.9655	1.0541	1.0357	.6561	.59049	.09890	.0098
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.11111	0.0123
1.1	0.8264	0.7570	.0488	.0323	0.9535	.9687	1.4641	1.6105	.12360	.0153
1.2	.6944	.5787	.0954	.0627	.9129	.9410	2.0736	2.4883	.13636	.0186
1.3	.5917	.4552	.1402	.0914	.8771	.9163	2.8561	3.7129	.14943	.0223
1.4	.5105	.3644	.1832	.1187	.8452	.8939	3.8416	5.3782	.16279	.0265
1.5	0.4444	0.2963	1.2247	1.1447	0.8165	0.8736	5.062	7.594	.17647	0.0311
1.6	.3906	.2441	.2649	.1696	.7906	.8550	6.554	10.486	.19048	.0363
1.7	.3460	.2035	.3038	.1935	.7670	.8379	8.352	14.199	.20482	.0420
1.8	.3086	.1715	.3416	.2164	.7454	.8221	10.498	18.896	.21951	.0482
1.9	.2770	.1458	.3784	.2386	.7255	.8074	13.032	24.761	.23457	.0550
2.0	0.2500	0.1250	1.4142	1.2599	0.7071	0.7937	16.000	32.000	.25000	0.0625
2.1	.2268	.1080	.4491	.2806	.6901	.7809	19.448	40.841	.26582	.0707
2.2	.2066	.0939	.4832	.3006	.6742	.7689	23.426	51.536	.28205	.0796
2.3	.1890	.0822	.5166	.3200	.6594	.7576	27.984	64.363	.29870	.0892
2.4	.1736	.0723	.5492	.3389	.6455	.7469	33.178	79.626	.31579	.0997
2.5	0.1600	0.0640	1.5811	1.3572	0.6325	0.7368	39.062	97.66	.33333	0.1111
2.6	.1479	.0569	.6125	.3751	.6201	.7272	45.698	118.81	.35135	.1234
2.7	.1372	.0508	.6432	.3925	.6086	.7181	53.144	143.49	.36986	.1368
2.8	.1276	.0456	.6733	.4095	.5976	.7095	61.466	172.10	.38889	.1512
2.9	.1189	.0410	.7029	.4260	.5872	.7012	70.728	205.11	.40845	.1668
3.0	0.1111	0.0370	1.7321	1.4422	0.5774	0.6934	81.00	243.00	.42857	0.1837
3.1	.1041	.0336	.7607	.4581	.5679	.6858	92.35	286.29	.44928	.2019
3.2	.0977	.0305	.7889	.4736	.5592	.6786	104.86	335.54	.47059	.2215
3.3	.0918	.0278	.8166	.4888	.5505	.6717	118.59	391.35	.49254	.2426
3.4	.0865	.0254	8439	.5037	.5423	.6650	133.63	454.35	.51515	.2654
3.5	0.0816	0.0233	1.8708	1.5183	0.5345	0.6586	150.06	525.22	.53846	0.2899
3.6	.0772	.0214	.8974	.5326	.5270	.6525	167.96	604.66	.56250	.3164
3.7	.0730	.0197	.9235	.5467	.5199	.6466	187.42	693.44	.58730	.3449
3.8	.0693	.0182	.9494	.5605	.5130	.6408	208.51	792.35	.61290	.3756
3.9	.0657	.0169	.9748	.5741	.5064	.6353	231.34	902.24	.63934	.4088
4.0	0.0625	0.0156	2.0000	1.5874	0.5000	0.6300	256.00	1024.0	.66667	0.4444
4.1	.0595	.0145	.0248	.6005	.4939	.6248	282.57	1158.6	.69492	.4829
4.2	.0567	.0135	.0494	.6134	.4880	.6198	311.17	1306.9	.72414	.5244
4.3	.0541	.0126	.0736	.6261	.4822	.6150	341.88	1470.1	.75439	.5691
4.4	.0517	.0117	.0976	.6386	.4767	.6103	374.81	1649.2	.78571	.6175
4.5	0.0494	0.0110	2.1213	1.6510	0.4714	0.6057	410.06	1845.3	.81818	0.6694
4.6	.0473	.0103	.1448	.6631	.4662	.6013	447.75	2059.6	.85185	.7256
4.7	.0453	.0096	.1679	.6751	.4613	.5970	487.97	2293.5	.88679	.7864
4.8	.0434	.0090	.1909	.6869	.4564	.5928	530.84	2548.0	.92308	.8521
4.9	.0416	.0085	.2136	.6985	.4517	.5888	576.48	2824.8	.96078	.9231

4. Powers. $y = x^n$.

x	x^{-2}	x^{-3}	$x^{1/2}$	$x^{1/3}$	$x^{-1/2}$	$x^{-1/3}$	x^4	x^5	$\frac{x}{10-x}$	$\left(\frac{x}{10-x}\right)^2$
5.0	.04000	.00800	2.2361	1.7100	0.4472	0.5848	625.00	3125.0	1.0000	1.000
5.1	3845	0754	.2583	.7213	.4428	.5810	676.52	3450.3	.0408	.083
5.2	3698	0711	.2804	.7325	.4385	.5772	731.16	3802.0	.0833	.174
5.3	3560	0672	.3022	.7435	.4344	.5735	789.05	4182.0	.1277	.272
5.4	3429	0635	.3238	.7544	.4303	.5695	850.31	4591.7	.1739	.378
5.5	.03306	.00601	2.3452	1.7652	0.4264	0.5665	915.1	5032.8	1.2222	1.494
5.6	3189	0569	.3664	.7758	.4226	.5631	983.4	5507.3	.2727	.620
5.7	3078	0540	.3875	.7863	.4189	.5598	1055.6	6016.9	.3256	.757
5.8	2973	0513	.4083	.7967	.4152	.5566	1131.6	6563.6	.3810	.907
5.9	2873	0487	.4290	.8070	.4117	.5534	1211.7	7149.2	.4390	2.071
6.0	.02778	.00463	2.4495	1.8171	0.4082	0.5503	1296.0	7776	1.5000	2.250
6.1	2687	0441	.4698	.8272	.4049	.5473	1384.6	8446	.5641	.446
6.2	2601	0420	.4900	.8371	.4016	.5443	1477.6	9161	.6316	.662
6.3	2520	0400	.5100	.8469	.3984	.5414	1575.3	9924	.7027	.889
6.4	2441	0381	.5298	.8566	.3953	.5386	1677.7	10737	.7778	3.160
6.5	.02367	.00364	2.5495	1.8663	0.3922	0.5358	1785.1	11603	1.8571	3.449
6.6	2296	0348	.5690	.8758	.3892	.5331	1897.5	12523	.9412	3.768
6.7	2228	0332	.5884	.8852	.3863	.5304	2015.1	13501	2.0303	4.122
6.8	2163	0318	.6077	.8945	.3835	.5278	2138.1	14539	.1250	4.516
6.9	2100	0304	.6268	.9038	.3807	.5253	2266.7	15640	.2258	4.954
7.0	.02041	.00292	2.6458	1.9129	0.3780	0.5228	2401.0	16807	2.3333	5.444
7.1	1984	0279	.6646	.9220	.3753	.5203	2541.2	18042	.4483	5.994
7.2	1929	0268	.6833	.9310	.3727	.5179	2687.4	19349	.5714	6.612
7.3	1877	0257	.7019	.9399	.3701	.5155	2839.8	20731	.7037	7.310
7.4	1826	0247	.7203	.9487	.3675	.5132	2998.7	22190	.8462	8.101
7.5	.01778	.00237	2.7386	1.9574	0.3651	0.5109	3164.1	23730	3.0000	9.00
7.6	1731	0228	.7568	.9661	.3627	.5086	3336.2	25355	.1667	10.03
7.7	1687	0219	.7749	.9747	.3604	.5064	3515.3	27068	.3478	11.21
7.8	1644	0211	.7928	.9832	.3581	.5042	3701.5	28872	.5455	12.57
7.9	1602	0203	.8107	.9916	.3558	.5021	3895.0	30771	.7619	14.15
8.0	.01562	.00195	2.8284	2.0000	0.3536	0.5000	4096.0	32768	4.0000	16.00
8.1	1524	0188	.8460	.0083	.3514	.4979	4304.7	34868	4.2632	18.17
8.2	1487	0181	.8636	.0165	.3492	.4959	4521.2	37074	4.5556	20.75
8.3	1452	0175	.8810	.0247	.3471	.4939	4745.8	39390	4.8824	23.84
8.4	1417	0169	.8983	.0328	.3450	.4919	4978.7	41821	5.2500	27.56
8.5	.01384	.00163	2.9155	2.0408	0.3430	0.4900	5220.1	44371	5.6667	32.11
8.6	1352	0157	.9326	.0488	.3410	.4881	5470.1	47043	6.1428	37.73
8.7	1321	0152	.9496	.0567	.3390	.4862	5729.0	49842	6.6923	44.79
8.8	1291	0147	.9665	.0646	.3371	.4844	5997.0	52773	7.3333	53.78
8.9	1262	0142	.9833	.0724	.3352	.4826	6274.2	55841	8.0909	65.46
9.0	.01235	.00137	3.0000	2.0801	0.3333	0.4808	6561.0	59049	9.000	81.0
9.1	1208	0133	.0166	.0878	.3315	.4790	6857.5	62403	10.111	102.2
9.2	1181	0129	.0332	.0954	.3297	.4772	7163.9	65908	11.500	132.2
9.3	1156	0124	.0496	.1029	.3279	.4755	7480.5	69569	13.286	176.5
9.4	1132	0120	.0659	.1105	.3262	.4738	7807.5	73390	15.667	245.4
9.5	.01108	.00117	.0822	2.1179	0.3244	0.4722	8145.1	77378	19.000	361.0
9.6	1085	0113	.0984	.1253	.3227	.4705	8493.5	81537	24.000	576.0
9.7	1063	0110	.1145	.1327	.3211	.4689	8852.9	85873	32.333	1045.0
9.8	1041	0106	.1305	.1400	.3194	.4673	9223.7	90392	49.000	2401.0
9.9	1020	0103	.1464	.1472	.3178	.4657	9606.0	95099	99.000	9801.0

x.	1.	2.	3.	4.	5.	6.	7.	8.	9.	N.L10x
.00	0.0000	0.3010	0.4771	0.6021	0.6990	0.7782	0.8451	0.9031	0.9542	— infin.
.01	.0043	.3032	.4786	.6031	.6998	.7789	.8457	.9036	.9547	7.6974
.02	.0086	.3054	.4800	.6042	.7007	.7796	.8463	.9042	.9552	8.3906
.03	.0128	.3075	.4814	.6053	.7016	.7803	.8470	.9047	.9557	8.7960
.04	.0170	.3096	.4829	.6064	.7024	.7810	.8476	.9053	.9562	9.0837
.05	0.0212	0.3118	0.4843	0.6075	0.7033	0.7818	0.8482	0.9058	0.9566	9.3069
.06	.0253	.3139	.4857	.6085	.7042	.7825	.8488	.9063	.9571	.4892
.07	.0294	.3160	.4871	.6096	.7050	.7832	.8494	.9069	.9576	.6433
.08	.0334	.3181	.4886	.6107	.7059	.7839	.8500	.9074	.9581	.7769
.09	.0374	.3201	.4900	.6117	.7067	.7846	.8506	.9079	.9586	.8946
.10	0.0414	0.3222	0.4914	0.6128	0.7076	0.7853	0.8513	0.9085	0.9590	0.0000
.11	.0453	.3243	.4928	.6138	.7084	.7860	.8519	.9090	.9595	.0953
.12	.0492	.3263	.4942	.6149	.7093	.7868	.8525	.9096	.9600	.1823
.13	.0531	.3284	.4955	.6160	.7101	.7875	.8531	.9101	.9605	.2624
.14	.0569	.3304	.4969	.6170	.7110	.7882	.8537	.9106	.9609	.3365
.15	0.0607	0.3324	0.4983	0.6180	0.7118	0.7889	0.8543	0.9112	0.9614	0.4055
.16	.0645	.3345	.4997	.6191	.7126	.7896	.8549	.9117	.9619	.4700
.17	.0682	.3365	.5011	.6201	.7135	.7903	.8555	9122	.9624	.5306
.18	.0719	.3385	.5024	.6212	.7143	.7910	.8561	.9128	.9628	.5878
.19	.0755	.3404	.5038	.6222	.7152	.7917	.8567	.9133	.9633	.6418
.20	0.0792	0.3424	0.5051	0.6232	0.7160	0.7924	0.8573	0.9138	0.9638	0.6931
.21	.0828	.3444	.5065	.6243	.7168	.7931	.8579	.9143	.9643	.7419
.22	.0864	.3464	.5079	.6253	.7177	.7938	.8585	.9149	.9647	.7885
.23	.0899	.3483	.5092	.6263	.7185	.7945	.8591	.9154	.9652	.8339
.24	.0934	.3502	.5105	.6274	.7193	.7952	.8597	.9159	.9657	.8755
.25	0.0969	0.3522	0.5119	0.6284	0.7202	0.7959	0.8603	0.9165	0.9661	0.9163
.26	.1004	.3541	.5132	.6294	.7210	.7966	.8609	.9170	.9666	.9555
.27	.1038	.3560	.5145	.6304	.7218	.7973	.8615	.9175	.9671	.9933
.28	.1072	.3579	.5159	.6314	.7226	.7980	.8621	.9180	.9675	1.0296
.29	.1106	.3598	.5172	.6325	.7235	.7987	.8627	.9186	.9680	.0647
.30	0.1139	0.3617	0.5185	0.6335	0.7243	0.7993	0.8633	0.9191	0.9685	1.0986
.31	.1173	.3636	.5198	.6345	.7251	.8000	.8639	.9196	.9689	.1314
.32	.1206	.3655	.5211	.6355	.7259	.8007	.8645	.9201	.9694	.1632
.33	.1239	.3674	.5224	.6365	.7267	.8014	.8651	.9206	.9699	.1939
.34	.1271	.3692	.5237	.6375	.7275	.8021	.8657	.9212	.9703	.2238
.35	0.1303	0.3711	0.5250	0.6385	0.7284	0.8028	0.8663	0.9217	0.9708	1.2528
.36	.1335	.3729	.5263	.6395	.7292	.8035	.8669	.9222	.9713	.2809
.37	.1367	.3747	.5276	.6405	.7300	.8041	.8675	.9227	.9717	.3083
.38	.1399	.3766	.5289	.6415	.7308	.8048	.8681	.9232	.9722	.3350
.39	.1430	.3784	.5302	.6425	.7316	.8055	.8686	.9238	.9727	..3610
.40	0.1461	0.3802	0.5315	0.6435	0.7324	0.8062	0.8692	0.9243	0.9731	1.3863
.41	.1492	.3820	.5328	.6444	.7332	.8069	.8698	.9248	.9736	.4110
.42	.1523	.3838	.5340	.6454	.7340	.8075	.8704	.9253	.9741	.4351
.43	.1553	.3856	.5353	.6464	.7348	.8082	.8710	.9258	.9745	.4586
.44	:1584	.3874	.5366	.6474	.7356	.8089	.8716	.9263	.9750	.4816
.45	0.1614	0.3892	0.5378	0.6484	0.7364	0.8096	0.8722	0.9269	0.9754	1.5041
.46	.1644	.3909	.5391	.6493	.7372	.8102	.8727	.9274	.9759	.5261
.47	.1673	.3927	.5403	.6503	.7380	.8109	.8733	.9279	.9763	.5476
.48	.1703	.3945	.5416	.6513	.7388	.8116	.8739	.9284	.9768	.5686
.49	.1732	.3962	.5428	.6522	.7396	.8122	.8745	.9289	.9773	.5892

x.	1.	2.	3.	4.	5.	6.	7.	8.	9.	N.L10x
.50	0.1761	0.3979	0.5441	0.6532	0.7404	0.8129	0.8751	0.9294	0.9777	1.6094
.51	.1790	.3997	.5453	.6542	.7412	.8136	.8756	.9299	.9782	.6292
.52	.1818	.4014	.5465	.6551	.7419	.8142	.8762	.9304	.9786	.6487
.53	.1847	.4031	.5478	.6561	.7427	.8149	.8768	.9309	.9791	.6677
.54	.1875	.4048	.5490	.6571	.7435	.8156	.8774	.9315	.9795	.6864
.55	0.1903	0.4065	0.5502	0.6580	0.7443	0.8162	0.8779	0.9320	0.9800	1.7047
.56	.1931	.4082	.5514	.6590	.7451	.8169	.8785	.9325	.9805	.7228
.57	.1959	.4099	.5527	.6599	.7459	.8176	.8791	.6330	.9809	.7405
.58	.1987	.4116	.5539	.6609	.7466	.8182	.8797	.9335	.9814	.7579
.59	.2014	.4133	.5551	.6618	.7474	.8189	.8802	.9340	.9818	.7750
.60	0.2041	0.4150	0.5563	0.6628	0.7482	0.8195	0.8808	0.9345	0.9823	1.7918
.61	.2068	.4166	.5575	.6637	.7490	.8202	.8814	.9350	.9827	.8083
.62	.2095	.4183	.5587	.6646	.7497	.8209	.8820	.9355	.9832	.8245
.63	.2122	.4200	.5599	.6656	.7505	.8215	.8825	.9360	.9836	.8405
.64	.2148	.4216	.5611	.6665	.7513	.8222	.8831	.9365	.9841	.8563
.65	0.2175	0.4232	0.5623	0.6675	0.7520	0.8228	0.8837	0.9370	0.9845	1.8718
.66	.2201	.4249	.5635	.6684	.7528	.8235	.8842	.9375	.9850	.8871
.67	.2227	.4265	.5647	.6693	.7536	.8241	.8848	.9380	.9854	.9021
.68	.2253	.4281	.5658	.6702	.7543	.8248	.8854	.9385	.9859	.9169
.69	.2279	.4298	.5670	.6712	.7551	.8254	.8859	.9390	.9863	.9315
.70	0.2304	0.4314	0.5682	0.6721	0.7559	0.8261	0.8865	0.9395	0.9868	1.9459
.71	.2330	.4330	.5694	.6730	.7566	.8267	.8871	.9400	.9872	.9601
.72	.2355	.4346	.5705	.6739	.7574	.8274	.8876	.9405	.9877	.9741
.73	.2380	.4362	.5717	.6749	.7582	.8280	.8882	.9410	.9881	.9879
.74	.2405	.4378	.5729	.6758	.7589	.8287	.8887	.9415	.9886	2.0015
.75	0.2430	0.4393	0.5740	0.6767	0.7597	0.8293	0.8893	0.9420	0.9890	2.0149
.76	.2455	.4409	.5752	.6776	.7604	.8299	.8899	.9425	.9894	.0281
.77	.2480	.4425	.5763	.6785	.7612	.8306	.8904	.9430	.9899	.0412
.78	.2504	.4440	.5775	.6794	.7619	.8312	.8910	.9435	.9903	.0541
.79	.2529	.4456	.5786	.6803	.7627	.8319	.8915	.9440	.9908	.0669
.80	0.2553	0.4472	0.5798	0.6812	0.7634	0.8325	0.8921	0.9445	0.9912	2.0794
.81	.2577	.4487	.5809	.6821	.7642	.8331	.8927	.9450	.9917	.0919
.82	.2601	.4502	.5821	.6830	.7649	.8338	.8932	.9455	.9921	.1041
.83	.2625	.4518	.5832	.6839	.7657	.8344	.8938	.9460	.9926	.1163
.84	.2648	.4533	.5843	.6848	.7664	.8351	.8943	.9465	.9930	.1282
.85	0.2672	0.4548	0.5855	0.6857	0.7672	0.8357	0.8949	0.9469	0.9934	2.1401
.86	.2695	.4564	.5866	.6866	.7679	.8363	.8954	.9474	.9939	.1518
.87	.2718	.4579	.5877	.6875	.7686	.8370	.8960	.9479	.9943	.1633
.88	.2742	.4594	.5888	.6884	.7694	.8376	.8965	.9484	.9948	.1748
.89	.2765	.4609	.5900	.6893	.7701	.8382	.8971	.9489	.9952	.1861
.90	0.2788	0.4624	0.5911	0.6902	0.7709	0.8388	0.8976	0.9494	0.9956	2.1972
.91	.2810	.4639	.5922	.6911	.7716	.8395	.8982	.9499	.9961	.2083
.92	.2833	.4654	.5933	.6920	.7723	.8401	.8987	.9504	.9965	.2192
.93	.2856	.4669	.5944	.6928	.7731	.8407	.8993	.9509	.9969	.2300
.94	.2878	.4683	.5955	.6937	.7738	.8414	.8998	.9513	.9974	.2407
.95	0.2900	0.4698	0.5966	0.6946	0.7745	0.8420	0.9004	0.9518	0.9978	2.2513
.96	.2923	.4713	.5977	.6955	.7752	.8426	.9009	.9523	.9983	.2618
.97	.2945	.4728	.5988	.6964	.7760	.8432	.9015	.9528	.9987	.2721
.98	.2967	.4742	.5999	.6972	.7767	.8439	.9020	.9533	.9991	.2824
.99	.2989	.4757	.6010	.6981	.7774	.8445	.9025	.9538	.9996	.2925

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
0.0	0.0000	0.1736	0.3420	0.5000	0.6428	0.7660	0.8660	0.9397	0.9848	
0.1	.0017	.1754	.3437	.5015	.6441	.7672	.8669	.9403	.9851	9.9
0.2	.0035	.1771	.3453	.5030	.6455	.7683	.8678	.9409	.9854	9.8
0.3	.0052	.1788	.3469	.5045	.6468	.7694	.8686	.9415	.9857	9.7
0.4	.0070	.1805	.3486	.5060	.6481	.7705	.8695	.9421	.9860	9.6
0.5	0.0087	0.1822	0.3502	0.5075	0.6494	0.7716	0.8704	0.9426	0.9863	9.5
0.6	.0105	.1840	.3518	.5090	.6508	.7727	.8712	.9432	.9866	9.4
0.7	.0122	.1857	.3535	.5105	.6521	.7738	.8721	.9438	.9869	9.3
0.8	.0140	.1874	.3551	.5120	.6534	.7749	.8729	.9444	.9871	9.2
0.9	.0157	.1891	.3567	.5135	.6547	.7760	.8738	.9449	.9874	9.1
1.0	0.0175	0.1908	0.3584	0.5150	0.6561	0.7771	0.8746	0.9455	0.9877	9.0
1.1	.0192	.1925	.3600	.5195	.6574	.7782	.8755	.9461	.9880	8.9
1.2	.0209	.1942	.3616	.5180	.6587	.7793	.8763	.9466	.9882	8.8
1.3	.0227	.1959	.3633	.5165	.6600	.7804	.8771	.9472	.9885	8.7
1.4	.0244	.1977	.3649	.5210	.6613	.7815	.8780	.9478	.9888	8.6
1.5	0.0262	0.1994	0.3665	0.5225	0.6626	0.7826	0.8788	0.9483	0.9890	8.5
1.6	.0279	.2011	.3681	.5240	.6639	.7837	.8796	.9489	.9893	8.4
1.7	.0297	.2028	.3697	.5255	.6652	.7848	.8805	.9494	.9895	8.3
1.8	.0314	.2045	.3714	.5270	.6665	.7859	.8813	.9500	.9898	8.2
1.9	.0332	.2062	.3730	.5284	.6678	.7869	.8821	.9505	.9900	8.1
2.0	0.0349	0.2079	0.3746	0.5299	0.6691	0.7880	0.8829	.9511	0.9903	8.0
2.1	.0366	.2096	.3762	.5314	.6704	.7891	.8838	.9516	.9905	7.9
2.2	.0384	.2113	.3778	.5329	.6717	.7902	.8846	.9521	.9907	7.8
2.3	.0401	.2130	.3795	.5344	.6730	.7912	.8854	.9527	.9910	7.7
2.4	.0419	.2147	.3811	.5358	.6743	.7923	.8862	.9532	.9912	7.6
2.5	0.0436	0.2164	0.3827	0.5373	0.6756	0.7934	0.8870	0.9537	0.9914	7.5
2.6	.0454	.2181	.3843	.5388	.6769	.7944	.8878	.9542	.9917	7.4
2.7	.0471	.2198	.3859	.5402	.6782	.7955	.8886	.9548	.9919	7.3
2.8	.0488	.2215	.3875	.5417	.6794	.7965	.8894	.9553	.9921	7.2
2.9	.0506	.2233	.3891	.5432	.6807	.7976	.8902	.9558	.9923	7.1
3.0	0.0523	0.2250	c.3907	0.5446	0.6820	0.7986	0.8910	0.9563	0.9925	7.0
3.1	.0541	.2267	.3923	.5461	.6833	.7997	.8918	.9568	.9928	6.9
3.2	.0558	.2284	.3939	.5476	.6845	.8007	.8926	.9573	.9930	6.8
3.3	.0576	.2300	.3955	.5490	.6858	.8018	.8934	.9578	.9932	6.7
3.4	.0593	.2317	.3971	.5505	.6871	.8028	.8942	.9583	.9934	6.6
3.5	0.0610	0.2334	0.3987	0.5519	0.6884	0.8039	0.8949	0.9588	0.9936	6.5
3.6	.0628	.2351	.4003	.5534	.6896	.8049	.8957	.9593	.9938	6.4
3.7	.0645	.2368	.4019	.5548	.6909	.8059	.8965	.9598	.9940	6.3
3.8	.0663	.2385	.4035	.5563	.6921	.8070	.8973	.9603	.9942	6.2
3.9	.0680	.2402	.4051	.5577	.6934	.8080	.8980	.9608	.9943	6.1
4.0	0.0698	0.2419	0.4067	0.5592	0.6947	0.8090	0.8988	0.9613	0.9945	6.0
4.1	.0715	.2436	.4083	.5606	.6959	.8100	.8996	.9617	.9947	5.9
4.2	.0732	.2453	.4099	.5621	.6972	.8111	.9003	.9622	.9949	5.8
4.3	.0750	.2470	.4115	.5635	.6984	.8121	.9011	.9627	.9951	5.7
4.4	.0767	.2487	.4131	.5650	.6997	.8131	.9018	.9632	.9952	5.6
4.5	0.0785	0.2504	0.4147	0.5664	0.7009	0.8141	0.9026	0.9636	0.9954	5.5
4.6	.0802	.2521	.4163	.5678	.7022	.8151	.9033	.9641	.9956	5.4
4.7	.0819	.2538	.4179	.5693	.7034	.8161	.9041	.9646	.9957	5.3
4.8	.0837	.2554	.4195	.5707	.7046	.8171	.9048	.9650	.9959	5.2
4.9	.0854	.2571	.4210	.5721	.7059	.8181	.9056	.9655	.9960	5.1
5.0	.0872	.2588	.4226	.5736	.7071	.8192	.9063	9659	.9962	5.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z.

6. Natural Cosines. y = cos z.

x.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
5.0	0.0872	0.2588	0.4226	0.5736	0.7071	0.8192	0.9063	0.9659	0.9962	5.0
5.1	.0889	.2605	.4242	.5750	.7083	.8202	.9070	.9664	.9963	4.9
5.2	.0906	.2622	.4258	.5764	.7096	.8211	.9078	.9668	.9965	4.8
5.3	.0924	.2639	.4274	.5779	.7108	.8221	.9085	.9673	.9966	4.7
5.4	.0941	.2656	.4289	.5793	.7120	.8231	.9092	.9677	.9968	4.6
5.5	0.0958	0.2672	0.4305	0.5807	0.7133	0.8241	0.9100	0.9681	0.9969	4.5
5.6	.0976	.2689	.4321	.5821	.7145	.8251	.9107	.9686	.9971	4.4
5.7	.0993	.2706	.4337	.5835	.7157	.8261	.9114	.9690	.9972	4.3
5.8	.1011	.2723	.4352	.5850	.7169	.8271	.9121	.9694	.9973	4.2
5.9	.1028	.2740	.4368	.5864	.7181	.8281	.9128	.9699	.9974	4.1
6.0	0.1045	0.2756	0.4384	0.5878	0.7193	0.8290	0.9135	0.9703	0.9976	4.0
6.1	.1063	.2773	.4399	.5892	.7206	.8300	.9143	.9707	.9977	3.9
6.2	.1080	.2790	.4415	.5906	.7218	.8310	.9150	.9711	.9978	3.8
6.3	.1097	.2807	.4431	.5920	.7230	.8320	.9157	.9715	.9979	3.7
6.4	.1115	.2823	.4446	.5934	.7242	.8329	.9164	.9720	.9980	3.6
6.5	0.1132	0.2840	0.4462	0.5948	0.7254	0.8339	0.9171	0.9724	0.9981	3.5
6.6	.1149	.2857	.4478	.5962	.7266	.8348	.9178	.9728	.9982	3.4
6.7	.1167	.2874	.4493	.5976	.7278	.8358	.9184	.9732	.9983	3.3
6.8	.1184	.2890	.4509	.5990	.7290	.8368	.9191	.9736	.9984	3.2
6.9	.1201	.2907	.4524	.6004	.7302	.8377	.9198	.9740	.9985	3.1
7.0	0.1219	0.2924	0.4540	0.6018	0.7314	0.8387	0.9205	0.9744	0.9986	3.0
7.1	.1236	.2940	.4555	.6032	.7325	.8396	.9212	.9748	.9987	2.9
7.2	.1253	.2657	.4571	.6046	.7337	.8406	.9219	.9751	.9988	2.8
7.3	.1271	.2974	.4586	.6060	.7349	.8415	.9225	.9755	.9989	2.7
7.4	.1288	.2990	.4602	.6074	.7361	.8425	.9232	.9759	.9990	2.6
7.5	0.1305	0.3007	0.4617	0.6088	0.7373	0.8434	0.9239	0.9763	0.9990	2.5
7.6	.1323	.3024	.4633	.6101	.7385	.8443	.9245	.9767	.9991	2.4
7.7	.1340	.3040	.4648	.6115	.7396	.8453	.9252	.9770	.9992	2.3
7.8	.1357	.3057	.4664	.6129	.7408	.8462	.9259	.9774	.9993	2.2
7.9	.1374	.3074	.4679	.6143	.7420	.8471	.9265	.9778	.9993	2.1
8.0	0.1392	0.3090	0.4695	0.6157	0.7431	0.8480	0.9272	0.9781	0.9994	2.0
8.1	.1409	.3107	.4710	.6170	.7443	.8490	.9278	.9785	.9995	1.9
8.2	.1426	.3123	.4726	.6184	.7455	.8499	.9285	.9789	.9995	1.8
8.3	.1444	.3140	.4741	.6198	.7466	.8508	.9191	.9792	.9996	1.7
8.4	.1461	.3156	.4756	.6211	.7478	.8517	.9298	.9796	.9996	1.6
8.5	0.1478	0.3173	0.4772	0.6225	0.7490	0.8526	0.9304	0.9799	0.9997	1.5
8.6	.1495	.3190	.4787	.6239	.7501	.8536	.9311	.9803	.9997	1.4
8.7	.1513	.3206	.4802	.6252	.7513	.8545	.9317	.9806	.9997	1.3
8.8	.1530	.3223	.4818	.6266	.7524	.8554	.9323	.9810	.9998	1.2
8.9	.1547	.3239	.4833	.6280	7536	.8563	.9330	.9813	.9998	1.1
9.0	0.1564	0.3256	0.4848	0.6293	0.7547	0.8572	0.9336	0.9816	0.9998	1.0
9.1	.1582	.3272	.4863	.6307	.7559	.8581	.9342	.9820	.9999	0.9
9.2	.1599	.3289	.4879	.6320	.7570	.8590	.9348	.9823	.9999	0.8
9.3	.1616	.3305	.4894	.6334	.7581	.8599	.9354	.9826	.9999	0.7
9.4	.1633	.3322	.4909	.6347	.7593	.8607	.9361	.9829	.9999	0.6
9.5	0.1950	0.3338	0.4924	0.6361	0.7604	0.8616	0.9367	0.9833	1.0000	0.5
9.6	.1668	.3355	.4939	.6374	.7615	.8625	.9373	.9836	.0000	0.4
9.7	.1685	.3371	.4955	.6388	.7627	.8634	.9379	.9839	.0000	0.3
9.8	.1702	.3387	.4970	.6401	.7638	.8643	.9385	.9842	.0000	0.2
9.9	.1719	.3404	.4985	.6414	.7649	.8652	.9391	.9845	.0000	0.1
	.1736	.3420	.5000	.6428	.7660	.8660	.9397	.9848	.0000	0.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	z.

6. Natural Cosines. y = cos z.

7. Natural Tangents. $y = \tan x$.

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
0.0	0.0000	0.1763	0.3640	0.5774	0.8391	1.1918	1.7321	2.7475	5.6713	
0.1	.0017	.1781	.3659	.5797	.8421	.1960	.7391	.7625	.7297	9.9
0.2	.0035	.1799	.3679	.5820	.8451	.2002	.7461	.7776	.7894	9.8
0.3	.0052	.1817	.3699	.5844	.8481	.2045	.7532	.7929	.8502	9.7
0.4	.0070	.1835	.3719	.5867	.8511	.2088	.7603	.8083	.9124	9.6
0.5	0.0087	0.1853	0.3739	0.5890	0.8541	1.2131	1.7675	2.8239	5.9758	9.5
0.6	.0105	.1871	.3759	.5914	.8571	.2174	.7747	.8397	6.0405	9.4
0.7	.0122	.1890	.3779	.5938	.8601	.2218	.7820	.8556	.1066	9.3
0.8	.0140	.1908	.3799	.5961	.8632	.2261	.7893	.8716	.1742	9.2
0.9	.0157	.1926	.3819	.5985	.8662	.2305	.7966	.8878	.2432	9.1
1.0	0.0175	0.1944	0.3839	0.6009	0.8693	1.2349	1.8040	2.9042	6.3138	9.0
1.1	.0192	.1962	.3859	.6032	.8724	.2393	.8115	.9208	.3859	8.9
1.2	.0209	.1980	.3879	.6056	.8754	.2437	.8190	.9375	.4596	8.8
1.3	.0227	.1998	.3899	.6080	.8785	.2482	.8265	.9544	.5350	8.7
1.4	.0244	.2016	.3919	.6104	.8816	.2527	.8341	.9714	.6122	8.6
1.5	0.0262	0.2035	0.3939	0.6128	0.8847	1.2572	1.8418	2.9887	6.6912	8.5
1.6	.0279	.2053	.3959	.6152	.8878	.2617	.8495	3.0061	.7720	8.4
1.7	.0297	.2071	.3979	.6176	.8910	.2662	.8572	.0237	.8548	8.3
1.8	.0314	.2089	.4000	.6200	.8941	.2708	.8650	.0415	.9395	8.2
1.9	.0332	.2107	.4020	.6224	.8972	.2753	.8728	.0595	7.0264	8.1
2.0	0.0349	0.2126	0.4040	0.6249	0.9004	1.2799	1.8807	3.0777	7.1154	8.0
2.1	.0367	.2144	.4061	.6273	.9036	.2846	.8887	.0961	.2066	7.9
2.2	.0384	.2162	.4081	.6297	.9067	.2892	.8967	.1146	.3002	7.8
2.3	.0402	.2180	.4101	.6322	.9099	.2938	.9047	.1334	.3962	7.7
2.4	.0419	.2199	.4122	.6346	.9131	.2985	.9128	.1524	7.4947	7.6
2.5	0.0437	0.2217	0.4142	0.6371	0.9163	1.3032	1.9210	3.1716	7.5958	7.5
2.6	.0454	.2235	.4163	.6395	.9195	.3079	.9292	.1910	.6996	7.4
2.7	.0472	.2254	.4183	.6420	.9228	.3127	.9375	.2106	.8062	7.3
2.8	.0489	.2272	.4204	.6445	.9260	.3175	.9458	.2305	.9158	7.2
2.9	.0507	.2290	.4224	.6469	.9293	.3222	.9542	.2506	8.0285	7.1
3.0	0.0524	0.2309	0.4245	0.6494	0.9325	1.3270	1.9626	3.2709	8.1443	7.0
3.1	.0542	.2327	.4265	.6519	.9358	.3319	.9711	.2914	.2636	6.9
3.2	.0559	.2345	.4286	.6544	.9391	.3367	.9797	.3122	.3863	6.8
3.3	.0577	.2364	.4307	.6569	.9424	.3416	.9883	.3332	.5126	6.7
3.4	.0594	.2382	.4327	.6594	.9457	.3465	.9970	.3544	.6427	6.6
3.5	0.0612	0.2401	0.4348	0.6619	0.9490	1.3514	2.0057	3.3759	8.7769	6.5
3.6	.0629	.2419	.4369	.6644	.9523	.3564	.0145	.3977	.9152	6.4
3.7	.0647	.2438	.4390	.6669	.9556	.3613	.0233	.4197	9.0579	6.3
3.8	.0664	.2456	.4411	.6694	.9590	.3663	.0323	.4420	.2052	6.2
3.9	.0682	.2475	.4431	.6720	.9623	.3713	.0413	.4646	.3572	6.1
4.0	0.0699	0.2493	0.4452	0.6745	0.9657	1.3764	2.0503	3.4874	9.514	6.0
4.1	.0717	.2512	.4473	.6771	.9691	.3814	.0594	.5105	.677	5.9
4.2	.0734	.2530	.4494	.6796	.9725	.3865	.0686	.5339	.845	5.8
4.3	.0752	.2549	.4515	.6822	.9759	.3916	.0778	.5576	10.019	5.7
4.4	.0769	.2568	.4536	.6847	.9793	.3968	.0872	.5816	.199	5.6
4.5	0.0787	0.2586	0.4557	0.6873	0.9827	1.4019	2.0965	3.6059	10.385	5.5
4.6	.0805	.2605	.4578	.6899	.9861	.4071	.1060	.6305	.579	5.4
4.7	.0822	.2623	.4599	.6924	.9896	.4124	.1155	.6554	.780	5.3
4.8	.0840	.2642	.4621	.6950	.9930	.4176	.1251	.6806	.988	5.2
4.9	.0857	.2661	.4642	.6976	.9965	.4229	.1348	.7062	11.205	5.1
5.0	.0875	.2679	.4663	.7002	1.0000	.4281	.1445	.7321	.430	5.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z.

7. Natural Cotangents. $y = \cot z$.

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
5.0	0.0875	0.2679	0.4663	0.7002	1.0000	1.4281	2.1445	3.7321	11.430	5.0
5.1	.0892	.2698	.4684	.7028	.0035	.4335	.1543	.7583	.664	4.9
5.2	.0910	.2717	.4706	.7054	.0070	.4388	.1642	.7848	.909	4.8
5.3	.0928	.2736	.4727	.7080	.0105	.4442	.1742	.8118	12.163	4.7
5.4	.0945	.2754	.4748	.7107	.0141	.4496	.1842	.8391	.429	4.6
5.5	0.0963	0.2773	0.4770	0.7133	1.0176	1.4550	2.1943	3.8667	12.706	4.5
5.6	.0981	.2792	.4791	.7159	.0212	.4605	.2045	.8947	.996	4.4
5.7	.0998	.2811	.4813	.7186	.0247	.4659	.2148	.9232	13.300	4.3
5.8	.1016	.2830	.4834	.7212	.0283	.4715	.2251	.9520	.617	4.2
5.9	.1033	.2849	.4856	.7239	.0319	.4770	.2355	.9812	.951	4.1
6.0	0.1051	0.2867	0.4877	0.7265	1.0355	1.4826	2.2460	4.0108	14.301	4.0
6.1	.1069	.2886	.4899	.7292	.0392	.4882	.2566	.0408	.669	3.9
6.2	.1086	.2905	.4921	.7319	.0428	.4938	.2673	.0713	15.056	3.8
6.3	.1104	.2924	.4942	.7346	.0464	.4994	.2781	.1022	.464	3.7
6.4	.1122	.2943	.4964	.7373	.0501	.5051	.2889	.1335	.895	3.6
6.5	0.1139	0.2962	0.4986	0.7400	1.0538	1.5108	2.2998	4.1653	16.350	3.5
6.6	.1157	.2981	.5008	.7427	.0575	.5166	.3109	.1976	.832	3.4
6.7	.1175	.3000	.5029	.7454	.0612	.5224	.3220	.2303	17.343	3.3
6.8	.1192	.3019	.5051	.7481	.0649	.5282	.3332	.2635	.886	3.2
6.9	.1210	.3038	.5073	.7508	.0686	.5340	.3445	.2972	18.464	3.1
7.0	0.1228	0.3057	0.5095	0.7536	1.0724	1.5399	2.3559	4.3315	19.081	3.0
7.1	.1246	.3076	.5117	.7563	.0761	.5458	.3673	.3662	.740	2.9
7.2	.1263	.3096	.5139	.7590	.0799	.5517	.3789	.4015	20.446	2.8
7.3	.1281	.3115	.5161	.7618	.0837	.5577	.3906	.4373	21.205	2.7
7.4	.1299	.3134	.5184	.7646	.0875	.5637	.4023	.4737	22.022	2.6
7.5	0.1317	0.3153	0.5206	0.7673	1.0913	1.5697	2.4142	4.5107	22.904	2.5
7.6	.1334	.3172	.5228	.7701	.0951	.5757	.4262	.5483	23.859	2.4
7.7	.1352	.3191	.5250	.7729	.0990	.5818	.4383	.5864	24.898	2.3
7.8	.1370	.3211	.5272	.7757	.1028	.5880	.4504	.6252	26.031	2.2
7.9	.1388	.3230	.5295	.7785	.1067	.5941	.4627	.6646	27.271	2.1
8.0	0.1405	0.3249	0.5317	0.7813	1.1106	1.6003	2.4751	4.7046	28.636	2.0
8.1	.1423	.3269	.5340	.7841	.1145	.6066	.4876	.7453	30.145	1.9
8.2	.1441	.3288	.5362	.7869	.1184	.6128	.5002	.7867	31.821	1.8
8.3	.1459	.3307	.5384	.7898	.1224	.6191	.5129	.8288	33.694	1.7
8.4	.1477	.3327	.5407	.7926	.1263	.6255	.5257	.8716	35.801	1.6
8.5	0.1495	0.3346	0.5430	0.7954	1.1303	1.6319	2.5386	4.9152	38.188	1.5
8.6	.1512	.3365	.5452	.7983	.1343	.6383	.5517	.9594	40.917	1.4
8.7	.1530	.3385	.5475	.8012	.1383	.6447	.5649	5.0045	44.066	1.3
8.8	.1548	.3404	.5498	.8040	.1423	.6512	.5782	.0504	47.740	1.2
8.9	.1566	.3424	.5520	.8069	.1463	.6577	.5916	.0970	52.081	1.1
9.0	0.1584	0.3443	0.5543	0.8098	1.1504	1.6643	2.6051	5.1446	57.290	1.0
9.1	.1602	.3463	.5566	.8127	.1544	.6709	.6187	.1929	63.657	0.9
9.2	.1620	.3482	.5589	.8156	.1585	.6775	.6325	.2422	71.615	0.8
9.3	.1638	.3502	.5612	.8185	.1626	.6842	.6464	.2924	81.847	0.7
9.4	.1655	.3522	.5635	.8214	.1667	.6909	.6605	.3435	95.489	0.6
9.5	0.1673	0.3541	0.5658	0.8243	1.1708	1.6977	2.6746	5.3955	114.59	0.5
9.6	.1691	.3561	.5681	.8273	.1750	.7045	.6889	.4486	143.24	0.4
9.7	.1709	.3581	.5704	.8302	.1792	.7113	.7034	.5026	190.98	0.3
9.8	.1727	.3600	.5727	.8332	.1833	.7182	.7179	.5578	286.48	0.2
9.9	.1745	.3620	.5750	.8361	.1875	.7251	.7326	.6140	572.96	0.1
	.1763	.3640	.5774	.8391	.1918	.7321	.7475	.6713	∞	0.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z.

7. Natural Cotangents. y = cot z.

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
0.0	−∞	9.2397	9.5341	9.6990	9.8081	9.8843	9.9375	9.9730	9.9934	
0.1	7.2419	.2439	.5361	.7003	.8090	.8849	.9380	.9733	.9935	9.9
0.2	.5429	.2482	.5382	.7016	.8099	.8855	.9384	.9735	.9936	9.8
0.3	.7190	.2524	.5402	.7029	.8108	.8862	.9388	.9738	.9937	9.7
0.4	.8439	.2565	.5423	.7042	.8117	.8868	.9393	.9741	.9939	9.6
0.5	7.9408	9.2606	9.5443	9.7055	9.8125	9.8874	9.9397	9.9743	9.9940	9.5
0.6	8.0200	.2647	.5463	.7068	.8134	.8880	.9401	.9746	.9941	9.4
0.7	.0870	.2687	.5484	.7080	.8143	.8887	.9406	.9749	.9943	9.3
0.8	.1450	.2727	.5504	.7093	.8152	.8893	.9410	.9751	.9944	9.2
0.9	.1961	.2767	.5523	.7106	.8161	.8899	.9414	.9754	.9945	9.1
1.0	8.2419	9.2806	9.5543	9.7118	9.8169	9.8905	9.9418	9.9757	9.9946	9.0
1.1	.2832	.2845	.5563	.7131	.8178	.8911	.9422	.9759	.9947	8.9
1.2	.3210	.2883	.5583	.7144	.8187	.8917	.9427	.9762	.9949	8.8
1.3	.3558	.2921	.5602	.7156	.8195	.8923	.9431	.9764	.9950	8.7
1.4	.3880	.2959	.5621	.7168	.8204	.8929	.9435	.9767	.9951	8.6
1.5	8.4179	9.2997	9.5641	9.7181	9.8213	9.8935	9.9439	9.9770	9.9952	8.5
1.6	.4459	.3034	.5660	.7193	.8221	.8941	.9443	.9772	.9953	8.4
1.7	.4723	.3070	.5679	.7205	.8230	.8947	.9447	.9775	.9954	8.3
1.8	.4971	.3107	.5698	.7218	.8238	.8953	.9451	.9777	.9955	8.2
1.9	.5206	.3143	.5717	.7230	.8247	.8959	.9455	.9780	.9956	8.1
2.0	8.5428	9.3179	9.5736	9.7242	9.8255	9.8965	9.9459	9.9782	9.9958	8.0
2.1	.5640	.3214	.5754	.7254	.8264	.8971	.9463	.9785	.9959	7.9
2.2	.5842	.3249	.5773	.7266	.8272	.8977	.9467	.9787	.9960	7.8
2.3	.6035	.3284	.5792	.7278	.8280	.8983	.9471	.9789	.9961	7.7
2.4	.6220	.3319	.5810	.7290	.8289	.8989	.9475	.9792	.9962	7.6
2.5	8.6397	9.3353	9.5828	9.7302	9.8297	9.8995	9.9479	9.9794	9.9963	7.5
2.6	.6567	.3387	.5847	.7314	.8305	.9000	.9483	.9797	.9964	7.4
2.7	.6731	.3421	.5865	.7326	.8313	.9006	.9487	.9799	.9965	7.3
2.8	.6889	.3455	.5883	.7338	.8322	.9012	.9491	.9801	.9966	7.2
2.9	.7041	.3488	.5901	.7349	.8330	.9018	.9495	.9804	.9967	7.1
3.0	8.7188	9.3521	9.5919	9.7361	9.8338	9.9023	9.9499	9.9806	9.9968	7.0
3.1	.7330	.3554	.5937	.7373	.8346	.9029	.9503	.9808	.9968	6.9
3.2	.7468	.3586	.5954	.7384	.8354	.9035	.9506	.9811	.9969	6.8
3.3	.7602	.3618	.5972	.7396	.8362	.9041	.9510	.9813	.9970	6.7
3.4	.7731	.3650	.5990	.7407	.8370	.9046	.9514	.9815	.9971	6.6
3.5	8.7857	9.3682	9.6007	9.7419	9.8378	9.9052	9.9518	9.9817	9.9972	6.5
3.6	.7979	.3713	.6024	.7430	.8386	.9057	.9522	.9820	.9973	6.4
3.7	.8098	.3745	.6042	.7442	.8394	.9063	.9525	.9822	.9974	6.3
3.8	.8213	.3775	.6059	.7453	.8402	.9069	.9529	.9824	.9975	6.2
3.9	.8326	.3806	.6076	.7464	.8410	.9074	.9533	.9826	9.9975	6.1
4.0	8.8436	9.3837	9.6093	9.7476	9.8418	9.9080	9.9537	9.9828	9.9976	6.0
4.1	.8543	.3867	.6110	.7487	.8426	.9085	.9540	.9831	.9977	5.9
4.2	.8647	.3897	.6127	.7498	.8433	.9091	.9544	.9833	.9978	5.8
4.3	.8749	.3927	.6144	.7509	.8441	.9096	.9548	.9835	.9978	5.7
4.4	.8849	.3957	.6161	.7520	.8449	.9101	.9551	.9837	.9979	5.6
4.5	8.8946	9.3986	9.6177	9.7531	9.8457	9.9107	9.9555	9.9839	9.9980	5.5
4.6	.9042	.4015	.6194	.7542	.8464	.9112	.9558	.9841	.9981	5.4
4.7	.9135	.4044	.6210	.7553	.8472	.9118	.9562	.9843	.9981	5.3
4.8	.9226	.4073	.6227	.7564	.8480	.9123	.9566	.9845	.9982	5.2
4.9	.9315	.4102	.6243	.7575	.8487	.9128	.9569	.9847	.9983	5.1
5.0	.9403	.4130	.6259	.7586	.8495	.9134	.9573	.9849	.9983	5.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z

8. Logarithmic Cosines. y = log cos z.

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
5.0	8.9403	9.4130	9.6259	9.7586	9.8495	9.9134	9.9573	9.9849	9.9983	5.0
5.1	.9489	.4158	.6276	.7597	.8502	.9139	.9576	.9851	.9984	4.9
5.2	.9573	.4186	.6292	.7607	.8510	.9144	.9580	.9853	.9985	4.8
5.3	.9655	.4214	.6308	.7618	.8517	.9149	.9583	.9855	.9985	4.7
5.4	.9736	.4242	.6324	.7629	.8525	.9155	.9587	.9857	.9986	4.6
5.5	8.9816	9.4269	9.6340	9.7640	9.8532	9.9160	9.9590	9.9859	9.9987	4.5
5.6	.9894	.4296	.6356	.7650	.8540	.9165	.9594	.9861	.9987	4.4
5.7	.9970	.4323	.6371	.7661	.8547	.9170	.9597	.9863	.9988	4.3
5.8	9.0046	.4350	.6387	.7671	.8555	.9175	.9601	.9865	.9988	4.2
5.9	.0120	.4377	.6403	.7682	.8562	.9181	.9604	.9867	.9989	4.1
6.0	9.0192	9.4403	9.6418	9.7692	9.8569	9.9186	9.9607	9.9869	9.9989	4.0
6.1	.0264	.4430	.6434	.7703	.8577	.9191	.9611	.9871	.9990	3.9
6.2	.0334	.4456	.6449	.7713	.8584	.9196	.9614	.9873	.9990	3.8
6.3	.0403	.4482	.6465	.7723	.8591	.9201	.9617	.9875	.9991	3.7
6.4	.0472	.4508	.6480	.7734	.8598	.9206	.9621	.9876	.9991	3.6
6.5	9.0539	9.4533	9.6495	9.7744	9.8606	9.9211	9.9624	9.9878	9.9992	3.5
6.6	.0605	.4559	.6510	.7754	.8613	.9216	.9627	.9880	.9992	3.4
6.7	.0670	.4584	.6526	.7764	.8620	.9221	.9631	.9882	.9993	3.3
6.8	.0734	.4609	.6541	.7774	.8627	.9226	.9634	.9884	.9993	3.2
6.9	.0797	.4634	.6556	.7785	.8634	.9231	.9637	.9885	.9994	3.1
7.0	9.0859	9.4659	9.6570	9.7795	9.8641	9.9236	9.9640	9.9887	9.9994	3.0
7.1	.0920	.4684	.6585	.7805	.8648	.9241	.9643	.9889	.9994	2.9
7.2	.0981	.4709	.6600	.7815	.8655	.9246	.9647	.9891	.9995	2.8
7.3	.1040	.4733	.6615	.7825	.8662	.9251	.9650	.9892	.9995	2.7
7.4	.1099	.4757	.6629	.7835	.8669	.9255	.9653	.9894	.9996	2.6
7.5	9.1157	9.4781	9.6644	9.7844	9.8676	9.9260	9.9656	9.9896	9.9996	2.5
7.6	.1214	.4805	.6659	.7854	.8683	.9265	.9659	.9897	.9996	2.4
7.7	.1271	.4829	.6673	.7864	.8690	9270	.9662	.9899	.9996	2.3
7.8	.1326	.4853	.6687	.7874	.8697	.9275	.9665	.9901	.9997	2.2
7.9	.1381	.4876	.6702	.7884	.8704	.9279	.9669	.9902	.9997	2.1
8.0	9.1436	9.4900	9.6716	9.7893	9.8711	9.9284	9.9672	9.9904	9.9997	2.0
8.1	.1489	.4923	.6730	.7903	.8718	.9289	.9675	.9906	.9998	1.9
8.2	.1542	.4946	.6744	.7913	.8724	.9294	.9678	.9907	.9998	1.8
8.3	.1594	.4969	.6759	.7922	.8731	.9298	.9681	.9909	.9998	1.7
8.4	.1646	.4992	.6773	.7932	.8738	.9303	.9684	.9910	.9998	1.6
8.5	9.1697	9.5015	9.6787	9.7941	9.8745	9.9308	9.9687	9.9912	9.9999	1.5
8.6	.1747	.5037	.6801	.7951	.8751	.9312	.9690	.9913	.9999	1.4
8.7	.1797	.5060	.6814	.7960	.8758	.9317	.9693	.9915	.9999	1.3
8.8	.1847	.5082	.6828	.7970	.8765	.9322	.9696	.9916	.9999	1.2
8.9	.1895	.5104	.6842	.7979	.8771	.9326	.9699	.9918	.9999	1.1
9.0	9.1943	9.5126	9.6856	9.7989	9.8778	9.9331	9.9702	9.9919	9.9999	1.0
9.1	.1991	.5148	.6869	.7998	.8784	.9335	.9704	.9921	.9999	0.9
9.2	.2038	.5170	.6883	.8007	.8791	.9340	.9707	.9922	0.0000	0.8
9.3	.2085	.5192	.6896	.8017	.8797	.9344	.9710	.9924	.0000	0.7
9.4	2131	.5213	.6910	.8026	.8804	.9349	.9713	.9925	.0000	0.6
9.5	9.2176	9.5235	9.6923	9.8035	9.8810	9.9353	9.9716	9.9927	0.0000	0.5
9.6	.2221	.5256	.6937	.8044	.8817	.9358	.9719	.9928	.0000	0.4
9.7	.2266	.5278	.6950	.8053	.8823	.9362	.9722	.9929	.0000	0.3
9.8	.2310	.5299	.6963	.8063	.8830	.9367	.9724	.9931	.0000	0.2
9.9	.2353	.5320	.6977	.8072	.8836	.9371	.9727	.9932	.0000	0.1
	.2397	.5341	.6990	.8081	.8843	.9375	.9730	.9934	.0000	0.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z.

8. Logarithmic Cosines. y = log cos z.

9. Logarithmic Tangents. $y = \log \tan x$.

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
0.0	— ∞	9.2463	9.5611	9.7614	9.9238	0.0762	0.2386	0.4389	0.7537	
0.1	7.2419	.2507	.5634	.7632	.9254	.0777	.2403	.4413	.7581	9.9
0.2	.5429	.2551	.5658	.7649	.9269	.0793	.2421	.4437	.7626	9.8
0.3	.7190	.2594	.5681	.7667	.9284	.0808	.2438	.4461	.7672	9.7
0.4	.8439	.2637	.5704	.7684	.9300	.0824	.2456	.4484	.7718	9.6
0.5	7.9409	9.2680	9.5727	9.7701	9.9315	0.0839	0.2474	0.4509	0.7764	9.5
0.6	8.0200	.2722	.5750	.7719	.9330	.0854	.2491	.4533	.7811	9.4
0.7	.0870	.2764	.5773	.7736	.9346	.0870	.2509	.4557	.7858	9.3
0.8	.1450	.2805	.5796	.7753	.9361	.0885	.2527	.4581	.7906	9.2
0.9	.1962	.2846	.5819	.7771	.9376	.0901	.2545	.4606	.7954	9.1
1.0	8.2419	9.2887	9.5842	9.7788	9.9392	0.0916	0.2562	0.4630	0.8003	9.0
1.1	.2833	.2927	.5864	.7805	.9407	.0932	.2580	.4655	.8052	8.9
1.2	.3211	.2967	.5887	.7822	.9422	.0947	.2598	.4680	.8102	8.8
1.3	.3559	.3006	.5909	.7839	.9438	.0963	.2616	.4705	.8152	8.7
1.4	.3881	.3046	.5932	.7856	.9453	.0978	.2634	.4730	.8203	8.6
1.5	8.4181	9.3085	9.5954	9.7873	9.9468	0.0994	0.2652	0.4755	0.8255	8.5
1.6	.4461	.3123	.5976	.7890	.9483	.1010	.2670	.4780	.8307	8.4
1.7	.4725	.3162	.5998	.7907	.9499	.1025	.2689	.4805	.8360	8.3
1.8	.4973	.3200	.6020	.7924	.9514	.1041	.2707	.4831	.8413	8.2
1.9	.5208	.3237	.6042	.7941	.9529	.1056	.2725	.4857	.8467	8.1
2.0	8.5431	9.3275	9.6064	9.7958	9.9544	0.1072	0.2743	0.4882	0.8522	8.0
2.1	.5643	.3312	.6086	.7975	.9560	.1088	.2762	.4908	.8577	7.9
2.2	.5845	.3349	.6108	.7992	.9575	.1103	.2780	.4934	.8633	7.8
2.3	.6038	.3385	.6129	.8008	.9590	.1119	.2798	.4960	.8690	7.7
2.4	.6223	.3422	.6151	.8025	.9605	.1135	.2817	.4986	.8748	7.6
2.5	8.6401	9.3458	9.6172	9.8042	9.9621	0.1150	0.2835	0.5013	0.8806	7.5
2.6	.6571	.3493	.6194	.8059	.9636	.1166	.2854	.5039	.8865	7.4
2.7	.6736	.3529	.6215	.8075	.9651	.1182	.2872	.5066	.8924	7.3
2.8	.6894	.3564	.6236	.8092	.9666	.1197	.2891	.5093	.8985	7.2
2.9	.7046	.3599	.6257	.8109	.9681	.1213	.2910	.5120	.9046	7.1
3.0	8.7194	9.3634	9.6279	9.8125	9.9697	0.1229	0.2928	0.5147	0.9109	7.0
3.1	.7337	.3668	.6300	.8142	.9712	.1245	.2947	.5174	.9172	6.9
3.2	.7475	.3702	.6321	.8158	.9727	.1260	.2966	.5201	.9236	6.8
3.3	.7609	.3736	.6341	.8175	.9742	.1276	.2985	.5229	.9301	6.7
3.4	.7739	.3770	.6362	.8191	.9757	.1292	.3004	.5256	.9367	6.6
3.5	8.7865	9.3804	9.6383	9.8208	9.9772	0.1308	0.3023	0.5284	0.9433	6.5
3.6	.7988	.3837	.6404	.8224	.9788	.1324	.3042	.5312	.9501	6.4
3.7	.8107	.3870	.6424	.8241	.9803	.1340	.3061	.5340	.9570	6.3
3.8	.8223	.3903	.6445	.8257	.9818	.1356	.3080	.5368	.9640	6.2
3.9	.8336	.3935	.6465	.8274	.9833	.1371	.3099	.5397	.9711	6.1
4.0	8.8446	9.3968	9.6486	9.8290	9.9848	0.1387	0.3118	0.5425	0.9784	6.0
4.1	.8554	.4000	.6506	.8306	.9864	.1403	.3137	.5454	.9857	5.9
4.2	.8659	.4032	.6527	.8323	.9879	.1419	.3157	.5483	.9932	5.8
4.3	.8762	.4064	.6547	.8339	.9894	.1435	.3176	.5512	1.0008	5.7
4.4	.8862	.4095	.6567	.8355	.9909	.1451	.3196	.5541	.0085	5.6
4.5	8.8960	9.4127	9.6587	9.8371	9.9924	0.1467	0.3215	0.5570	1.0164	5.5
4.6	.9056	.4158	.6607	.8388	.9939	.1483	.3235	.5600	.0244	5.4
4.7	.9150	.4189	.6627	.8404	.9955	.1499	.3254	.5629	.0326	5.3
4.8	.9241	.4220	.6647	.8420	.9970	.1516	.3274	.5659	.0409	5.2
4.9	.9331	.4250	.6667	.8436	.9985	.1532	.3294	.5689	.0494	5.1
5.0	.9420	.4281	.6687	.8452	0.0000	.1548	.3313	.5719	.0580	5.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z.

9. Logarithmic Cotangents. $y = \log \cot z$.

X.	0°	10°	20°	30°	40°	50°	60°	70°	80°	
5.0	8.9420	9.4281	9.6687	9.8452	0.0000	0.1548	0.3313	0.5719	1.0580	5.0
5.1	.9506	.4311	.6706	.8468	.0015	.1564	.3333	.5750	.0669	4.9
5.2	.9591	.4341	.9726	.8484	.0030	.1580	.3353	.5780	.0759	4.8
5.3	.9674	.4371	.6746	.8501	.0045	.1596	.3373	.5811	.0850	4.7
5.4	.9756	.4400	.6765	.8517	.0061	.1612	.3393	.5842	.0944	4.6
5.5	8.9836	9.4430	9.6785	9.8533	0.0076	0.1629	0.3413	0.5873	1.1040	4.5
5.6	.9915	.4459	.6804	.8549	.0091	.1645	.3433	.5905	.1138	4.4
5.7	.9992	.4488	.6824	.8565	.0106	.1661	.3453	.5936	.1238	4.3
5.8	9.0068	.4517	.6843	.8581	.0121	.1677	.3473	.5968	.1341	4.2
5.9	.0143	.4546	.6863	.8597	.0136	.1694	.3494	.6000	.1446	4.1
6.0	9.0216	9.4575	9.6882	9.8613	0.0152	0.1710	0.3514	0.6032	1.1554	4.0
6.1	.0289	.4603	.6901	.8629	.0167	.1726	.3535	.6065	.1664	3.9
6.2	.0360	.4632	.6920	.8644	.0182	.1743	.3555	.6097	.1777	3.8
6.3	.0430	.4660	.6939	.8660	.0197	.1759	.3576	.6130	.1893	3.7
6.4	.0499	.4688	.6958	.8676	.0212	.1776	.3596	.6163	.2012	3.6
6.5	9.0567	9.4716	9.6977	9.8692	0.0228	0.1792	0.3617	0.6196	1.2135	3.5
6.6	.0633	.4744	.6996	.8708	.0243	.1809	.3638	.6230	.2261	3.4
6.7	.0699	.4771	.7015	.8724	.0258	.1825	.3659	.6264	.2391	3.3
6.8	.0764	.4799	.7034	.8740	.0273	.1842	.3679	.6298	.2525	3.2
6.9	.0828	.4826	.7053	.8755	.0288	.1858	.3700	.6332	.2663	3.1
7.0	9.0891	9.4853	9.7072	9.8771	0.0303	0.1875	0.3721	0.6366	1.2806	3.0
7.1	.0954	.4880	.7090	.8787	.0319	.1891	.3743	.6401	.2954	2.9
7.2	.1015	.4907	.7109	.8803	.0334	.1908	.3764	.6436	.3106	2.8
7.3	.1076	.4934	.7128	.8818	.0349	.1925	.3785	.6471	.3264	2.7
7.4	.1135	.4961	.7146	.8834	.0364	.1941	.3806	.6507	.3429	2.6
7.5	9.1194	9.4987	9 7165	9.8850	0.0379	0.1958	0.3828	0.6542	1.3599	2.5
7.6	.1252	.5014	.7183	.8865	.0395	.1975	.3849	.6578	.3777	2.4
7.7	.1310	.5040	.7202	.8881	.0410	.1992	.3871	.6615	.3962	2.3
7.8	.1367	.5066	.7220	.8897	.0425	.2008	.3892	.6651	.4155	2.2
7.9	.1423	.5092	.7238	.8912	.0440	.2025	.3914	.6688	.4357	2.1
8.0	9.1478	9.5118	9.7257	9.8928	0.0456	0.2042	0.3936	0.6725	1.4569	2.0
8.1	.1533	.5143	.7275	.8944	.0471	.2059	.3958	.6763	.4792	1.9
8.2	.1587	.5169	.7293	.8959	.0486	.2076	.3980	.6800	.5027	1.8
8.3	.1640	.5195	.7311	.8975	.0501	.2093	.4002	.6838	.5275	1.7
8.4	.1693	.5220	.7330	.8990	.0517	.2110	.4024	.6877	.5539	1.6
8.5	9.1745	9.5245	9.7348	9.9006	0.0532	0.2127	0.4046	.6915	1.5819	1.5
8.6	.1797	.5270	.7366	.9022	.0547	.2144	.4068	.6954	.6119	1.4
8.7	.1848	.5295	.7384	.9037	.0562	.2161	.4091	.6994	.6441	1.3
8.8	.1898	.5320	.7402	.9053	.0578	.2178	.4113	.7033	.6789	1.2
8.9	.1948	.5345	.7420	.9068	.0593	.2195	.4136	.7073	.7167	1.1
9.0	9.1997	9.5370	9.7438	9.9084	0.0608	0.2212	0.4158	0.7113	1.7581	1.0
9.1	.2046	.5394	.7455	.9099	.0624	.2229	.4181	.7154	.8038	0.9
9.2	.2094	.5419	.7473	.9115	.0639	.2247	.4204	.7195	.8550	0.8
9.3	.2142	.5443	.7491	.9130	.0654	.2264	.4227	.7236	.9130	0.7
9.4	.2189	.5467	.7509	.9146	.0670	.2281	.4250	.7278	.9800	0.6
9.5	9.2236	9.5491	9.7526	9.9161	0.0685	0.2299	0.4273	0.7320	2.0591	0.5
9.6	.2282	.5516	.7544	.9176	.0700	.2316	.4296	.7363	.1561	0.4
9.7	.2328	.5539	.7562	.9192	.0716	.2333	.4319	.7406	.2810	0.3
9.8	.2374	.5563	.7579	.9207	.0731	.2351	.4342	.7449	.4571	0.2
9.9	.2419	.5587	.7597	.9223	.0746	.2368	.4366	.7493	.7581	0.1
	.2463	.5611	.7614	.9238	.0762	.2386	.4389	.7537	∞	0.0
	80°	70°	60°	50°	40°	30°	20°	10°	0°	Z.

9. Logarithmic Cotangents. y = log cot z.

	Num. Val.	Recip.	Log.
Circumference in diameters, π	3.1415926	.3183099	0.49715
$\pi^2 = 9.8696044$, $\sqrt{\pi} = 1.7724538$			
Base of Naperian logarithms, e	2.7182818	.3678893	0.43429
Modulus of common logarithms, M	.4342945	2.302584	9.63778
Square root of 2	1.4142136	.7071068	0.15051
Square root of 3	1.7320508	.5773508	0.23856
Ratio of probable to mean error	.67449	.14826	9.82898
Radius in degrees	57.29578	.01745329	1.75812
Radius in minutes	3437.7468	.00029089	3.53627
Radius in seconds	206264.81	.00000485	5.31443
Solar parallax	8″.94		0.9523
Aberration constant	20″.44		1.3105
Mean time in sidereal time	.9972696	1.0027379	9.99881
Tropical year in mean days = 365d. 5h. 48m. 46s	365.24221	.0027379	2.56258
Obliquity of ecliptic 1877	23°27′ 24″		
Equatorial radius of earth in kilometres	6377.4	.0001568	3.80464
Polar radius of earth in kilometres	6356.1	.0001573	3.80319
Force of gravity at equator in centimetres	978.0	.001022	2.9903
Force of gravity at pole in centimetres	983.2	.001917	2.9926
Seconds pendulum at equator in centimetres	99.09	.01009	1.9960
Seconds pendulum at pole in centimetres	99.62	.01004	1.9983
Seconds pendulum at London in inches (Kater)	39.1393	.02555	1.59261
Toise of Peru in centimetres	194.897	.0051309	2.28981
Inch in centimetres	2.5400	.39370	0.4048
Foot in centimetres	30.480	.03281	1.4840
Statute miles in kilometres	1.6093	.6214	0.2066
Square inch in centimetres	6.451	.1550	0.8097
Cubic inch in centimetres	16.387	.061025	1.2145
Imperial quart in litres	1.13551	.88066	0.0552
Imperial gallon in cubic inches	.0036075	277.274	7.55721
Grain in grammes	.06480	15.432	8.81157
Pound Avoirdupois in kilogrammes	.45359	2.20462	9.65667
Pound Troy in kilogrammes	.37324	2.67951	9.57198
Weight of 1 cu. inch of water in grains at 62°F.	252.458	.003961	2.40219
Weight of 1 litre of air in grammes	1.293	.7734	0.1116
Foot-pound in kilogrammetres	.13825	7.2331	9.1407
Dyne in grammes, approx.	.00102	980	7.009
Erg in gramme-centimetres, approx.	.00102	980	7.009
Horse power in erg-nines, approx.	7.46	.134	0.873
Pound per sq. inch in grammes per sq. centimetre	70.308	.01422	1.8470
Specific gravity of mercury	13.596	.07355	1.13341
Interval of semitone in isotonic scale	1.05949	.94386	0.02509
Velocity of sound in air in centimetres at 0° C.	33030		4.5189
Velocity of light in vacuo in kilometres	300400		5.4777
Wave-length of sodium line D in centimetres	.00005895	16963.5	5.7705
Rotation of 1 mm. of quartz (D line)	21°.67	.04615	1.3359
Mechanical equivalent of heat in French units	425	.00235	2.6277
Mechanical equivalent of heat in English units	773	.00129	2.8882
Mechanical equivalent of heat in megalergs, app.	41.6	.02404	1.619
Expansion of gases per degree Centigrade	.003665	273	7.5641
Ratio of two specific heats of gases	1.417	.7057	0.1514
Atomic weight ÷ vapor density	28.88	.03463	1.4606
Latent heat of fusion of ice	79.4	.0126	1.900
Latent heat of vaporization of steam at 100° C.	537	.001862	2.730
Siemens unit in ohms	.9536	1.0487	9.9794
Farad liberates of H in milligrammes	.01024	97.656	8.0107
Electromotive force of a Daniell's cell in volts	1.079	.9268	0.0330
Resist. of Cu. wire 1 inch long, wt. 1 grn., in ohms	.001516	659.6	7.1807

Name.	Symbol.	Atomic Weight.	Specific Gravity.	Modulus of Elast.	Hardness.	Specific Heat.	Point of Fusion.	Expans.	Conduct.	Electrical Resist.	Thermo-Electric.	Refractive Equiv.
Aluminum	Al	27.4	2.6		.0821	.214		2336		.030	+.7	8.4
Antimony	Sb	122.0	6.7			.051	450	1055		.359	—2.8	24.5
Arsenic	As	75.0	5.7			.081		602			—13.6	15?
Barium	Ba	137.0	4.									15.8
Bismuth	Bi	210.0	9.8			.031	270	1316	2.4	.133	+89	39.2
Cadmium	Cd	112.0	8.6		.0760	.054	320	3102		.068	—3.9	13.6
Cæsium	Cs	133.0										13?
Calcium	Ca	40.0	1.6		.0405	.167				.036	—15.1	10.4
Cerium	Ce	42.	5.5									13?
Chromium	Cr	52.2	7.3		.1450							16?
Cobalt	Co	58.8	8.5		.1360	.107	1250	1244			+22.4	10.8
Copper	Cu	63.4	8.9	12000		.094	1100	1698	100	.017		11.6
Didymium	D	95.0										16?
Erbium	E	112.6										
Gallium	Ga											
Glucinum	Gl	9.4	2.1									5.7
Gold	Au	197.0	19.3	8000	.0979	.032	1250	1451	72	.021	—1.2	24.0
Hydrogen	H	1.0				3.41						1?
Indium	In	198	7.2		.0984	.057	176	4594				
Iridium	Ir	198	22.4			.033		708			—2.5	
Iron	Fe	56.0	7.7	20000	.1375	.113	1500	1228	16	.098	—17.5	12?
Lanthanum	La	93.6										
Lead	Pb	207.0	11.4	1700	.0570	.031	330	2948	11	.198	0.0	24.8
Lithium	Li	7.0	.6			.941	180			.080	—13.7	3.8
Magnesium	Mg	24.0	1.7		.0726	.247		2762		.031	+.4	70?
Manganese	Mr	.55.0	7.		.1456	.122	1600					12?
Mercury	Hg	200.0	13.6			.033	—39			.956	+4.2	21?
Molybdenum	Mo	96.0	8.6			.072	1600					
Nickel	Ni	58.8	8.3		.1410	.109		1286			+11.4	10.4
Niobium	Nb	94.0	7.									
Osmium	Os	199.2	21.4			.031		679				
Palladium	Pd	106.0	11.7	10000	.1200	.059		1190	8.6	.138	+7.2	22.2
Platinum	Pt	197.4	21.5	16000	.1107	.032		907	11	.092	—.9	26.0
Potassium	K	39.1	.9		.0230	.170	60			.072	+12.7	8.1
Rhodium	R	104.4	11.2			.058		858				24?
Rubidium	Rb	85.4	1.5				38					14.0
Ruthenium	Ru	104.4	11.2			.061		991				
Selenium	Se	79.4	4.8		.0990	.084	217	3791			—807	
Silver	Ag	108.0	10.4	7000	.0400	.057	1000	1935	136	.015	—3	13?
Sodium	Na	23.0	1.0			.293	96			.021	+5.9	4.8
Strontium	Sr	87.6	2.5							.227	—8.7	13.6
Tantalum	Ta	182.0	10.5									
Tellurium	Te	128.0	6.2			.047	500	1732			—502	
Thallium	Tl	204.0	11.8		.0565	.034	290	3135		.183		21.6
Thorinum	Th	57.9	7.7									
Tin	Sn	118.0	7.2	4000	.0651	.056	230	2270	20	.134	—.1	27?
Titanium	Ti	50.0										25?
Tungsten	W	184.0	17.4			.033						
Uranium	U	240.0	18.4			.062						10.8
Vanadium	V	51.2	5.5									25?
Yttrium	Y	61.6										
Zinc	Zn	65.2	6.8	9000	.1077	.093	420	2905	26	.057	—3.7	10?
Zirconium	Zr	89.6	4.1									22?
Steel			7.8	20000			1400	1200	16			
Brass			8.3	9000		.094	900	1900	32	.058		

12. Properties of Liquids.

Name.	Symbol.	Specific Gravity.	Capillarity.	Compress.	Velocity of Sound.	Specific Heat.	Expans.	Boiling Point.	Latent Heat.	Index of Refract.	Dispers.	Magnet.
Water	H_2O	1.000	29.3	477	1437	1.000	.0466	100	536	1.334	.012	—1.00
Alcohol	C_2H_6O	.792	11.4	904	1160	.595	.111	78	209	1.372	.011	—.81
Ether	$C_4H_{10}O$	.715	9.5	1110	1160	.540	.0714	35	91	1.35	.012	—.80
Bisulph. Carb.	C_2S	1.263	9.7			.238		46		1.678	.077	—1.03
Turpentine	$C_{10}H_{16}$	.869	12.7	714	1212	.432	.0714	159	69	1.474	.022	
Mercury	Hg	13.60	—9.2	30		.033	.0182	357				
Bromine	Br	2.966	9.0			.113		47				
Sulph. Acid	H_2SO_4	1.841		320		.343	.0588	338		1.434	.014	—1.08
Nitric Acid	H_2NO_6	1.55		322			.111	86		1.410	.019	—.91

13. Properties of Gases.

Name.	Symbol.	Molec. Wt.	Density.	Weight 1 litre.	Sp. Ht. eq. Wt.	Sp. Ht. eq. Vol.	Boiling Point.	Velocity Transpir.	Velocity of Sound.	Index Refract.	Dispers.
Hydrogen	H_2	2	.069	.089	3.409	.236	—	2.288	1269	.1388	.0044
Marsh Gas	CH_4	16	.555	.717	.593	.328	—	1.815		.443	
Ammonia	NH_3	17	.596	.770	.508	.'300	—38.5	1.955		.385	
Steam	H_2O	18	.623	.805	.480	.299	100				
Carbonic Ox.	CO	28	.957	1.236	.245	.237	—	r.145	337	.3336	.0075
Nitrogen	N_2	28	.972	1.256	.244	.237	—	1.141		.2972	.0069
Ethylene	C_2H_4	28	.978	1.263	.404	.411		1.980	314	.678	
Air			1.000	1.292	.237	.237	—	1.107	330	.2923	.0058
Binox. Nit.	NO	30	1.039	1.342	.231	.238	—	1.141		.2967	
Oxygen	O_2	32	1.106	1.432	.218	.240	—	1.000	317	.272	
Sulph. Hyd.	H_2S	32	1.191	1.538	.243	.286	—61.8	1.614		.644	
Nitrous Oxide	N_2O	44	1.520	1.963	.224	.245	—87.9	1.335	262	.5084	.0127
Carbonic Acid	CO_2	48	1.529	1.975	.217	.331	—78.2	1.370	262	.4494	.0052
Cyanogen	C_2N_2	54	1.806	2.336			—35	1.976		.8202	.0100
Sulph. Anhyd.	SO_2	64	2.234	2.886	.154	.341	—10.1	1.538		.6820	
Chlorine	Cl_2	71	2.470	3.191	.121	.296	—33.6	1.500		.772	

14. Hydrometer Tables.

	Baumé heavy liq.	Baumé light liq.	Beck heavy liq.	Beck light liq.	Cartier.	Twaddell.
0	1.000		1.000	1.000		1.00
5	1.035		1.030	.971		1.02
10	1.073	1.000	1.062	.944		1.05
15	1.114	.967	1.097	.919	.970	1.07
20	1.158	.936	1.133	.895	.936	1.10
25	1.205	.907	1.172	.872	.905	1.12
30	1.257	.880	1.214	.850	.876	1.15
35	1.313	.854	1.259	.829	.849	1.17
40	1.375	.830	1.308	.810	.824	1.20
45	1.442	.807	1.360	.791		1.22
50	1.517	.785	1.417	.773		1.25
55	1.599	.764	1.478	.756		1.27
60	1.691	.745	1.545	.739		1.30
65	1.795		1.619	.723		1.32
70	1.912		1.700	.708		1.35
75	2.045		1.790			1.37

15. Temperatures.

	C.	F.
Absolute zero	—273	—460
Lowest temperature yet attained.	140	220
Lowest observed temp. of air .	60	76
Mercury freezes	39	38
Water freezes	0	+32
Average temp. of earth's surface.	+15	60
Temperature of human body .	37	99
Highest observed temp. of air .	56	133
Wood's metal, 1 Cd, 2 Sn, 4 Pb,.	70	158
Rose's metal, 4 Bi, 1 Pb, 1 Sn, .	94	200
Boiling point of water . . .	100	212
Highest temp. sustained by man.	129	264
Boiling point of mercury. . .	357	677
Boiling point of sulphur . . .	440	824
Dark red heat (Draper) . . .	525	977
Boiling point of cadmium . .	860	1580
Cherry red	900	1650
Boiling point of zinc	1040	1900
Yellow heat.	1200	2200
White heat	1300	2400

16. Pressure of Vapors.

T.	Water.	Alcohol.	Ether.	C_2S	Oil Turp.	SO_2	NH_3	CO_2	Hg
−20	.91	3.34	68.90	47.30		479.46	1392.1	15142.4	
10	2.08	6.47	114.72	79.44		762.49	2144.6	20340.2	
0	4.60	12.70	184.39	127.91	2.07	1165.06	3183.3	26906.6	.02
+10	9.16	24.23	286.83	198.46	2.94	1799.55	4574.0	44716.6	.03
20	17.39	44.46	432.78	298.03	4.45	2462.05	6387.8	56119.0	.04
30	31.55	78.52	634.80	434.62	6.87	3431.80	8700.9	69184.4	.05
40	54.91	133.69	907.04	617.53	10.80	4670.23	11595.3		.08
50	91.98	219.90	1264.83	857.07	16.98	6220.01	15158.3		.11
60	148.79	350.21	1725.01	1164.51	26.46	8123.80	19482.1		.16
70	233.09	541.15	2304.90	1552.09	40.64		24675.5		.24
80	354.64	812.91	3022.79	2032.53	61.30		30843.1		.35
90	525.45	1189.30	3898.26	2619.08	90.61		38109.2		.51
100	760.00	1697.55	4953.30	3325.15	131.11		46608.2		.75
120	1491.28	3231.73	7719.20	5148.79	257.21				1.53
140	2717.63	5674.59		7603.96	464.02				3.06
160	4651.62				775.09				5.90
180	7546.39				1207.92				11.00
200	11689.0				1771.47				19.90
220	17390.4								34.70

17. Wet and Dry Bulb.

T.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	14°
−10	2.1	1.6	1.0	.5	.4									
8	2.5	1.9	1.4	.9	.4									
6	2.9	2.4	1.8	1.3	.8	.3								
4	3.4	2.9	2.3	1.8	1.3	.8	.3							
2	4.0	3.4	2.9	2.4	1.9	1.4	.9	.4						
0	4.6	4.0	3.4	2.8	2.2	1.6	1.0	.4						
+2	5.3	4.7	4.1	3.5	2.9	2.3	1.7	1.1	.5					
4	6.1	5.5	4.9	4.3	3.7	3.1	2.5	1.9	1.3	.7	.1			
6	7.0	6.4	5.8	5.2	.4.6	4.0	3.4	2.8	2.2	1.6	1.0	.4		
8	8.0	7.4	6.8	6.2	5.6	5.0	4.4	3.8	3.2	2.6	2.0	1.4	.8	.2
10	9.2	8.6	8.0	7.3	6.7	6.1	5.5	4.9	4.3	3.7	3.1	2.5	1.9	1.3
12	10.5	9.8	9.2	8.6	8.0	7.4	6.8	6.2	5.6	5.0	4.4	3.8	3.2	2.6
14	11.9	11.3	10.7	10.1	9.5	8.9	8.3	7.6	7.0	6.4	5.8	5.2	4.6	4.0
16	13.5	12.9	12.3	11.7	11.1	10.5	9.9	9.3	8.7	8.0	7.4	6.8	6.2	5.6
18	15.4	14.7	14.1	13.5	12.9	12.3	11.7	11.1	10.5	9.8	9.2	8.6	8.0	7.4
20	17.4	16.8	16.2	15.5	14.9	14.3	13.7	13.1	12.5	11.9	11.2	10.6	10.0	9.4
22	19.7	19.0	18.4	17.8	17.2	16.6	16.0	15.3	14.7	14.1	13.5	12.9	12.3	11.6
24	22.2	21.6	20.9	20.3	19.7	19.1	18.5	17.9	17.2	16.6	16.0	15.4	14.8	14.2
26	25.0	24.4	23.7	23.1	22.5	21.9	21.3	20.6	20.0	19.4	18.8	18.2	17.5	16.9
28	28.1	27.5	26.9	26.2	25.6	25.0	24.4	23.7	23.1	22.5	21.8	21.2	20.6	
30	31.5	30.9	30.3	29.7	29.0	28.4	27.8	27.2	26.5	25.9	25.3			
32	35.4	34.7	34.1	33.5	32.8	32.2	31.6	31.0	30.3					
34	39.6	38.9	38.3	37.7	37.0	36.4	35.8							

18. Solar System.

Name.	Sy.	Miles dist.	E = 1.	Time rev.	Ecc.	Incl.	Asc. node	Diam.	Mass.	S.G.
						° ′	° ′			
Sun	☉							852584	314760	1.4
Mercury	☿	35393000	.3871	87.97	.2056	7 0.4	45 57.5	2962	.065	6.8
Venus	♀	66131000	.7233	224.70	.0068	3 23.6	74 54.2	7510	.785	5.1
Earth	⊕	91430000	1.0000	365.26	.0168	0 0.0	00.0	7901	1.000	5.5
Moon	☽	238800	.0026	27.32	.0548	5 8.8	13 53.3	2153	.011	3.1
Mars	♂	139312000	1.5237	686.98	.0933	1 51.1	4 80.5	4920	.124	5.1
Jupiter	♃	475693000	5.2028	4332.53	.0482	1 18.5	98 26.3	85390	300.86	1.2
Saturn	♄	872135000	9.5388	10759.22	.0560	2 29.4	111 56.6	71904	90.032	.7
Uranus	♅	1753851000	19.1824	30686.82	.0466	0 46.9	72 59.6	33094	12.641	1.0
Neptune	♆	2746270000	30.0363	60126.71	.0087	1 47.0	13 05.2	36620	16.761	.9

R. A.		Dec.			Constellation.	Name	Mags.		Dist.	Angle	Color.
h.	m.		°	″					″	°	
O	30	N	33	0	Andromeda	π	4.5	9	36	173.9	w. bl.
I	12	N	88	37	Ursa Minor	α	2.5	9.5	18.6	210.1	ye. l bl.
I	17	N	67	27	Cassiopea	ψ	4.5	9	29	106	d ye. w.
I	46	N	18	40	Aries	γ	4.5	5	8.8	359.8	
I	56	N	41	42	Andromeda	γ	3.5	5.5	11	61.6	d ye. gr.
II	41	N	55	21	Perseus	η	5	8.5	28	300.4	o. bl.
III	48	S	3	20	Eridanus	32	5	7	6.6	346.5	d ye. d gr.
III	49	N	39	38	Perseus	ε	3.5	9	8.4	9.1	w. lil.
IV	10	S	7	50	Eridanus	o^2	5	9.5	82	107.6	o. bl.
IV	28	N	16	14	Taurus	α	1	12	108	35.9	
IV	29	N	9	54	Taurus	88	5	8.5	68	300.4	B.? w. bl.
IV	34	N	22	42	Taurus	τ	5	8	62	209.8	w. vi.
IV	50	N	37	42	Auriga	ω	5	9	7	352.6	w. l bl.
V	7	N	32	32	Auriga	14	5	7.5	13.5	224.5	
V	8	S	8	21	Orion	β	1	9	9.5	199.4	p ye. d bl.
V	16	N	3	25	Orion	23	5	7	32	27.9	w. l bl.
V	25	S	0	24	Orion	δ	2	7	53	359.9	w. w.
V	28	N	9	51	Orion	λ	4	6	4.5	43.0	p ye. pur.
V	29	S	6	0	Orion	ι	3.5	8.5	11.5	141.7	T. w. bl. r.
V	30	N	30	25	Auriga	26	5	8	12.3	267.8	w. p bl.
V	32	S	2	40	Orion	σ	4	8	12.5	84.2	T. w. bl. r.
V	39	S	22	29	Lepus	γ	4	6.5	93	349.0	p ye. p gr.
VI	36	N	25	15	Gemini	ε	3	9.5	111	94.1	w. d bl.
VII	12	N	22	13	Gemini	δ	3.5	9	7.2	200	w. pur.
VII	26	N	32	10	Gemini	α	3	3.5	5	240	B. w. w.
X	49	N	25	29	Leo	54	4.5	7	6.2	102.7	
X	56	N	62	27	Ursa Major	α	1.5	8	381	203.8	
XII	16	N	26	34	ComaBerenices	12	5	8	66	168.2	y. r.
XII	23	S	15	47	Corvus	δ	3	8.5	24	210.9	l ye. pur.
XII	35	S	0	44	Virgo	γ	4	4	5	150	B. w. p gr.
XII	50	N	39	4	Canes Venatici	12	2.5	6.5	19.8	227.0	w. lil.
XIII	19	N	55	36	Ursa Major	ζ	3	5	14.4	147.4	w. gr.
XIV	12	N	51	58	Boötes	ι	4.5	8	38	33.4	l ye. w.
XIV	35	N	16	59	Boötes	π	3.5	6	6	102	w. p ye.
XIV	44	S	15	30	Libra	α	3	6	229	314.3	p ye. gr.
XIV	45	N	19	39	Boötes	ξ	3.5	6.5	5	290	B.
XV	10	N	33	48	Boötes	δ	3.5	8.5	110	75.0	ye. lil.
XV	34	N	37	4	CoronaBorealis	ζ	5	6	6.1	301.2	w. gr.
XV	58	S	19	27	Scorpio	β	2	5.5	13.1	24.9	p ye. lil.
XVI	4	S	19	7	Scorpio	ν	4	7	40	338.5	p ye. w.
XVI	13	S	25	17	Scorpio	σ	4	9.5	20	271.6	w. r.
XVII	3	N	54	39	Draco	μ	4	4.5	3	180	B. w. w.
XVII	7	S	26	24	Ophiuchus	36	4.5	6.5	5	200	B. p r. p ye.
XVII	9	N	14	32	Hercules	α	3.5	5.5	4.5	118.7	o. gr.
XVII	10	N	25	0	Hercules	δ	4	8.5	19	180	
XVII	14	S	12	43	Serpens	ν	4.5	9	51	31.3	p gr. lil.
XVII	19	N	37	16	Hercules	ρ	4	5.5	3.7	308.9	w. p gr.
XVII	30	N	55	16	Draco	ν^1	5	5	62	311.8	w. w.
XVII	54	N	2	56	Ophiuchus	67	4	8	55	143.6	p ye. pur.
XVII	59	N	2	32	Ophiuchus	70	4.5	7	4	90	B. pur.
XVIII	33	N	38	40	Lyra	α	1	11	47	152.0	w. bl.
XVIII	40	N	37	28	Lyra	ζ	5	5.5	44	149.6	ye. p gr.
XVIII	40	N	39	32	Lyra	ε^1	5	6.5	3	18	B. ye. p r.
XVIII	40	N	39	32	Lyra	ε^2	5	5.5	2.5	145	B. w. w.
XVIII	50	N	4	2	Serpens	θ	4.5	5	22	103.9	p ye. d ye.
XIX	9	N	38	55	Lyra	η	5	9	28	84.8	bl. vi.
XIX	25	N	27	41	Cygnus	β	3	7	34	55.6	d ye. d bl.
XIX	41	N	33	26	Cygnus	χ	5	9	26	72.9	d ye. p bl.
XIX	43	N	18	49	Sagitta	ζ	5	9	8.6	312.3	w. bl.

R. A.		Dec.			Constellation.	Name	Mags.		Dist.	Angle	Color.
h.	m.		°	′					″	°	
XX	10	N	46	21	Cygnus	o^2	4	7.5	107	174.1	T. o. bl. bl.
XX	13	N	77	19	Cepheus	κ	4.5	8.5	7.5	123.8	p ye. bl.
XX	14	S	15	11	Capriconus	β	3.5	7	205	267.2	o. bl.
XX	41	N	15	40	Delphinus	γ	4	7	11.8	273.3	ye. p gr.
XXI	1	N	38	5	Cygnus	61	5.5	6	20	120	B. ye. d ye.
XXI	16	N	19	15	Pegasus	I	4	9	36	310.8	d ye. lil.
XXI	27	N	70	0	Cepheus	β	3	8	14	251.0	w. bl.
XXI	38	N	9	17	Pegasus	ε	2.5	9	138	324.3	ye. bl.
XXI	38	N	28	10	Cygnus	μ	5	6	5.4	114.3	T. w. bl. bl.
XXII	0	N	64	0	Cepheus	ξ	5	7	6	286	
XXII	24	N	57	45	Cepheus	δ	4.5	7	41	192.2	d ye. d. bl.

20. Clusters and Nebulæ.

No.	R. A.		Dec.			Constellation.	Name.	Remarks.
	h.	m.						
116	O	36	N	40	30	Andromeda	M 31	C. E. large, oval.
352	I	27	N	30	1	Triangulum	M 33	O. large, faint cluster.
392	I	37	N	60	35	Cassiopea	♅ VI 31	O. cluster.
512	II	10	N	56	34	Perseus	♅ VI 33	E. brilliant cluster.
584	II	34	N	42	11	Perseus	M 34	E. fine.
	III	40	N	23	45	Taurus		Pleiades.
826	IV	8	S	13	4	Eridanus	♅ IV 26	C. planetary nebula.
1157	V	27	N	21	56	Taurus	M 1	Crab nebula.
1179	V	29	S	5	29	Orion	M 42	G. E. brightest nebula.
1295	V	44	N	32	31	Auriga	M 37	Fine. 500 stars.
1360	VI	1	N	24	21	Gemini	M 35	E. cluster stars uniform.
1424	VI	24	N	5	2	Monoceros	♅ VII 2	E. cluster.
1454	VI	41	S	20	37	Canis Major	M 41	E. fine group.
1551	VII	31	S	14	12	Argo Navis	♅ VIII 38	E. large group.
1564	VII	36	S	14	31	Argo Navis	M 46	Circular clust. diam. 30′
1571	VII	39	S	23	33	Argo Navis	M 93	Bright cluster.
1611	VII	54	S	10	25	Argo Navis	♅ VI 37	Fine vicinity.
1681	VIII	33	N	20	24	Cancer	M 44	Praesepe.
1712	VIII	44	N	12	17	Cancer	M 67	O. cluster.
2102	X	18	S	18	0	Hydra	♅ IV 27	Plan. neb. like Jupiter.
2343	XI	7	N	55	43	Ursa Major	M 97	Plan. neb. diam. 2′40″
2838	XII	12	N	15	15	Virgo	M 99	Spiral nebula.
3049	XII	25	N	15	8	Virgo	M 88	Dull, fine neb. vicinity.
3132	XII	33	S	10	54	Virgo	♅ I 43	Elongated.
3572	XIII	34	N	47	52	Canes Venatici	M 51	C. O. spiral.
3636	XIII	36	N	29	1	Canes Venatici	M 3	C. cluster.
4173	XVI	9	S	22	40	Scorpio	M 80	Like a comet.
4230	XVI	37	N	36	42	Hercules	M 13	C. E. cl. finest of kind.
4294	XVII	13	N	43	16	Hercules	M 92	C. E. cl. like M 13.
4346	XVII	49	S	18	59	Ophiuchus	M 23	Fine vicinity.
4361	XVII	56	S	24	21	Sagittarius	M 8	E. bright part Galaxy.
4373	XVII	59	N	66	38	Draco	♅ IV 37	G. plan. neb. diam. 35″
4397	XVIII	11	S	18	27	Clypeus	M 24	G. E. fine vicinity.
4400	XVIII	11	S	13	50	Clypeus	M 16	Fine cluster.
4401	XVIII	12	S	17	11	Clypeus	M 18	Fine vicinity.
4403	XVIII	13	S	16	15	Clypeus	M 17	G. O. Horseshoe neb.
4410	XVIII	21	N	6	29	Serpens	♅ VIII 72	E. fine.
4424	XVIII	28	S	24	0	Sagittarius	M 22	Br. cl. stars 11 mag.
4432	XVIII	38	S	9	32	Scutum	M 26	Coarse cluster.
4447	XVIII	49	N	32	52	Lyra	M 57	G. finest annular neb.
4532	XIX	54	N	22	22	Vulpecula	M 27	G. O. Dumb-bell neb.
4628	XX	57	S	11	52	Aquarius	♅ IV 1	G. plan. neb. diam. 20″
4670	XXI	24	N	11	35	Pegasus	M 15	C. insulated resolv. cl.
4678	XXI	27	S	1	24	Aquarius	M 2	C. structure granulated.
4687	XXI	33	S	23	44	Capricornus	M 30	Cluster.

Appendix C.

PHYSICAL LABORATORIES.

In the Preface to the first volume of this work is a brief description of the method recommended for conducting a Physical Laboratory. It is believed, however, that some suggestions regarding details, may prove of value. If the Laboratory is to be used simply for the current instruction of large classes it should consist of a large room with two or three of smaller size adjacent. One of the latter should be arranged so that it may be completely darkened for the Photometers, Expts. 67 and 69; another should be partially dark for Expts. 65, 72, 76, 78, 88, 94, 103, 104, 111 and 113, while a third room should be provided with a *porte-lumière* and a southern window for Expts. 77, 89, 90, 91, 131 and for Lantern Projections. In the larger room some of the experiments require a good light, which is best attained by tables in front of a northern window which the student faces. Expts. 71, 79, 92 and 93 should be so placed. Most of the other experiments may be performed on tables placed in the centre of the large room, so that students may work on both sides of them. They should be three feet high that the students may stand, or sit on high stools. Four feet is a convenient width, and the length will depend on the location. The space beneath them may be made available by drawers and cupboards. Most of the tables should be provided with gas and, for some, water is needed, as in a chemical laboratory. A considerable amount of wall space should be left free and covered with wood rather than plaster, as it is very convenient for Expts. 23, 24, 26, 27, 34, 39, 41, 42, 49, 50, 55, 63, 121, and 128. Curves, drawings and photographs may also be hung up on the wall for examination or consultation.

The indicator board, to show what work each student is doing, may be made of various forms. A convenient plan is to drive pins obliquely into a drawing board in rows so that they shall be separated about three inches horizontally, and two inches vertically. The heads of the pins are then cut off, and cards hung on them, those in the first vertical row bearing the names of the experiments, those in the other rows giving the names of the students. The class should be divided into sections of from fifteen to twenty-five students, though the smaller number is much to be preferred. The number of experiments should be considerably greater, that there may be no delay, and the simpler experiments should gradually be replaced by those of greater difficulty. The following list is a good one to begin with: Expts. 1, 2, 3, 4, 5, 7, 10, 11, 12, 13, 14, 15, 17, 18, 20, 23, 24, 25, 26, 28, 29, 30, 35, 36, 41, 42, 45, 46, 70, and 78. If the class is small, every student

may be required to perform the following Expts., 3 or 4, 10 or 11, 12, 14, 15, 23 or 24, 25 or 26, 28 or 29, 35, 41, 45, 46 and 78, all of which work well in practice. When the student has acquired some skill with these experiments, the following may be added, 19, 21, 44, 48, 67, 69, 71, 72, 76, 77, 79, 88, 91, 92, 93 and 94. Some experiments should next be given from the second volume, as 95, 96, 97, 98, 101, 102, 103, 104, 105, 109, 110, 111, 112, 114, 119, 120, 122, 126, 131, 133, 134, 138, 145 and 147. The later experiments should be taken up by the older students according to their respective wants, and form, in fact, several distinct courses.

The Graphical Method is used very largely in the discussion of the results of these experiments, as it possesses great advantages in many ways. It shows at a glance the accuracy of the work, and as modified, is exact enough to show the errors of the most carefully conducted experiments. The paper on which the curves are drawn may be prepared in various ways. It should be divided into squares by two sets of parallel lines of which every fifth should be more marked than the rest. The interval between the lines may be one millimetre, but generally a coarser ruling, as two millimetres or a tenth of an inch, is more convenient. The absolute interval is unimportant, but regularity is desirable, though by the method of residual curves the errors may be rendered so large that defects in the ruling will be quite imperceptible. The squares may be engraved on metal or stone, and the paper printed as in line engraving or lithography, but since the paper must be wet in these processes, the irregular contraction will introduce errors. A number of printer's *rules* may also be set up at the required intervals, and the lines printed from them on a common printing press. By taking two impressions on the paper, turning it 90°, the squares are formed by the intersection of the two sets of lines. This method would be cheap for a very large number of copies, but if the squares are not to be very small the best method seems to be to rule the paper like writing paper. A set of pens is obtained properly spaced and every fifth one is spread slightly, so that it shall make a broader line. The paper is then ruled twice at right angles. The sheets should be about 16 × 21 inches, and be cut into six parts, 7 × 8 inches. Each student should be provided with a note book, on the cover of which, his name should be marked. A convenient size of page is 7 × 8 inches so that the paper on which the curves are drawn, may be pasted into the book. Students in taking notes may follow the rule that the observations, method of discussing them, and the results, should be entered in full, so that any one understanding the experiment may see exactly what has been done. Each student, after performing an experiment, should check it off in the Index, as it is thus easy to see at a glance what others he must still perform.

The cost of establishing a Physical Laboratory need not be very great, and a prominent object throughout this work has been to devise apparatus which will be efficient, without being expensive. A large portion of it may be made by any carpenter, and the expense should be mainly in the moving portions, as the micrometer screws, joints and slides. A great saving may also be effected in the graduations, which may be made in many cases on paper instead of on metal. When great accuracy is essential, steel scales may be procured divided either into millimetres or fractions of an inch. Where, however, the readings are made simply by the eye, sufficient accuracy is often attainable with paper or cardboard scales. If these are lithographed or engraved, an error is introduced from the shrinkage of the paper, since it must be printed wet. A better method, therefore, is to cut out the scale from a plate of type metal or set it up with printer's rules.

Great care must be taken as regards the intervals, which may be adjusted with sheet-metal or tinfoil and the whole *justified*, or continually compared with a steel graduated scale. The scales may then be printed at trifling expense on a common printing press on dry paper.

Graduated circles may also be printed on paper or cardboard at small expense; but a simple and inexpensive way to graduate metallic circles is on a lathe with the index wheel used for cutting gears. The average error in this case is only about 2′ which is of no consequence when the readings are made by the eye to tenths of a degree only.

Small microscopes and telescopes are required in many of the experiments for reading scales and for other purposes. These instruments are made very cheaply in France, and though not suitable for the most accurate work, yet for ordinary purposes are all that is required.

By adopting the method of weighing described in Experiment 19, great accuracy is attainable at small expense. With large balances the index may be replaced by a spirit-level attached to the beam, which will show very small variations in the load. The knife edges may also be replaced by pieces of steel watch spring, like the suspension of a pendulum, with the advantage of freedom from friction. This is especially convenient where acids are to be weighed, since their fumes would soon dull the knife edges, while, as the springs are straight, when the beam is level, variations in their elasticity will not affect the result.

Reference has frequently been made to a simple form of galvanometer which combines at the same time efficiency and cheapness. A circular box is turned out of wood, having an interior diameter of about four inches and a depth of an inch and a half. On the bottom of this is placed a circular piece of looking-glass and on it a cardboard graduated circle, with the central portion removed. The top of the box is formed of a circular plate of glass sunk in the wood so that its upper surface shall be flush with the top of the box. The magnet consists of a piece of watch-spring about three eighths of an inch long, straightened by bending, and magnetized by rubbing it on a powerful magnet. A piece of fine wire nearly four inches long, is now straightened by rolling it between two plates of glass and is attached to the magnet by enclosing both in a small stirrup of paper. The latter is then suspended by a single filament of silk from the centre of the glass at such a distance that as the magnet turns it will approach but not touch the graduated circle. To find the centre of the glass, or point exactly over the centre of the graduated circle, lay a rule on the glass so that when the eye is brought into the plane passing through its edge and its reflection, the reading of both ends shall be the same. Draw a short line near the centre of the circle on the glass with common ink. Repeat, turning the ruler nearly at right angles. The intersection will give the required centre. Now turn the glass over, put a drop of varnish on the centre, dip the end of the filament of silk in it, and fasten it in place with a small piece of paper bringing its edge just over the cross. The exactness of the centering may be tested by Expt. 7. In this form the instrument makes an excellent compass. The reading may be taken to tenths of a degree by placing the eye so that the wire index shall just cover its reflection, and estimating the tenths. To convert this into a galvanometer a piece of covered wire is wound on a flat, square block of wood, and inserted in a square hole or mortise cut in the bottom of the box. The ends of the wire are then connected with binding screws in the sides of the box. The advantages of this instrument are, that, as there is no friction, minute deflections may be observed with accuracy, the error from parallax is eliminated

by the mirror, and the tangents of the angular readings are nearly proportional to the currents on account of the short length of needle. The lower half of the coil tends to counteract the effect of the upper portion, but its distance being greater the effect is slight. The principal objection to this instrument, if great accuracy is required, is the torsion of the silk fibre, which introduces an error. To avoid this, the needle should never be allowed to swing completely around, and if it deviates from the magnetic north, the cover should be turned until the filament is untwisted.

The current expenses of the Laboratory need not be great, since the apparatus is easily replaced and not easily injured. It was anticipated that the loss by breakage would be considerable, but in practice it has proved to be very trifling, in fact, almost nothing, except from causes beyond the control of the student. The annual expenses compare very favorably with those of a chemical laboratory, as so little material is consumed, and the apparatus can in general be used over and over again.

Where the more advanced work of the second volume is to be done, a number of small rooms are much more convenient than a single large laboratory. One is needed as a workshop, and should be provided with carpenter's tools, a lathe, a table with blast-lamp for glass-work, and tools for working in metal, soldering and other similar work. Another room is needed for experiments in Mechanical Engineering, which should be, if possible, on the ground floor, and should contain an engine and boiler. The measurements of terrestrial magnetism should also be made on the ground floor, or at least on stone piers disconnected from the building, or preferably in a small separate building. Great care should be taken that no iron is near, especially if it is liable to be moved. The astronomical work should be done in an observatory which may be on top of the building if the instruments rest on stone piers. It is difficult, however, to attain steadiness at a great height. The need of a clear horizon is much less than is commonly supposed, except in special cases. Generally if we can observe to within 20° or 30° of the horizon to the south, and even to within 30° or 40° in other directions, it is all that is really needed. The effect of the heated air from chimneys, etc., however, extends much beyond their apparent altitude. The Lantern Projections should be conducted in a lecture room, and students may acquire practice in addressing an audience and performing experiments in their presence, by inviting their friends to an exhibition of the various phenomena at the close of their course.

One of the principal objections made to the introduction of the Laboratory method of teaching Physics, was the amount of time that would be required for each experiment. It was said that as it takes an entire day to measure a temperature accurately with the air thermometer, that in a whole term the student would become familiar with but very few methods of experiment. While this might be the case with a certain class of experiments, it is wholly different with the work described in this book. When the students first entered our Laboratory, the average time per experiment, including absences, was 1.8 hours. By the introduction of Volume I, so that they could read over the descriptions of the experiments at home, and by the gradual improvement in the apparatus, the average time has been materially reduced. Probably with small sections and abundant means for keeping the apparatus in perfect condition, the time would not greatly exceed one hour.

Books of Reference.

A good Library, even if not very extensive, is an exceedingly valuable adjunct to a Physical Laboratory, provided the selection is properly made. A list is accordingly appended of a few books useful for frequent reference and forming a working library, such as should be at the command of every physicist. First are given the principal German, French, English and American periodicals, that relate to physics; y. denotes that the numbers are published yearly, q. quarterly, m. monthly, w. weekly, and i. at irregular intervals. Then follow works on general physics and its branches, works on kindred subjects, and finally the catalogues of instrument makers, which are sufficiently complete to render them valuable for reference. To these should be added a good Encyclopædia, German, French and English dictionaries, an atlas and seven-place logarithmic tables.

Periodicals.

Poggendorff's Annalen. m.
Fortschrifte der Physik. y.
Carl's Repetitorium. m.
Astronomische Nachrichten. w.
Bulletin of the Royal Academy of of St Petersburg. q.
Memoirs of the Royal Academy of Vienna. i.
Memoirs of the Swedish Roy. Acad.
Bibliothèque Universelle. m.
Comptes Rendus. w.
Memoirs of the French Academy. i.
Annales de Chim. et de Physique. m.
Journal de Physique. m.
Annales de l'École norm. supér. m.
Les Mondes. w.
London Philosophical Magazine. m.
Philosophical Transactions of the Royal Society. i.
Nature. w.
Report of the British Association. y.
Quarterly Journal of Science. q.
Quarterly Journal of Microscopical Science. q.
Monthly Microscopical Journal. m.
Astronomical Register. m.
Symons' Meteorological Journal. m.
Electrical News. w.
Photographic News. w.
Amer. Journ. of Arts and Sci. m.
Journal of the Franklin Instit. m.
Proc. of the Amer. Association. y.
Proc. of the American Academy. i.
Proc. of the Amer. Philos. Soc. $\frac{1}{2}$y.
Smithsonian Reports. y.
Coast Survey Reports. y.
Annual Reports of the Signal Service Department.
Popular Science Monthly. m.
The Lens. q.
The Magic Lantern. m.
The Telegrapher. m.

General Physics.

Wüllner. Lehrbuch des Exper. Physik. 4v.
Hessler, Pisko. Lehrbuch der Technische Physik. 2v.
Verdet. Œuvres. 8v.
Jamin. Cours de Physique. 3v.
Jamin. Petit traité de Physique.
Daguin. Traitè elementaire de Physique. 4v.
Boutan, Almeida. Cours elem. de Physique. 2v.
Pouillet. Elemens de Physique. 3v.
Deschanel. Elementary Treatise on Nat. Philos. 4v.
Ganot. Elem. Treatise on Physics.
Muller. Lehrbuch der Physik und Meteorologie. 3v.
Mayer. Lecture Notes on Physics.
Stewart. Lessons in Elementary Physics.
Lardner. Handbook of Natural Philosophy. 4v.
Silliman. Principles of Physics.

Mechanics.

Thomson, Tait. Treatise on Natural Philosophy.
Weisbach. The Mechanics of Engineering and Machinery.
Rankine. A Manual of Applied Mechanics.
Todhunter. Analytical Statics.
Tait, Steele. Dynamics of a Particle.
Goodeve. Principles of Mechanics.
Goodeve. Elements of Mechanism.
Cross. Course in Elemen. Physics.
Smith. An Elem. Treatise on Mechanics.
Ball. Experimental Mechanics.
D'Aubuisson. Hydraulics.
Francis. Lowell Hydraulic Exper.
Bunsen. Gasometry.
Williamson. Use of the Barometer.
Clegg. Treatise on the Manufacture of Coal Gas.

Sound.

Helmholtz. Die Lehre von den Tonempfindungen.
Tyndall. On Sound.
Airy. On Sound and Atmos. Vibrat.
Donkin. Acoustics.
Taylor. Sound and Music.
Peirce. An Elem. Treatise on Sound.
Radau. Wonders of Sound.
Hopkins and Rimbault. The Organ.

Light.

Billet. Traité d'Optique phys. 2v.
Moigno. Répertoire d'Optique moderne. 4v.
Nugent. Treatise on Optics.
Parkinson. A Treatise on Optics.
Airy. Undul. Theory of Optics.
Fresnel. Œuvres complétes. 3v.
Potter. An Elem. Treat. on Optics.
Potter. Physical Optics.
Brewster. A Treatise on Optics.
Brewster. New Philos. Instruments.
Tyndall. Light and Electricity.
Tyndall. Six Lectures on Light.
Lommel. Light. (Int. Sci. Ser.)
Helmholtz. Physiological Optics.
Kirchhoff. Researches on the Solar Spectrum.
Angström. Spectre Normale.
Schellen. Spectrum Analysis.
Roscoe. Spectrum Analysis.
Watts. Index of Spectra.
Lockyer. The Spectroscope and its Applications.
Grandeau. Instruction pratique sur l'Analyse Spectrale.
Chevreul. De la loi du contraste simultané des Couleurs.
Bezold. Die Farbenlehre.
Brücke. Die Physiol. der Farben.
Pereira. Lectures on Polar. Light.
Spottiswoode. Polariz. of Light.
Woodward. Familiar Introduction to the study of Polarized Light.
Carpenter. The Microscope and its Revelations.
Beale. How to work with the Microscope.
Griffith, Henfrey. The Micrographic Dictionary.
Hogg. The Microscope.
Moigno. L'Art des Projections.
Monkhoven. A Popular Treatise on Photography.
Sutton. A Dictionary of Photog.
Hunt. A Manual of Photography.
Stillman. Photography.
Vogel. The Chemistry of Light and Photography (Int. Sci. Ser.)
Phipson. Phosphorescence.

Electricity.

De la Rive. A Treatise on Elec. 3v.
Faraday. Experimental Researches in Electricity. 3v.
Becquerel. Traité d'Elect. et de Magnétisme. 3v.
Maxwell. Electricity and Mag. 2v.
Thomson. Papers on Statical Electricity and Magnetism.
Wiedemann. Die Lehre von Galvanismus.

Electricity (Continued).

Jenkin. Electricity and Magnet.
Noad. A Manual of Electricity.
Noad. Student's Text Book of Electricity.
Guthrie. Electricity and Magnetism.
Jenkin. Reports on Elect. Standards.
Du Moncel. Applications d'Electricité. 4v.
Dub. Electromagnetismus.
Blavier. Nouveau Traité de Télég.
Schellen. Der electrom. Telegraph.
Sabine. The Electric Telegraph.
Culley. A Handbook of Practical Telegraphy.
Russell. History of the Elect. Teleg.
Russell. The Atlantic Telegraph.
Watts. Manual of Electro-Metal.
Napier. Electro-Metallurgy.
Rosaline. Galvanoplastic Manip.
Lamont. Der Erdström.
Airy. A Treatise on Magnetism.
Harris. Rudimentary Magnetism.
Tyndall. Researches on Diamag.

Heat.

Regnault. Memoirs of the French Academy. Vols. xxi, xxvi.
Hirn. Théorie mécanique de la Chaleur.
Zeuner. Théorie mécanique de la Chaleur.
Clausius. The Mechanical Theory of Heat.
Rankine. Steam Engine and other Prime Movers.
Briot. Théorie mécanique de la Chaleur.
Melloni. La Thermochrose.
Tait. Thermodynamics.
Maxwell. Theory of Heat.
Stewart. An Elementary Treatise on Heat.
Stewart. Conservation of Energy.
Tyndall. Heat as a Mode of Motion.
Tyndall. Contrib. to Molec. Sci. in the Domain of Radiant Heat.
Tyndall. On Radiation.
Péclet. Traité de la Chaleur. 3v.

Astronomy.

Watson. Theoretical Astronomy.
Chauvenet. Manual of Spherical and Practical Astronomy.
Loomis. An Introduction to Practical Astronomy.
Brünnow. Spherical Astronomy.
Coffin. Navigation and Nautical Astronomy.
Bowditch. Practical Navigator.
Grant. History of Physical Astron.
Arago. Popular Astronomy.
Chambers. Descriptive Astronomy.
Herschel. Outlines of Astronomy.
Loomis. A Treatise on Astronomy.
Lockyer. Elem. Lessons in Astron.
Guillemin. Le Ciel.
Smyth. Cycle of Celestial Objects.
Webb. Celestial Objects for Common Telescopes.
Secchi. Le Soleil.
Lockyer. Contrib. to Solar Physics.
Beer, Mädler. Der Mond.
Beer, Mädler. Mappa Selenograph.
Nasmyth, Carpenter. The Moon.
Proctor. Saturn and his System.
Heis. Atlas cœlestis novus.

Tables.

Hutton. Mathematical Tables.
Barlow. Tables of Squares, Cubes.
Crelle. Rechentafeln.
Clarke. Constants of Nature.
Rankine. Rules and Tables.
Guyot. Tables, Meteor. and Phys.
Sharples. Chemical Tables.
Sabine, Clark. Electrical Tables.
Hirsch. Definite Integrals.
Alexander. Universal Dictionary of Weights and Measures.

Catalogues.

Salleron. Paris.
Deleuil. Paris.
Alvergniat. Paris.
Hoffman. Optical. Paris.
Kœnig. Acoustic. Paris.
Beck. Optical and General. Lond.
Negretti and Zambra. London.
Elliott Brothers. Electrical. Lond.
Casella. Meteorological. London.
Griffith. Chemical. London.
Queen. Philadelphia, New York.

Miscellaneous.

Royal Society. Catal. of Sci. Papers.
Poggendorff. Handwörterbuch Biog.
Karmarsch. Technolog. Dictionary.
Whewell. Hist. of Induct. Sci. 2v.
Kohlrausch. Phys. Measurements.
Frick. Physical Technics.
Nicol. Cyclopædia of Sciences.
Davies, Peck. Mathemat. Diction.
Wurtz. Dictionaire de Chemie.
Watts. Dictionary of Chemistry.
Taylor. Scientific Memoirs.
Miller. Elements of Chemistry.
Cooke. Chemical Physics.
Cooke. The New Chemistry (Int. Sci. Ser.)
Eliot, Storer. Manual of Chemistry.
Helmholtz. Popular Lectures.
Youmans. Conservation and Correlation of Forces.
Barnard. Report on the Industrial Arts, Paris Expos., 1867.
Barnard. The Metric System.
Brook. French Measures and English Equivalents.
Gillespie. Land Surveying.
Buchan. Handy Book of Meteorol.
Dove. Law of Storms.
Herschel. Meteorology.
Loomis. A Treatise on Meteorology.

ADDITIONAL EXPERIMENTS.

One of the greatest advantages of a Physical Laboratory would, however, be lost if the work should be confined to what has been already described. The highest aim of every physicist should be to direct, not only his own utmost efforts, but those of his students, toward original investigations or the determination of new facts and laws. Without this, he is liable to become a mere machine, disseminating knowledge, but never advancing it. The whole aim of this book has been in this direction, and without it, we may educate followers, but never leaders in science. Originality is not, however, easily acquired, and seems to come to many persons as a natural gift. Nothing appears to be more difficult to many students than to attempt something new. Much, however, may be acquired by practice; the student should first be required to carry out some simple details, then to devise and plan an instrument of a given construction, and next to devise apparatus for producing some required effect. Much aid may often thus be obtained from students in planning and carrying out a Laboratory, or in increasing the number of experiments. Some problems should now be proposed to the student which he should be required to solve experimentally; and generally by this time his tastes will lead him towards some branch of the subject, or some difficulty will present itself which he should attempt to solve himself. He will now have attained the position of a true student of science, one who aims to study the unknown, as well as the known. Such work is, however, by no means easy for the instructor, especially when he has a large number of students to direct. It is believed that the following list of one hundred additional experiments will materially aid him in this matter. They cover a broad range of sub-

jects and serve as suggestions of problems for the student. Some of them are extremely simple, and serve as examples for the student to describe clearly and concisely how they should be performed. They may also be used where large classes are to be taught and a larger number of very elementary experiments are required. Others again are taken from published memoirs, and include various suggestive methods, as yet by no means exhausted. A thoughtful student will often, from an examination of these, see some new application or extension which may lead to most valuable results. These may be increased almost indefinitely, in fact, the current numbers of the scientific periodicals, especially *Poggendorff's Annalen* and the *Comptes Rendus* are full of them. Others again are new, and contain suggestions of valuable work which might be done by any one who will devote sufficient time and labor to them. The tables of Appendix *B*, especially Nos. 10, 11, 12, and 13, suggest many constants which need to be more accurately determined and point out many gaps which need to be filled. Enough has been said to show how vast the field is, and that the following list might have been almost indefinitely extended.

201. Measure any distance in metres and in inches, and reducing both to the same unit, determine the error of the scales. Place English and French scales edge to edge, and noting where the divisions coincide, as in the vernier, determine the ratio of the metre to the inch.

202. Weigh a pound Troy in grammes and determine the error, measure also the weight of a kilogramme in grains. Weigh any convenient object with French and with English weights, and, reducing the latter to grammes, determine the error.

203. Cut out a circle of cardboard and determine its area by the formula πr^2, by drawing parallel lines and applying the formula $A = \frac{1}{2}a\ (b_1 + 2b_2 + 2b_3 + \&c.\ .\ .\ .\ b_n)$ and $A = \frac{1}{3}a\ (b_1 + 4b_2 + 2b_3 + 4b_4 + 2b_5 + \&c.,\ .\ .\ .\ .\ b_n)$. Determine also the area by weighing the cardboard, and by drawing the circle on rectangular paper and counting the squares as in Vol. I, p. 22.

204. Form a triangle by making three pin holes in a sheet of cardboard. Measure the three angles by a table of chords, and by a protractor, and see if the sum equals 180°. Compute the angles also trigonometrically, after measuring the three sides.

205. Measure the thickness of some sheet-metal, the diameter of wires, and the exterior and interior diameters of tubes with gauges and calipers. Determine from this the numbers of the Birmingham and American wire gauges in inches or millimetres.

206. Measure the thickness of thin plates with Cornu's reflecting spherometer and compare the results with those obtained by a sheet-metal gauge (*Journ. de Phys.*, iv. 7).

207. Form a simple pendulum by suspending a heavy ball from the end of a string. Measure the time of a hundred vibrations, giving the string various lengths. Find the relation of the time to the length, by constructing a line with coördinates equal to the logarithms of these quantities. Finally, deduce the value of g from each observation.

208. Measure the force of gravity by Kater's pendulum, determining the time of vibration, as in Experiment 41, and the distance between the knife-edges by Experiment 20.

209. Measure on the floor of the laboratory a distance of twenty metres with the greatest possible accuracy, by the method of measuring base-lines, Vol. I, p. 21, or by Experiment 20. Mark each five metre point by a nail with a fine cross scratched upon it. Determine the variation of length of a steel or linen tape-measure when subjected to various strains. Measure also the sag when it is hung freely at both ends under various tensions and compare the observed length of the catenary with that given by theory.

210. Measure the strength of different kinds and sizes of thread and of various kinds of knots, by the following apparatus. Suspend a cannon ball by a fine wire and attach the thread to be tested to its centre. Draw the ball from the vertical by the thread, until the latter breaks. Its strength will then be nearly proportional to the distance through which the ball has been moved. This distance may be accurately measured by allowing a bent wire to trail from the ball, dragging a second wire while the ball moves, but detaching itself when the ball swings back on the breaking of the thread.

211. Find the position of the neutral axis of a bar bent transversely, by measuring the distance between two pairs of points near the upper and lower surfaces before and after the load is applied.

212. Determine the laws of torsion by suspending a magnet by a fine wire. Determine the angular deviation of the upper end by a graduated circle, and of the lower end by a mirror and graduated circle below the magnet. Measure the magnetic moment of the magnet, and determine its deviation as the upper end of the wire is turned. Repeat with wires of other lengths, diameters and materials. The torsion of a spider's thread, or filament of silk may be similarly measured. The compass described on p. 266 is well adapted to this experiment. Turn the magnet 360° by a large magnet, and notice the change of reading. Repeat until it is twisted nearly at right angles to the meridian. It will now slowly return owing to the permanent set of the fibre.

213. Hang two ivory balls, side by side, in front of a graduated scale. Draw one aside a known distance, and letting it fall back observe the motion of each ball after impact. Compare the results with theory, using balls first of the same, and then of different, sizes. Deduce thus the coefficient of elasticity.

214. Measure the velocity of the bullet from a revolver, parlor rifle, cross-bow or catapult with a ballistic pendulum. If the catapult is used attach it to the table, and determine the effect of drawing the spring by known amounts, and also by varying the weight of the ball.

215. Test the laws of impact in the case of a pile driver. The pile may be a stick an inch in diameter and be driven by a weight of 10 lbs., falling from various heights. Measure the height of fall, and descent of the pile after each blow and compare with the weight required to depress the pile by a dead pressure. The pile may be driven into clay, or the resistance may be produced by clamping it between two boards.

216. Cut a hole in the bottom or side of a box, and close it by a board fitting loosely. Fill the box with sand and measure the magnitude and point of application of the force required to prevent the sand from forcing the board outwards.

217. Make four pin holes in a sheet of drawing paper forming a square, and measure the distances of each from the others very exactly. Stretch

the paper on a drawing-board and measure again. Repeat after cutting the paper off the board, and then measure the variations, both parallel and at right angles to the fibre with various conditions of temperature and moisture.

218. Repeat the experiments of Clarke on the pressure of sap in plants (*Amer. Journ. Sci.*, cvii, 522).

219. Allow a stream of water to impinge upon a vertical disk attached to one end of a suspended beam, and measure the pressure exerted by the weight which must be added to the other end to keep the beam horizontal. Repeat, directing the stream at various angles and varying the pressure or velocity, the diameter of the stream, the size of the disk, and making it concave or convex.

220. Measure the flow of liquids through capillary tubes by the method of Poiseuille (*Ann. Chim. Phys.*, III, xxi, 76).

221. Determine the laws of osmotic action of liquids by the method of Graham (*Phil. Trans.*, 1854 and 1861).

222. Determine the amount of air carried down by adhesion to raindrops.

223. Measure the resistance of the air by making a disk of cardboard revolve at the end of a horizontal bar, and measure the force required on varying the size and form of the disk and its velocity.

224. Determine the flow of gases through small apertures by measuring the variation of pressure of a receiver from which the air has been exhausted, and into which it is allowed to return through a pin hole in a platinum plate.

225. Determine the flow of gases by Bunsen's method of determining the density of gases.

226. Determine the transpiration of gases by the method of Graham (*Phil. Trans.*, 1846 and 1849).

227. Determine the viscosity of gases by the method of Meyer (*Pogg. Ann.*, cxxv, 177), and Maxwell (*Phil. Trans.*, 1866, 249).

228. Study the laws of the passage of gases through porous plates. Suitable plates of any desired thickness may be made by moulding plaster of Paris between two plates of glass. Close a glass tube by such a plate, and immersing the open end in water measure the changes in level under different pressures. Repeat, filling the tube with other gases instead of air, and also replacing the plaster with rubber or bladder.

229. Measure the length of a rectangular organ pipe and add twice its depth. See how the result agrees with the wave-length computed from the velocity of sound and the pitch. Repeat with other pipes and with cylindrical pipes, adding to their length five-thirds of their diameter.

230. Measure on the monochord the length of string corresponding to the various notes of the scale, and compare with the computed pitch.

231. Repeat Melde's Experiment as modified by Lowery (*Amer. Journ. Sci.*, cvii, 493). See Experiment 63.

232. Determine the number of vibrations of a tuning fork from its dimensions, by the formulas of Mercadier. (*Comptes Rendus*, lxxix, 1001, 1069). Determine also the law connecting the position and magnitude of a weight on the prongs with the number of vibrations as given by Lissajous' method.

233. Measure electrically the number of vibrations of each string of the middle octave of a well-tuned piano by the method of Cooley (*Journ. Frank. Inst.*, lxxxvii, 44; lxxxviii, 341).

234. Determine by Lissajous' curves the number of vibrations of each reed of a well-tuned cabinet organ. A *comparateur* should be used, kept vibrating electrically, whose pitch may be slightly altered, without stopping it, by moving a weight.

235. Determine the relation between the velocity of translation of a moving sound and its change in pitch, as proposed by Mayer (*Amer. Journ. Sci.*, ciii, 267).

236. Determine the phase of vibration of the air surrounding a sounding body, as proposed by Mayer (*Amer. Journ. Sci.*, civ, 387, 504).

237. Measure the wave-lengths of sounds in air, and test the applications of the method, as proposed by Mayer (*Amer. Journ. Sci.*, civ, 425).

238. Measure the relative intensities of sounds, as proposed by Mayer (*Amer. Journ. Sci.*, cv, 44, 123).

239. Compare the methods of sonorous analysis proposed by Mayer (*Amer. Journ. Sci.*, viii, 170).

240. Measure the relative brightness of two lights by the Rumford photometer.

241. Devise a form of photometer for measuring the amount of light reflected at various angles by polished surfaces, in which a single light only shall be used, as in Expt. 67.

242. Determine the law of concave mirrors (see Expt. 78), and measure the size of the images as well as their position.

243. Determine the law of the enlargement of the images and positions of the conjugate foci of a combination of two lenses not in contact. (See Expt. 78.)

244. Measure the change in focus of a lens placed obliquely both for horizontal and vertical beams (*Proc. Amer. Acad.*, x, 300), and apply the same method to the case of mirrors.

245. Determine the distortion produced by the lens of a photographic camera by taking a picture of a scale of equal parts, and measuring by Expt. 21 the position of the divisions. The distortion may then be shown by a residual curve. Instead of a scale the equidistant vertical posts of a distant iron fence may be used, taking care to turn the plate exactly parallel to the fence.

246. Arrange a spectroscope so that the relative brightness of different portions of two spectra may be compared by the method of Vierordt, (*Amer. Journ. Sci.*, cii, 139), or by that of Trannin (*Comptes Rendus*, lxxvii, 1495). Determine the relative distribution of the light of the sky, of a white cloud, of a platinum wire heated to incandesence, of various flames, of various absorption spectra, and of the different lines of an incandescent metal or gas.

247. Measure the relative actinic effects of different parts of various spectra (See Expt. 246) by the method of Bunsen and Roscoe (*Phil. Trans.* 1863, 139).

248. Form Newton's rings between a lens and plane surface with monochromatic light, and measure the diameter of the rings accurately by the

Dividing Engine, Expt. 21. From this deduce the curvature of the surface and its distance from the lens under various pressures.

249. Measure the dispersion of thin plates of various materials by forming Talbot's bands with them in a spectroscope. The number of bands between any two solar lines of known wave-length serves to determine the difference in index of refraction. By turning the plate a known angle a second equation of condition may be formed. By polarizing the light the ordinary and extraordinary indices of doubly refracting media may be measured. The same method may be applied to liquids by using thin tanks partly filled with the substance to be examined.

250. Repeat the experiments with the interferential refractor, and apply this instrument to testing Mariotte's law and measuring the ratio of the two specific heats of gases.

251. Set a strip of thick glass on edge, support it at the ends and load it in the middle. Determine the strain of the different portions by their effect on polarized light.

252. Repeat Maxwell's experiments on the three primitive colors and their combinations (*Roy. Edin. Trans.*, xxi, 275; *Phil. Trans.*, 1860, 57).

253. Measure the variations in resistance of crystallized selenium when exposed to light (*Phil. Mag.*, xlvii, 216, xlviii, 161, l, 416). Test the results by the law that the intensity of light is inversely as the square of the distance, and apply this method of measurement to the spectra of Expt. 246.

254. Measure the resistance of liquids by the electrodynamometer as used by Kohlrausch and Nippoldt (*Pogg. Ann.*, cxxxviii, 280, 370). The polarization is eliminated since the currents are rapidly reversed.

255. Determine the forms of equipotential curves and surfaces by the method of Adams (*Proc. Roy. Soc.*, xxiv, 64; *Phil. Mag.*, l, 548).

256. Prove the laws of the attraction of currents and of solenoids by the horizontal pendulum.

257. When a circuit is closed through a very long wire, the current does not instantly arrive in its full intensity at the further end. Determine the form of the arrival wave. Study in the same way the form of induced currents.

258. Determine the magnitude of the ohm by the method of Weber (*Pogg. Ann.*, lxxxii, 337; *Rep. Brit. Assoc.*, 1863, 163 and 1864, 345; *Jenkin's Elect. Stand.*, 96).

259. Determine the ratio of the electrostatic, to the electrodynamic unit (*Rep. Brit. Assoc.*, 1869, 434; *Jenkin's Elect. Stand.*, 186).

260. Repeat the experiments of Lippman on the effects of electricity on capillarity (*Pogg. Ann.*, cxlix, 546).

261. Study the laws of electro-torsion by the method of Gore (*Proc. Roy. Soc.*, xxii, 57; *Amer. Journ. Sci.*, cvii, 418.)

262. Determine the specific inductive capacity of various di-electrics, by the method of Gibson and Barclay (*Phil. Trans.*, 1871, 573).

263. Study the laws of the electricity generated by belts. (See Joulin, *Ann. Chim. Phys.*, III, ii, 5.)

264. Repeat the experiments of Angot on the distribution of statical electricity.

265. Measure the electromotive force required to produce sparks of various lengths (*Proc. Roy. Soc.*, x, 326; *Reprint of Papers on Elect.*, *Thomson*, 247).

266. Measure the change in length and in volume of an iron bar when magnetized (*Phil. Mag.*, xxx, 76, 225, xlv, 350).

267. Measure the comparative efficiency of different cores for electro-magnets by the method of Mayer (*Amer. Journ. Sci.*, l, 195).

268. Measure the distribution of magnetism in soft iron by the method of Jamin (*Comptes Rendus* lxxviii, 95 et seq.).

269. Measure the distribution of magnetism in soft iron by the method of Rowland (*Amer. Journ. Sci.*, cx, 334).

270. Repeat the experiments of Bouty on the magnetic moments of minute magnets (*Phil. Mag.*, xlix, 81, 186).

271. Measure the strength of the various parts of the field of an electro-magnet by the magnetic proof plane of Rowland (*Amer. Journ. Sci.*, cx, 14).

272. Determine the coefficient of magnetism of various substances as proposed by Rowland (*Amer. Journ. Sci.*, cix, 358).

273. Determine the absolute conductibility of metals for heat by the method of Peclet (*Ann. Chim. Phys.*, III, ii, 107).

274. Measure the temperature of maximum density of water by the method of Joule (*Phil. Mag.*, xxx, 41).

275. Measure the velocity of evaporation of volatile liquids by the method of Stefan (*Phil. Mag.* xlvi, 483).

276. Repeat Expt. 246, using a thermopile instead of the photometer, and compare the relative amounts of heat of the various spectra with their relative amounts of light (Expt. 131).

277. Repeat Crooke's experiments with the radiometer (*Quart. Journ. Sci.*, xlii, 274, xlvii, 348· *Phil. Mag.*, xlviii, 65, 81, l, 177, 245) and apply this instrument to the accurate measure of radiant energy in Expt. 246.

278. Measure the specific heat of various substances by the mercury calorimeter of Favre and Silbermann (*Ann. Chim. Phys.*, III, xxxvi, 33).

279. Measure the specific heat of various substances by Bunsen's ice calorimeter (*Pogg. Ann.*, cxl, 1).

280. Measure the flow of coal gas through apertures of various forms and sizes and under different pressures, first when burning, and then when extinguished.

281. Determine the mechanical equivalent of heat by the methods of Joule and Hirn.

282. Determine the dynamical equivalent of heat from the thermal effects of electric currents by the method of Joule (*Brit. Assoc. Rep.*, 1867, 512; *Jenkin's Elect. Standards*, p. 175).

283. Measure the thermal equivalent of magnetism from the heating of the core of an electro magnet by the method of Cazin (*Ann. Chim. Phys.*, III, vi, 493).

284. Determine the relative efficiency or ratio of energy consumed to energy generated in a waterwheel, turbine or water-pressure engine. Measure the height of fall or pressure, the water employed, by Expt. 48, the number of turns per minute by Expt. 158, and the power generated, by Expts. 153, 154 and 155.

285. Determine the relative efficiency of a steam, gas or hot-air engine by Expts. 148, 153, 154, 155 and 158.

286. Determine the relative efficiency of an electro-magnetic engine, measuring the resistance by Expt. 102 and the current employed by Expt. 98. If the circuit is closed during a portion only of the revolution, a suitable allowance must be made. We can, by varying the number of cells of the battery, determine how many would be required to produce any given effect. Finally measure the power by Expts. 154, 155 and 158. Measure also by a spring balance the dead pull on the fly wheel in different portions of its revolution and by integration deduce the total value. See how this compares with that previously determined.

287. Determine the efficiency of a thermal battery, measuring the consumption of gas by a meter, and the quantity of electricity by Expt. 98. The experiment may be divided into three parts, while heating, after the temperature has become constant, and while cooling after the gas is extinguished. See how far the result will vary with changes in the rate of consumption of gas and in the outside resistance.

288. Determine the efficiency of a Planté battery (*Les Mondes*, 1873,) measuring the current which passes into it from the charging battery (Expt. 98), and again that which returns. Measure also the variations in electromotive force and resistance (Expt. 105).

289. Determine the efficiency of a magneto-electric machine. (*Proc. Amer. Acad.*, x, 432).

290. Determine the efficiency of a Holtz machine (*Ann. Chim. Phys.*, III, ii, 5). Measure by a transmission dynamometer, Expt. 155, the power required to drive the machine, the speed by Expt. 158, and the current by Expt. 98, using the secondary coil of an induction coil with a mirror and magnet hung inside, as a galvanometer.

291. Measure the height of clouds and the velocity of the winds moving them, compared with that on the earth's surface as described in the *Proc. Amer. Acad.*, xi.

292. Measure the density of fog, or the amount of light absorbed by layers of various thicknesses.

293. Suspend a Foucault's pendulum and measure the angular deviation per minute. See how much it differs from its theoretical value of $15' \sin L$, in which L is the latitude.

294. Determine the density of the earth by the method of Cavendish.

295. Repeat the experiments of Zöllner with the horizontal pendulum (*Pogg. Ann.*, xl, 134, 140).

296. Determine the limit of resolvability of close double stars by the following apparatus. A miniature telescope is formed with a microscope objective and a position and spider-line micrometer. By the side of this are placed two artificial stars formed of needle holes in sheet metal strongly illuminated by a light behind, and whose position and distance may be varied at will and measured by a micrometer screw and graduated circle. A vertical mirror of plane glass is placed opposite so as to reflect the image of the stars into the telescope. Determine the probable error of the position angle and distance of the stars when variously set.

297. Measure the light of the sky at different distances from the sun. This may be done by the photometer of Expt. 68, or better, by illuminating the two halves of the field of view of an eyepiece by allowing the light

of the sun, reduced by a lens, to fall on one, and then reflecting into the other half. the light of the sky. The position of the lens may then be varied until equality is obtained.

298. Compare the light of the sun with that of a candle by the photometer of Expt. 68. The light of the sun is easily reduced sufficiently by passing it through a short focus lens.

299. Measure the brightness of different portions of the sun's disk, as described in the *Proc. Amer. Acad.*, x, 428.

300. Determine the relative brightness of various portions of the larger nebulæ by attaching a Rood's photometer (*Amer. Journ. Sci.*, xlix.) to the eyepiece of a telescope and allowing the light to pass through a small hole in a diaphragm filling the field of view. A large telescope is needed for this experiment, or if this is not available, a large cosmorama lens of long focus may be used for an objective. Similar observations may also be made to excellent advantage on any bright comet.

INDEX.